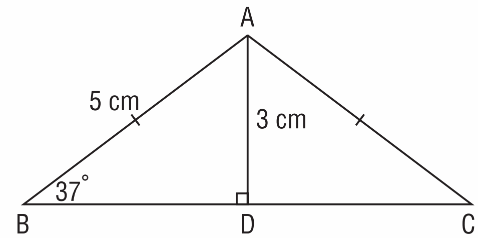
Start Where You Are

What Should I Recall?

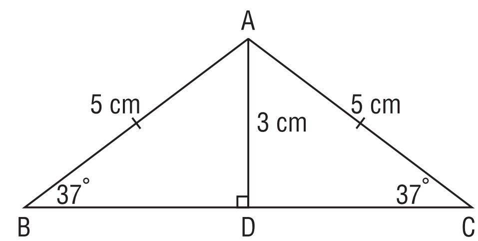
Suppose I have to solve this problem:
Determine the unknown measures of the angles and sides in ΔABC.
The given measures are rounded to the nearest whole number.

I think of what I already know about triangles.
I see that AB and AC have the same hatch marks; this means the sides are equal.
AC = AB
So, AC = 5 cm

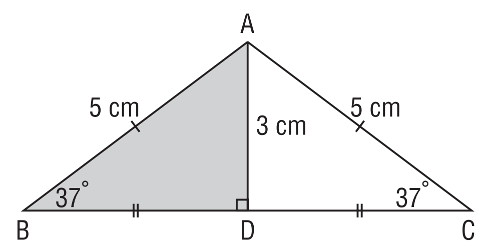


I know that a triangle with 2 equal sides is an isosceles triangle.
So, ΔABC is isosceles.
An isosceles triangle has 2 equal angles that are formed where the equal sides intersect the third side.
I use 3 letters to describe an angle.

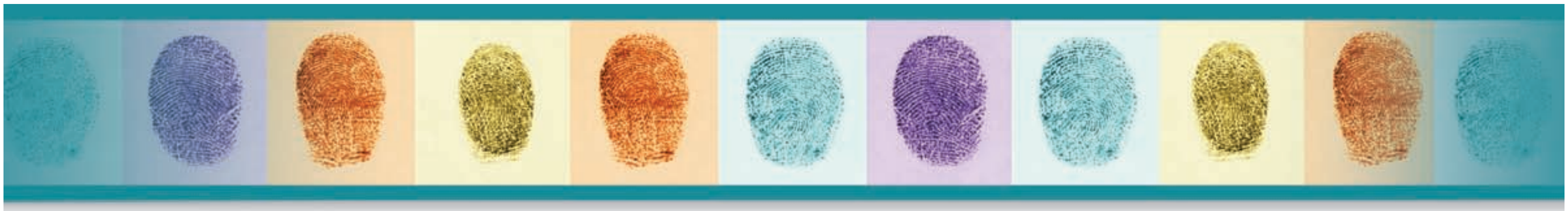
$$\begin{aligned} \text{So, } \angle ACD &= \angle ABD \\ &= 37° \end{aligned}$$



Since ΔABC is isosceles, the height AD is the perpendicular bisector of the base BC.
So, BD = DC and $\angle ADB = 90°$
I can use the Pythagorean Theorem in ΔABD to calculate the length of BD.



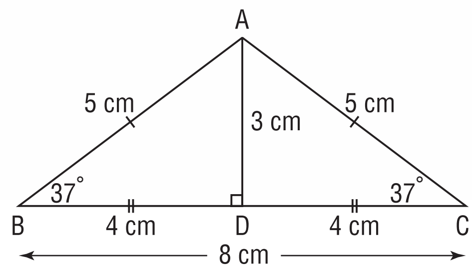
$$\begin{aligned} AD^2 + BD^2 &= AB^2 \\ 3^2 + BD^2 &= 5^2 \\ 9 + BD^2 &= 25 \\ 9 - 9 + BD^2 &= 25 - 9 \\ BD^2 &= 16 \\ BD &= \sqrt{16} \\ BD &= 4 \end{aligned}$$



BD = 4 cm
So, BC = 2 × 4 cm
= 8 cm

I know that the sum of the angles in a triangle is 180°.
So, I can calculate the measure of ∠BAC.

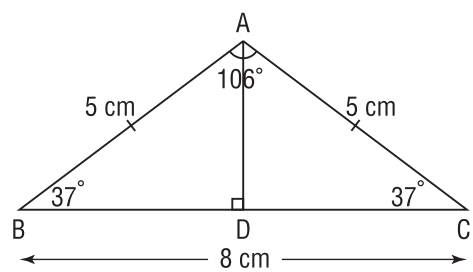
$\angle BAC + \angle ACD + \angle ABD = 180°$
$\angle BAC + 37° + 37° = 180°$
$\angle BAC + 74° = 180°$
$\angle BAC + 74° - 74° = 180° - 74°$
$\angle BAC = 106°$



My friend Janelle showed me a different way to calculate.
She recalled that the line AD is a line of symmetry for an isosceles triangle.
So, ΔABD is congruent to ΔACD.
This means that ∠BAD = ∠CAD
Janelle calculated the measure of ∠BAD in ΔABD.

$\angle BAD + 37° + 90° = 180°$
$\angle BAD + 127° = 180°$
$\angle BAD + 127° - 127° = 180° - 127°$
$\angle BAD = 53°$

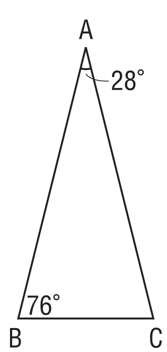
Then, $\angle BAC = 2 \times 53°$
$= 106°$



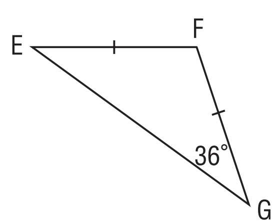
Check

1. Calculate the measure of each angle.

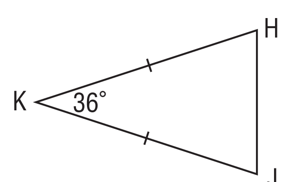
a) ∠ACB



b) ∠GEF and ∠GFE



c) ∠HJK and ∠KHJ

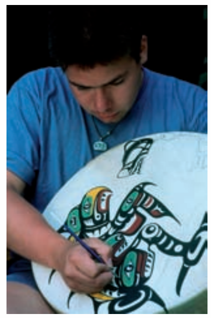


7.1 Scale Diagrams and Enlargements

FOCUS

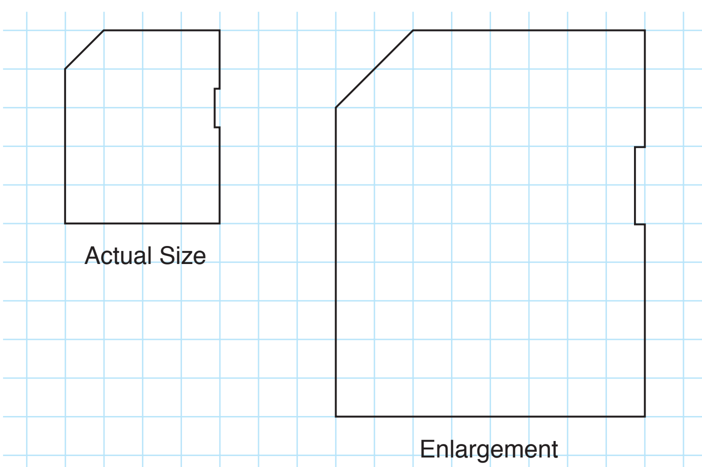
- Draw and interpret scale diagrams that represent enlargements.

How are these photos alike?
How are they different?



Investigate

You will need 0.5-cm grid paper.
Here is an actual size drawing of a memory card for a digital camera and an enlargement of the drawing.



➤ Copy the drawings on grid paper.
Measure the lengths of pairs of matching sides on the drawings.
Label each drawing with these measurements.

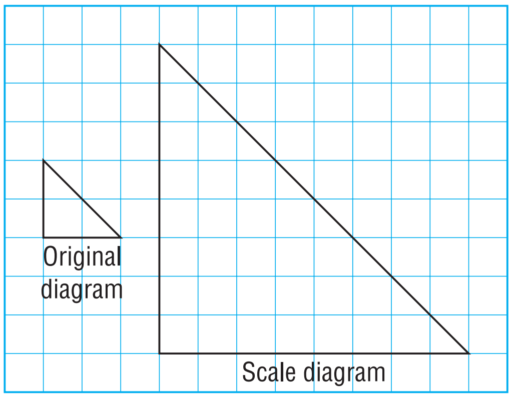
➤ For each measurement, write the fraction: $\frac{\text{Length on enlargement}}{\text{Length on actual size drawing}}$
Write each fraction as a decimal.
What do you notice about these numbers?

Practice

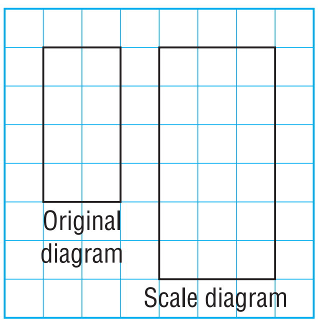
Check

4. Determine the scale factor for each scale diagram.

a)



b)



5. Scale diagrams of different squares are to be drawn. The side length of each original square and the scale factor are given. Determine the side length of each scale diagram.

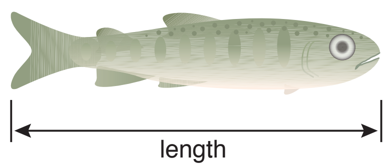
	Side length of original square	Scale factor
a)	12 cm	3
b)	82 mm	$\frac{5}{2}$
c)	1.55 cm	4.2
d)	45 mm	3.8
e)	0.8 cm	12.5

Apply

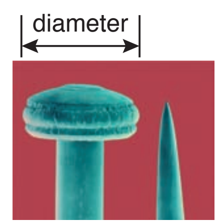
6. A photo of a surfboard has dimensions 17.5 cm by 12.5 cm. Enlargements are to be made with each scale factor below. Determine the dimensions of each enlargement. Round the answers to the nearest centimetre.

a) scale factor 12 **b)** scale factor 20
c) scale factor $\frac{7}{2}$ **d)** scale factor $\frac{17}{4}$

7. Here is a scale diagram of a salmon fry. The actual length of the salmon fry is 30 mm. Measure the length on the diagram to the nearest millimetre. Determine the scale factor for the scale diagram.



8. The head of a pin has diameter 2 mm. Determine the scale factor of this photo of the pinhead.



9. This view of the head of a bolt has the shape of a regular hexagon. Each angle is 120°. Use a protractor and ruler to draw a scale diagram of the bolt with scale factor 2.5.

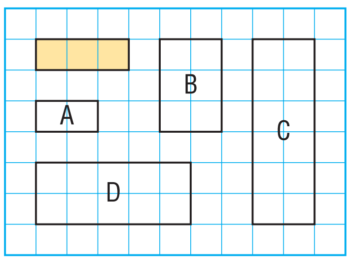
10. Draw your initials on 0.5-cm grid paper. Use different-sized grid paper to draw two different scale diagrams of your initials. For each scale diagram, state the scale factor.

11. Assessment Focus For each set of diagrams below, identify which of diagrams A, B, C, and D are scale diagrams of the shaded shape. For each scale diagram you identify:

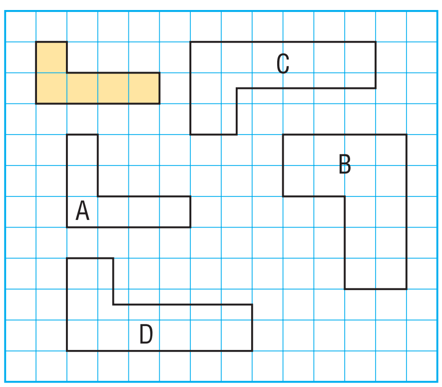
i) State the scale factor.

ii) Explain how it is a scale diagram.

a)



b)



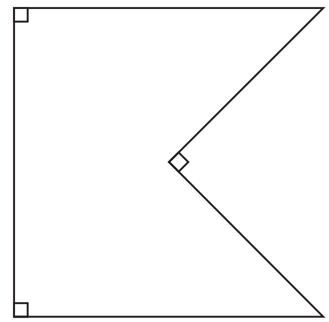
12. One frame of a film in a projector is about 50 mm high. The film is projected onto a giant screen. The image of the film frame is 16 m high.

a) What is the scale factor of this enlargement?

b) A penguin is 35 mm high on the film. How high is the penguin on the screen?

13. Look in a newspaper, magazine, or on the Internet. Find an example of a scale diagram that is an enlargement and has its scale factor given. What does the scale factor indicate about the original diagram or object?

14. Draw a scale diagram of the shape below with scale factor 2.5.



15. On a grid, draw ΔOAB with vertices O(0, 0), A(0, 3), and B(4, 0).

a) Draw a scale diagram of ΔOAB with scale factor 3 and one vertex at C(3, 3). Write the coordinates of the new vertices.

b) Is there more than one answer for part a? If your answer is no, explain why no other diagrams are possible. If your answer is yes, draw other possible scale diagrams.

Take It Further

16. One micron is one-millionth of a metre, or 1 m $= 10^6$ microns.

a) A human hair is about 200 microns wide. How wide is a scale drawing of a human hair with scale factor 400? Give your answer in as many different units as you can.

b) A computer chip is about 4 microns wide. A scale diagram of a computer chip is 5 cm wide. What is the scale factor?

Reflect

Suppose you are given a scale diagram. Why is it important to know the scale factor?

Practice

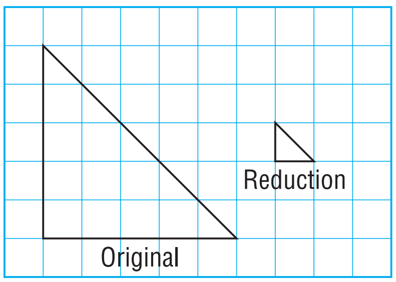
Check

4. Write each fraction in simplest form, then express it as a decimal.

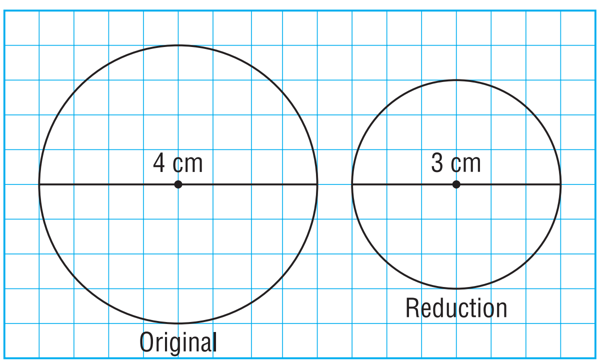
a) $\frac{25}{1000}$ b) $\frac{5}{125}$ c) $\frac{2}{1000}$ d) $\frac{3}{180}$

5. Determine the scale factor for each reduction as a fraction or a decimal.

a)



b)

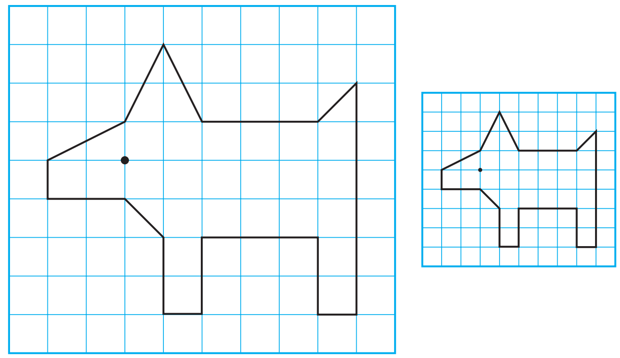


6. For each pair of circles, the original diameter and the diameter of the reduction are given. Determine each scale factor as a fraction or a decimal.

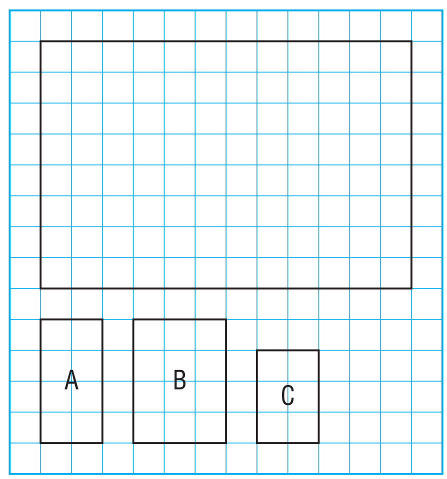
	Diameter of Actual Circle	Diameter of Reduction
a)	50 cm	30 cm
b)	30 cm	20 cm
c)	126 cm	34 cm
d)	5 m	2 cm
e)	4 km	300 m

Apply

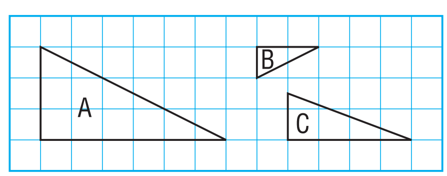
7. Here are two drawings of a dog. Determine the scale factor of the reduction as a fraction and as a decimal.



8. Which of rectangles A, B, and C is a reduction of the large rectangle? Justify your answer.



9. Which two polygons have pairs of corresponding lengths that are proportional? Identify the scale factor for the reduction.



➤ Copy the table below. Use your results from the first 2 tables to complete this table.
Write the ratios of the lengths of the sides as decimals to the nearest hundredth.

$\frac{AB}{A'B'}$	$\frac{BC}{B'C'}$	$\frac{CD}{C'D'}$	$\frac{DA}{D'A'}$	$\frac{AB}{A''B''}$	$\frac{BC}{B''C''}$	$\frac{CD}{C''D''}$	$\frac{DA}{D''A''}$

➤ What do you notice about the measures of the matching angles?
What do you notice about the ratios of matching sides?

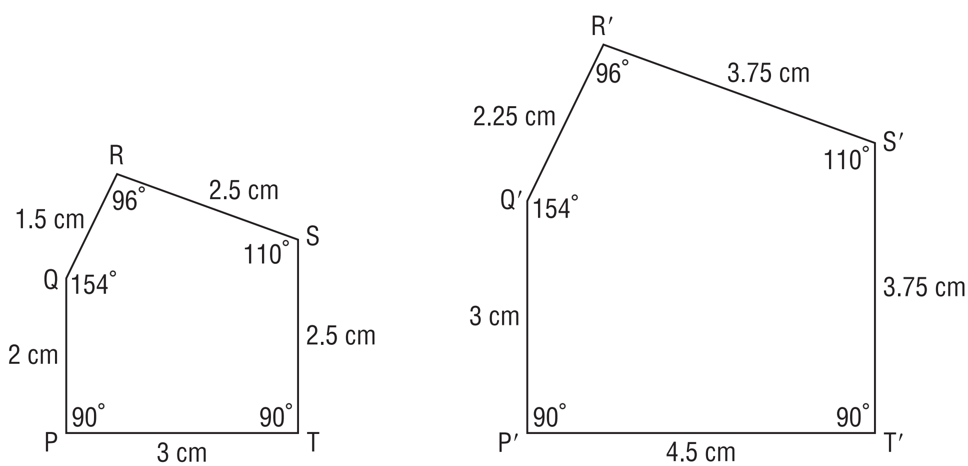
Reflect & Share

Compare your results with those of another pair of students.
Work together to draw two other quadrilaterals that have sides and angles related the same way as your quadrilaterals.
How does this work relate to scale drawings that show enlargements and reductions?

Connect

When one polygon is an enlargement or a reduction of another polygon, we say the polygons are **similar**. Similar polygons have the same shape, but not necessarily the same size.

Here are two similar pentagons.



Matching angles are **corresponding angles**.
Matching sides are **corresponding sides**.
We list the corresponding angles and the pairs of corresponding sides.

b) Draw a reduction. Choose a scale factor that is less than 1, such as $\frac{1}{2}$.
Let the similar pentagon be A′B′C′D′E′.
Multiply each side length of ABCDE by $\frac{1}{2}$ to get the corresponding side lengths of A′B′C′D′E′.

$$A'B' = \frac{1}{2} \times AB \qquad B'C' = \frac{1}{2} \times BC \qquad E'A' = \frac{1}{2} \times EA$$
$$= \frac{1}{2} \times 2.0 \text{ cm} \qquad = \frac{1}{2} \times 2.8 \text{ cm} \qquad = \frac{1}{2} \times 4.0 \text{ cm}$$
$$= 1.0 \text{ cm} \qquad = 1.4 \text{ cm} \qquad = 2.0 \text{ cm}$$

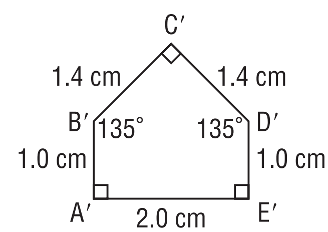
Since DE = AB, then D′E′ = A′B′ = 1.0 cm

Since CD = BC, then C′D′ = B′C′ = 1.4 cm

The corresponding angles are equal. So:

$$\angle A' = \angle A = 90° \qquad \angle B' = \angle B = 135° \qquad \angle C' = \angle C = 90° \qquad \angle D' = \angle D = 135° \qquad \angle E' = \angle E = 90°$$

Use a ruler and protractor to draw pentagon A′B′C′D′E′.
The pentagons are similar because corresponding angles are equal and corresponding sides are proportional.
That is, the length of each side of the reduction is $\frac{1}{2}$ the length of the corresponding side of the original pentagon.

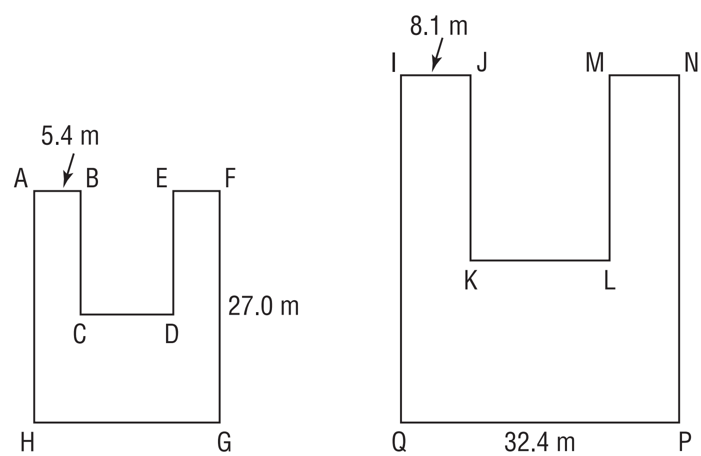


Example 3 Solving Problems Using the Properties of Similar Polygons

These two octagonal garden plots are similar.

a) Calculate the length of GH.

b) Calculate the length of NP.



A Solution

a) To calculate GH, consider polygon ABCDEFGH as a reduction of polygon IJKLMNPQ.

Practice

Check

4. Calculate the side length, in units, in each proportion.

a) $\frac{AB}{8} = \frac{3}{2}$ b) $\frac{BC}{25} = \frac{12}{15}$

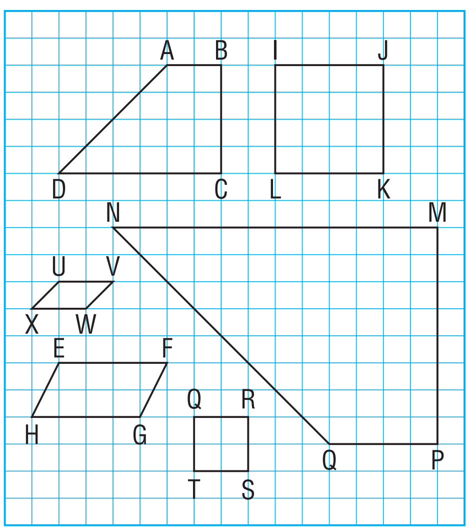
c) $\frac{CD}{4} = \frac{63}{28}$ d) $\frac{DE}{7} = \frac{24}{30}$

5. Calculate the value of the variable in each proportion.

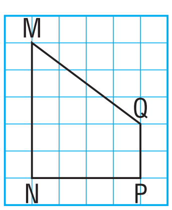
a) $\frac{x}{2.5} = \frac{7.5}{1.5}$ b) $\frac{y}{21.4} = \frac{23.7}{15.8}$

c) $\frac{z}{12.5} = \frac{0.8}{1.2}$ d) $\frac{a}{0.7} = \frac{1.8}{24}$

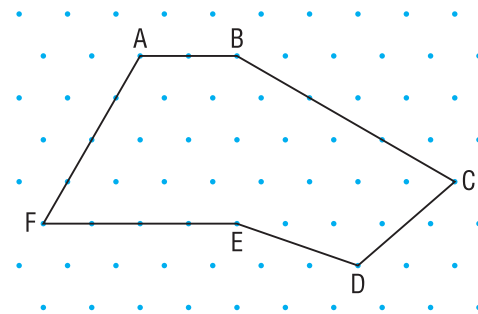
6. Identify similar quadrilaterals. List their corresponding sides and corresponding angles.



7. Use grid paper. Construct a quadrilateral similar to quadrilateral MNPQ.

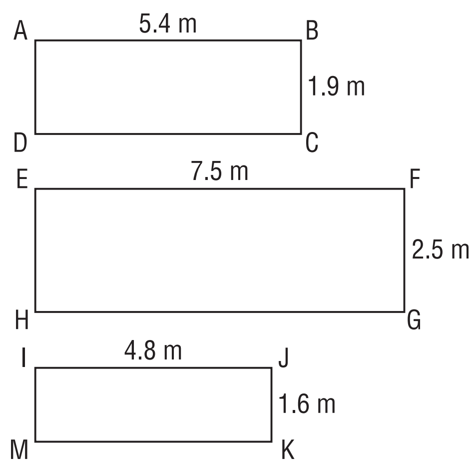


8. Use isometric dot paper. Construct a hexagon similar to hexagon ABCDEF.



Apply

9. Are any of these rectangles similar? Justify your answer.



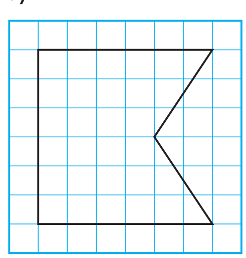
10. For each polygon below:

i) Draw a similar larger polygon.

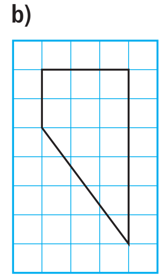
ii) Draw a similar smaller polygon.

Explain how you know the polygons are similar.

a)



b)



Connect

When two polygons are similar:

- the measures of corresponding angles must be equal *and*
- the ratios of the lengths of corresponding sides must be equal.

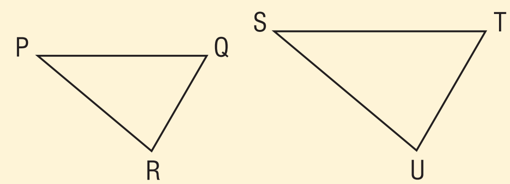
A triangle is a special polygon. When we check whether two triangles are similar:

- the measures of corresponding angles must be equal; *or*
- the ratios of the lengths of corresponding sides must be equal

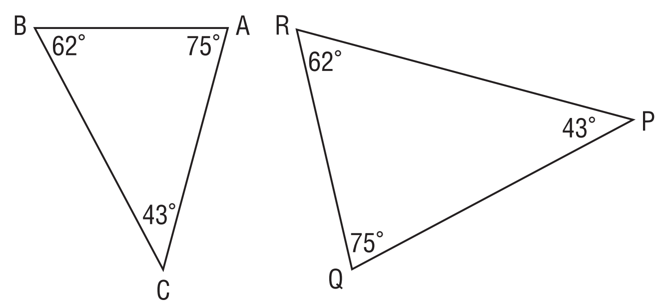
Properties of Similar Triangles

To identify that ΔPQR and ΔSTU are similar, we only need to know that:

- $\angle P = \angle S$ and $\angle Q = \angle T$ and $\angle R = \angle U$; *or*
- $\frac{PQ}{ST} = \frac{QR}{TU} = \frac{PR}{SU}$



These triangles are similar because:
$\angle A = \angle Q = 75°$
$\angle B = \angle R = 62°$
$\angle C = \angle P = 43°$
When we name two similar triangles, we order the letters to match the corresponding angles.
We write: $\Delta ABC \sim \Delta QRP$
Then we can identify corresponding sides:
AB corresponds to QR.
BC corresponds to RP.
AC corresponds to QP.



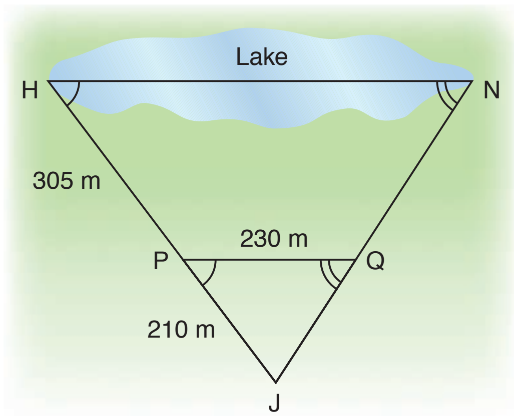
Math Link

Your World

Satellite imagery consists of photographs of Earth taken from space. The images are reductions of regions on Earth. The quality of an image depends upon the instrument used to obtain it and on the altitude of the satellite. The Landsat 7 satellite can create images of objects as small as 10 cm long.

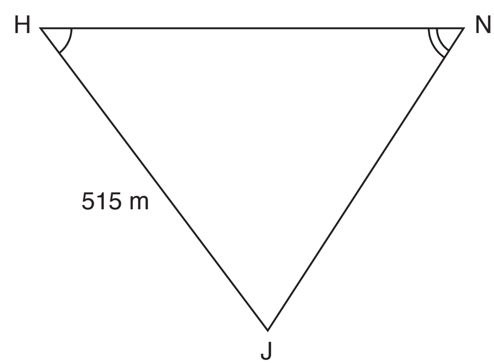
Example 3 Using Overlapping Similar Triangles to Determine a Length

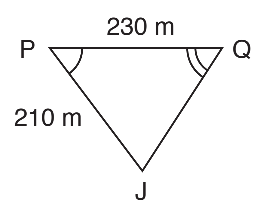
A surveyor wants to determine the width of a lake at two points on opposite sides of the lake. She measures distances and angles on land, then sketches this diagram. How can the surveyor determine the length HN to the nearest metre?



A Solution

Identify the two triangles, then draw them separately.





Consider ΔHNJ and ΔPQJ. From the diagram:

$\angle NHJ = \angle QPJ$

$\angle HNJ = \angle PQJ$

$\angle J$ is the common angle to both triangles.

Since 3 pairs of corresponding angles are equal,

$\Delta HNJ \sim \Delta PQJ$

Two corresponding sides are:

$HJ = 305 \text{ m} + 210 \text{ m}$ and $PJ = 210 \text{ m}$

$= 515 \text{ m}$

So, ΔHNJ is an enlargement of ΔPQJ with a scale factor of $\frac{515}{210}$.

Write a proportion that includes the unknown length HN.

$$\frac{HN}{PQ} = \frac{515}{210} \qquad \text{Substitute } PQ = 230.$$

$$\frac{HN}{230} = \frac{515}{210} \qquad \text{To solve for HN, multiply each side by 230.}$$

$$230 \times \frac{HN}{230} = \frac{515}{210} \times 230$$

$$HN = \frac{515 \times 230}{210}$$

$$\doteq 564.0476$$

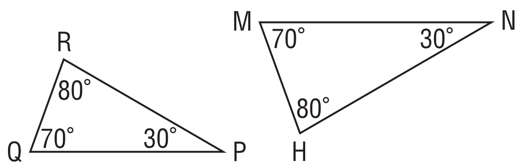
The width of the lake, HN, is about 564 m.

Practice

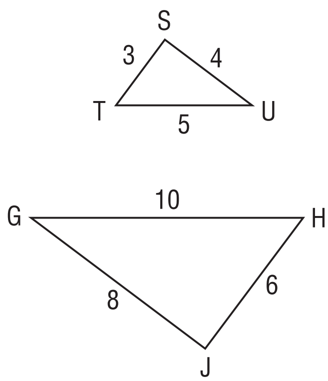
Check

4. Which triangles in each pair are similar? How do you know?

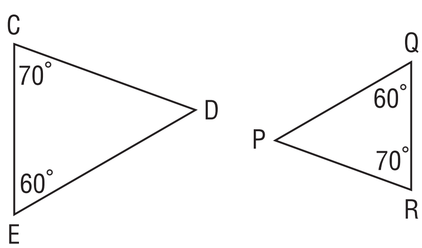
a)



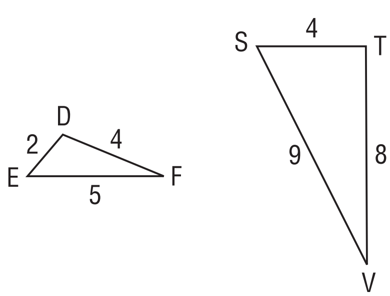
b)



c)

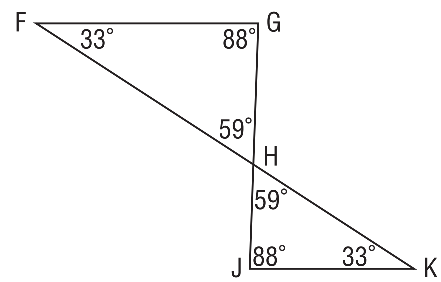


d)

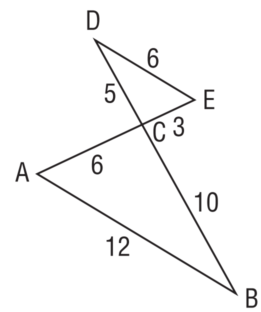


5. In each diagram, identify two similar triangles. Explain why they are similar.

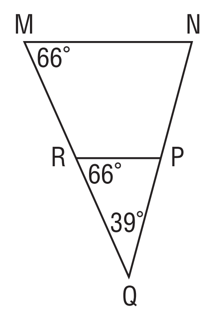
a)



b)



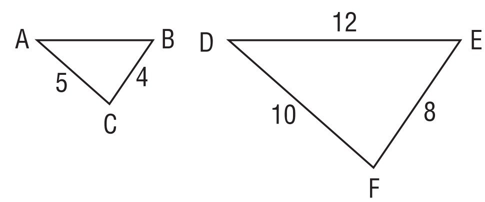
c)



Apply

6. Determine the length of AB in each pair of similar triangles.

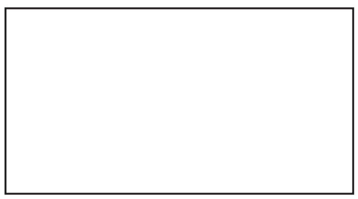
a)



Mid-Unit Review

7.1 **1.** A photo of a gymnast is to be enlarged. The dimensions of the photo are 15 cm by 10 cm. What are the dimensions of the enlargement with a scale factor of $\frac{7}{5}$?

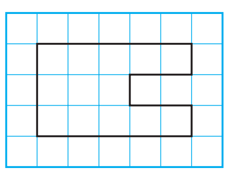
2. A computer chip has dimensions 15 mm by 8 mm. Here is a scale drawing of the chip.



a) Determine the scale factor of the diagram.

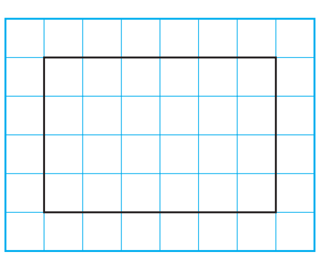
b) Draw a scale diagram of the chip with a scale factor of 8.

7.2 **3. a)** Copy this polygon on 1-cm grid paper.

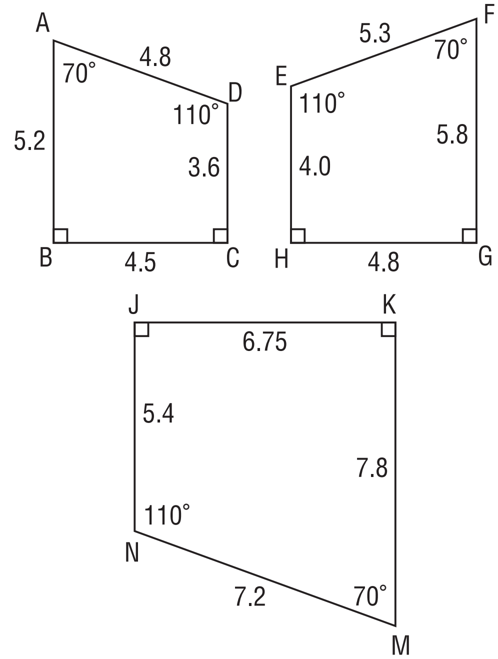


b) Draw a scale diagram of the polygon with a scale factor of $\frac{3}{5}$. Show any calculations you made.

4. This top view of a swimming pool is drawn on 0.5-cm grid paper. The dimensions of the pool are 60 m by 40 m. Determine the scale factor of the reduction as a fraction or a decimal.



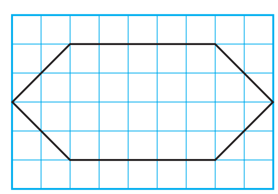
7.3 **5.** These quadrilaterals have corresponding angles equal.



a) Are any of these quadrilaterals similar? Justify your answer.

b) Choose one quadrilateral. Draw a similar quadrilateral. How do you know the quadrilaterals are similar?

6. A window has the shape of a hexagon.



Draw a hexagon that is similar to this hexagon. Explain how you know the hexagons are similar.

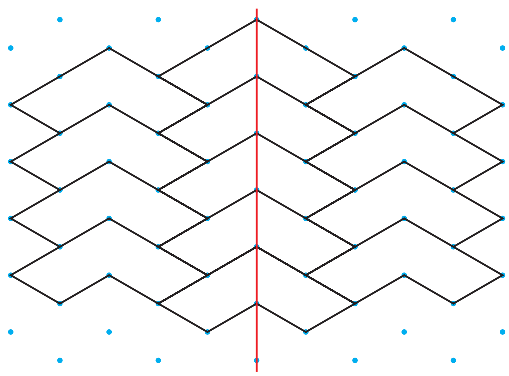
7.4 **7.** A tree casts a shadow 8 m long. At the same time a 2-m wall casts a shadow 1.6 m long.

a) Sketch a diagram.

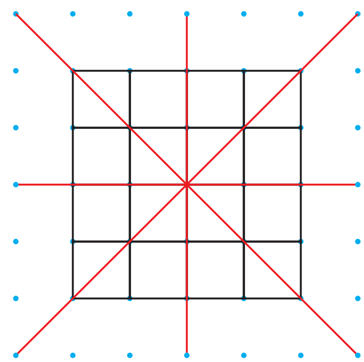
b) What is the height of the tree?

A Solution

a) The red line is the line of symmetry for this tessellation. Each point on one side of the line has a corresponding point on the other side. The pattern on one side of the line of symmetry is a mirror image of the pattern on the other side.



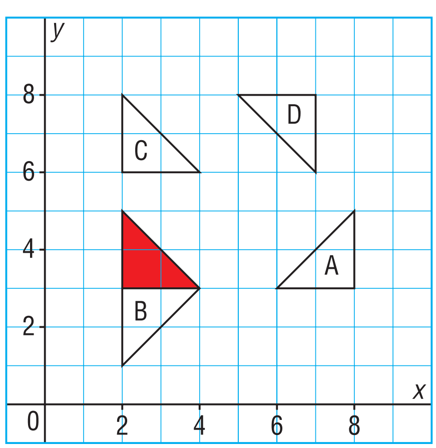
b) This tessellation has 4 lines of symmetry. For each line, a point on one side of the line has a matching point on the other side. And, the pattern on one side of the line is a mirror image of the pattern on the other side.



Two shapes may be related by a line of reflection.

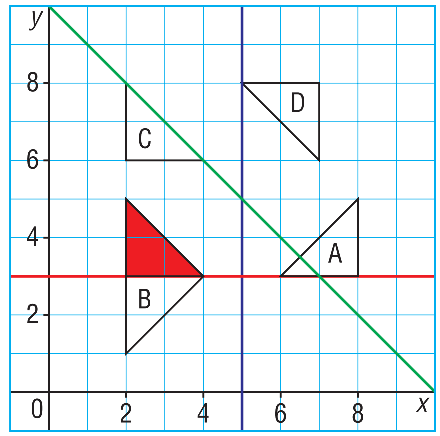
Example 2 Identifying Shapes Related by a Line of Reflection

Identify the triangles that are related to the red triangle by a line of reflection. Describe the position of each line of symmetry.



A Solution

Triangle A is the reflection image of the red triangle in the blue line through 5 on the *x*-axis.
Triangle B is the reflection image of the red triangle in the red line through 3 on the *y*-axis.
Triangle C is not a reflection image of the red triangle.
Triangle D is the reflection image of the red triangle in the green line through the points (9, 1) and (1, 9).



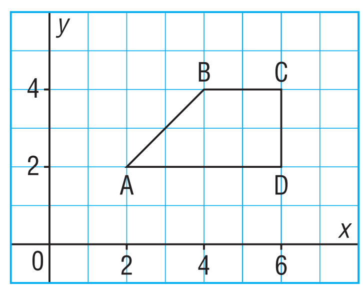
We can use a coordinate grid to draw shapes and their reflection images.

Example 3 Completing a Shape Given its Line of Symmetry

Quadrilateral ABCD is part of a larger shape.

- Draw the image of ABCD after each reflection below.
- Write the coordinates of the larger shape formed by ABCD and its image.
- Describe the larger shape and its symmetry.

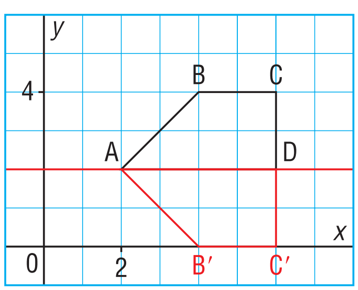
a) a reflection in the horizontal line through 2 on the *y*-axis
b) a reflection in the vertical line through 6 on the *x*-axis
c) a reflection in an oblique line through (0, 0) and (6, 6)



A Solution

The red line is the line of reflection. Each image point is the same distance from this line as the corresponding original point.

a)



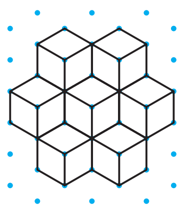
Point	Image
A(2, 2)	A(2, 2)
B(4, 4)	B′(4, 0)
C(6, 4)	C′(6, 0)
D(6, 2)	D(6, 2)

The larger shape ABCC′B′ has coordinates: A(2, 2), B(4, 4), C(6, 4), C′(6, 0), B′(4, 0)
This shape is a pentagon with line symmetry. The line of symmetry is the red line.

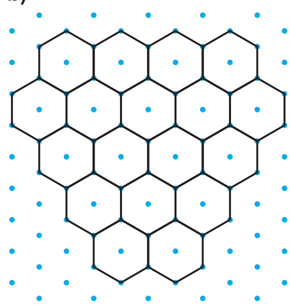
Apply

4. Identify the lines of symmetry in each tessellation.

a)

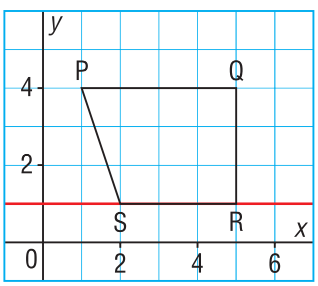


b)

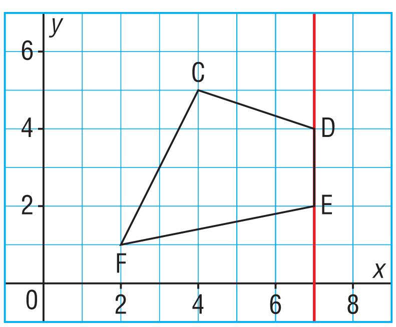


5. Copy each polygon on grid paper. It is one-half of a shape. Use the red line as a line of symmetry to complete the shape by drawing its other half. Label the shape with the coordinates of its vertices.

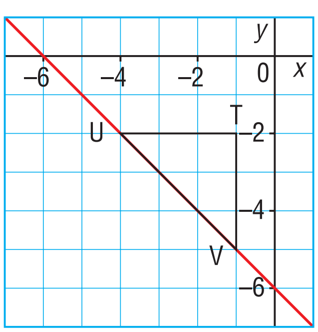
a)



b)



c)



6. State the number of lines of symmetry in each design.

a) a tessellation created by M.C. Escher

b) a Haida button blanket

7. **Assessment Focus**

a) Draw a triangle on a grid.

b) Choose one side of the triangle as a line of reflection.

  i) Draw the reflection image.

  ii) Label the vertices of the shape formed by the original triangle and its image.

  iii) Write the coordinates of each vertex.

  iv) How many lines of symmetry does the shape have?

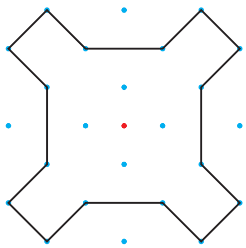
c) Repeat part b for each of the other two sides of the triangle. Do you always get the same shape? Explain.

d) Repeat parts a to c for different types of triangles.

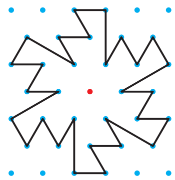
e) Which types of triangle always produce a shape that is a quadrilateral with line symmetry? Justify your answer.

8. Does each shape have rotational symmetry about the red dot? If it does, state the order and the angle of rotation symmetry.

a)

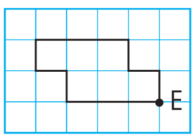


b)

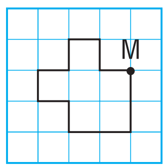


9. Copy each shape on grid paper. Draw the rotation image after each given rotation.

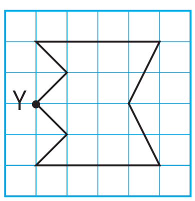
a) 90° clockwise about E



b) 180° about M

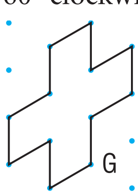


c) 270° counterclockwise about Y

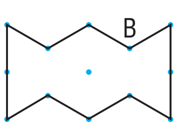


10. Copy each shape on isometric dot paper. Draw the rotation image after each given rotation.

a) 60° clockwise about G



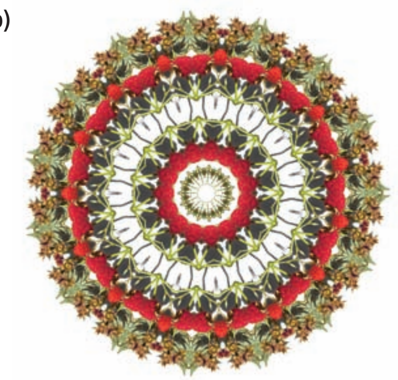
b) 120° counterclockwise about B



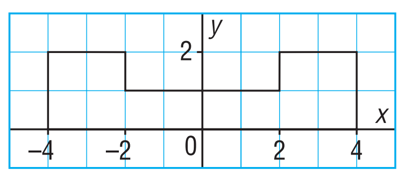
11. Identify and describe any rotational symmetry in each design.

a)

b)



12. This octagon is part of a larger shape that is to be completed by a rotation of 180° about the origin.

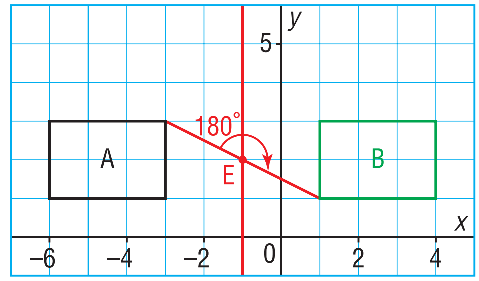


a) On a coordinate grid, draw the octagon and its image.

b) Outline the shape formed by the octagon and its image. Describe any rotational symmetry in this shape. Explain why you think the symmetry occurred.

Connect

On this grid, rectangle A has been rotated 180° about E(−1, 2) to produce its image, rectangle B.
We can extend our meaning of line symmetry to relate the two rectangles.
The line through −1 on the *x*-axis is a line of symmetry for the two rectangles.
Each point on rectangle A has a corresponding point on rectangle B.
These points are equidistant from the line of symmetry.



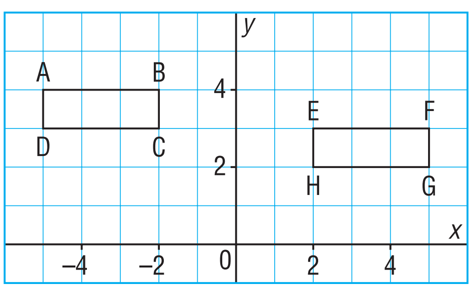
When a shape and its transformation image are drawn, the resulting diagram may show:

- no symmetry
- line symmetry
- rotational symmetry
- both line symmetry and rotational symmetry

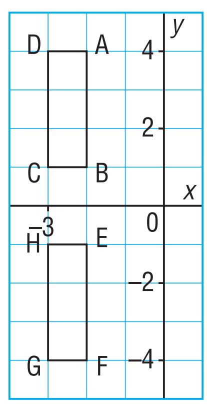
Example 1 Determining whether Shapes Are Related by Symmetry

For each pair of rectangles ABCD and EFGH, determine whether they are related by symmetry.

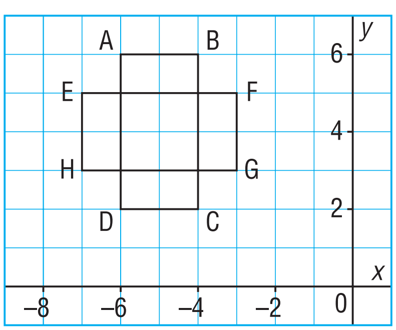
a)



b)

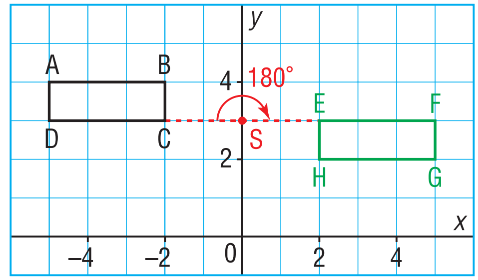


c)

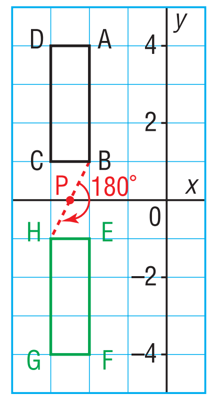


A Solution

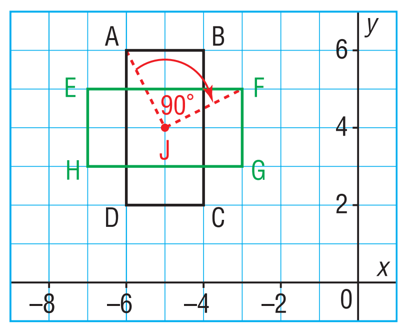
a) There is no line on which a mirror can be placed so that one rectangle is the reflection image of the other.
So, the rectangles are not related by line symmetry.
Trace the rectangles. Use guess and check to determine if a centre of rotation exists.
When ABCD is rotated 180° about the point S(0, 3), ABCD coincides with GHEF.
So, the rectangles are related by rotational symmetry of order 2 about S(0, 3).



b) Each point on ABCD has a corresponding point on EFGH.
These points are equidistant from the x-axis.
So, the two rectangles are related by line symmetry; the x-axis is the line of symmetry.
Trace the rectangles. Use guess and check to determine if a centre of rotation exists.
When a tracing of ABCD is rotated 180° about the point P(−2.5, 0), ABCD coincides with GHEF.
So, the two rectangles are related by rotational symmetry.



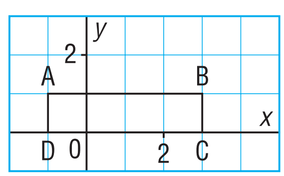
c) When ABCD is rotated 90° clockwise about point J(−5, 4), ABCD coincides with FGHE.
Then, the polygon formed by both rectangles together has rotational symmetry of order 4 about point J. So, the two rectangles are related by rotational symmetry.



Example 2 Identifying Symmetry in a Shape and Its Transformation Image

Draw the image of rectangle ABCD after each transformation.
Write the coordinates of each vertex and its image.
Identify and describe the type of symmetry that results.

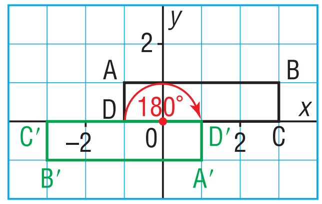
a) a rotation of 180° about the origin
b) a reflection in the x-axis
c) a translation 4 units right and 1 unit down



Solution

a) Use tracing paper to rotate ABCD 180° about the origin.

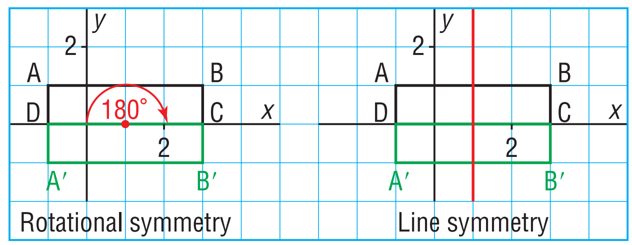
Point	Image
A(−1, 1)	A′(1, −1)
B(3, 1)	B′(−3, −1)
C(3, 0)	C′(−3, 0)
D(−1, 0)	D′(1, 0)



The octagon ABCD′A′B′C′D, formed by both rectangles together, has rotational symmetry of order 2 about the origin, and no line symmetry.

b) Reflect ABCD in the *x*-axis.

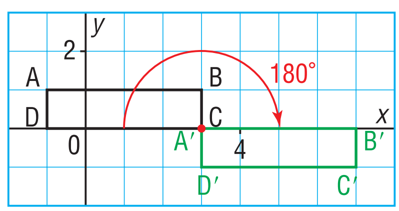
Point	Image
A(−1, 1)	A′(−1, −1)
B(3, 1)	B′(3, −1)
C(3, 0)	C(3, 0)
D(−1, 0)	D(−1, 0)



The rectangle ABB′A′, formed by both rectangles, has rotational symmetry of order 2 about the point (1, 0). It also has 2 lines of symmetry: the *x*-axis and the vertical line through 1 on the *x*-axis.

c) Translate ABCD 4 units right and 1 unit down.

Point	Image
A(−1, 1)	A′(3, 0)
B(3, 1)	B′(7, 0)
C(3, 0)	C′(7, −1)
D(−1, 0)	D′(3, −1)



The two rectangles do not form a shape; but they have a common vertex at C (or A′). The two rectangles are related by rotational symmetry of order 2 about the point C(3, 0). There is no line of symmetry relating the rectangles.

In *Example 2,* we could write the translation 4 units right and 1 unit down in a shorter form as R4, D1. In this shorter form, a translation of 7 units left and 2 units up would be written as L7, U2.

Discuss the ideas

1. How can you tell if two shapes are related by line symmetry?

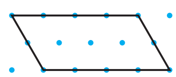
2. How can you tell if two shapes are related by rotational symmetry?

Practice

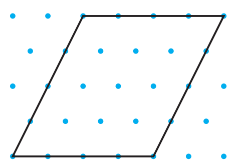
Check

3. Describe the rotational symmetry and line symmetry of each shape.

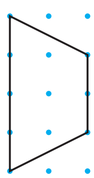
a) a parallelogram



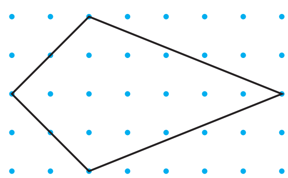
b) a rhombus



c) an isosceles trapezoid



d) a kite



4. Describe the rotational symmetry and line symmetry of each wheel cover. On a copy of the wheel covers, mark the centre of rotation and the line of reflection.

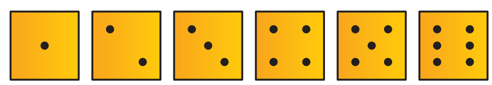
a)

b)

c)

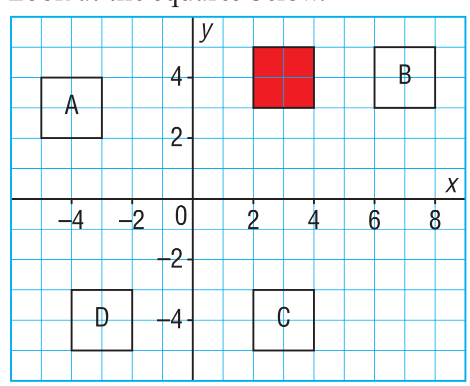
d)

5. Describe the symmetry of each face of a die. Copy each face. Mark the centre of rotation and the lines of symmetry.



Apply

6. Look at the squares below.



Which of squares A, B, C, and D are related to the red square:

a) by rotational symmetry about the origin?

b) by line symmetry?

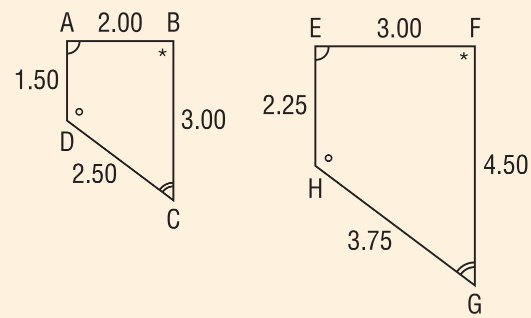
Study Guide

Scale Diagrams

For an enlargement or reduction, the scale factor is: $\frac{\text{Length on scale diagram}}{\text{Length on original diagram}}$

An enlargement has a scale factor > 1. A reduction has a scale factor < 1.

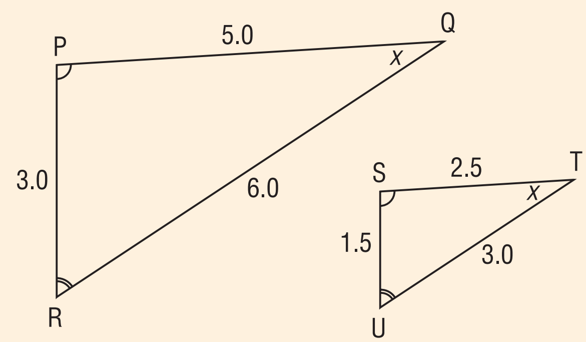
Similar Polygons



Similar polygons are related by an enlargement or a reduction. When two polygons are similar:

- their corresponding angles are equal:
  $\angle A = \angle E$; $\angle B = \angle F$; $\angle C = \angle G$; $\angle D = \angle H$
  and
- their corresponding sides are proportional:
  $\frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{DA}{HE}$
  Any of the ratios $\frac{AB}{EF}$, $\frac{BC}{FG}$, $\frac{CD}{GH}$, and $\frac{DA}{HE}$ is the scale factor.

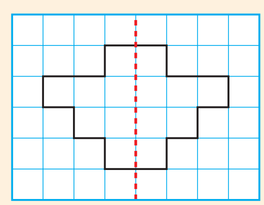
Similar Triangles



When we check whether two triangles are similar:

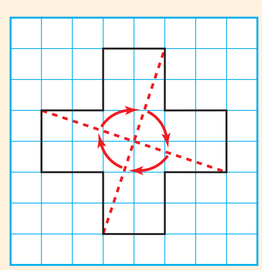
- their corresponding angles must be equal:
  $\angle P = \angle S$ and $\angle Q = \angle T$ and $\angle R = \angle U$
  or
- their corresponding sides must be proportional:
  $\frac{PQ}{ST} = \frac{QR}{TU} = \frac{PR}{SU}$
  Any of the ratios $\frac{PQ}{ST}$, $\frac{QR}{TU}$, and $\frac{PR}{SU}$ is the scale factor.

Line Symmetry



A shape has line symmetry when a line divides the shape into two congruent parts so that one part is the image of the other part after a reflection in the line of symmetry.

Rotational Symmetry



A shape has rotational symmetry when it coincides with itself after a rotation of less than 360° about its centre. The number of times the shape coincides with itself is the order of rotation.

The angle of rotation symmetry $= \frac{360°}{\text{the order of rotation}}$

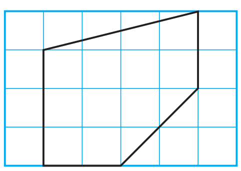
Review

7.1 **1.** This photo of participants in the Arctic Winter Games is to be enlarged.

Measure the photo. What are the dimensions of the enlargement for each scale factor?

a) 3 **b)** 2.5 **c)** $\frac{3}{2}$ **d)** $\frac{21}{5}$

2. Draw this pentagon on 1-cm grid paper. Then draw an enlargement of the shape with a scale factor of 2.5.

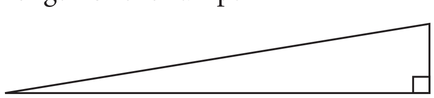


7.2 **3.** A full-size pool table has dimensions approximately 270 cm by 138 cm. A model of a pool table has dimensions 180 cm by 92 cm.

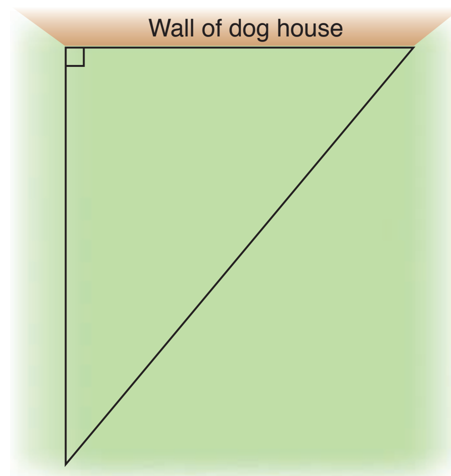
a) What is the scale factor for this reduction?

b) A standard-size pool cue is about 144 cm long. What is the length of a model of this pool cue with the scale factor from part a?

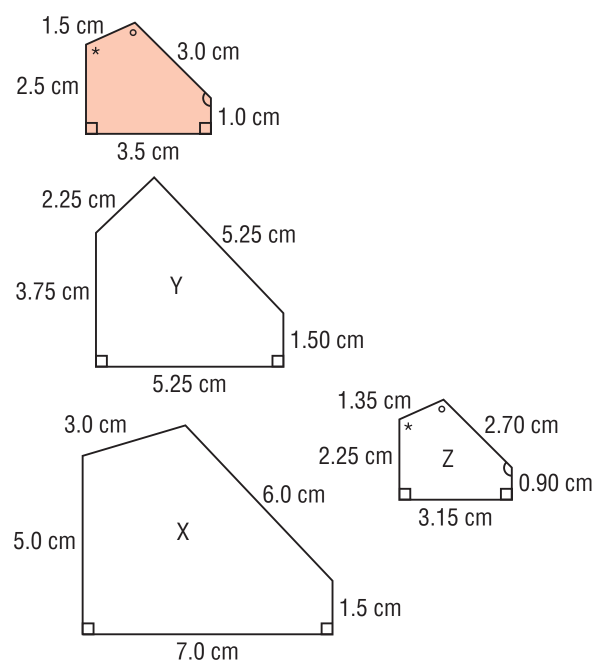
4. Here is a scale diagram of a ramp. The height of the ramp is 1.8 m. Measure the lengths on the scale diagram. What is the length of the ramp?



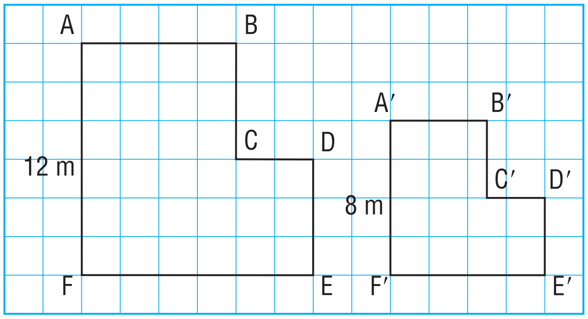
5. Gina plans to build a triangular dog run against one side of a dog house. Here is a scale diagram of the run. The wall of the dog house is 2 m long. Calculate the lengths of the other two sides of the dog run.



7.3 **6.** Which pentagon is similar to the red pentagon? Justify your answer.



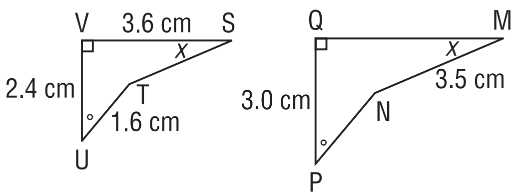
7. These two courtyards are similar.



Determine each length.

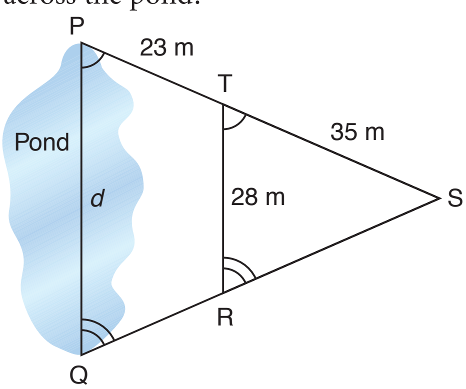
a) BC **b)** B′C′ **c)** A′B′

8. These two quadrilaterals are similar.

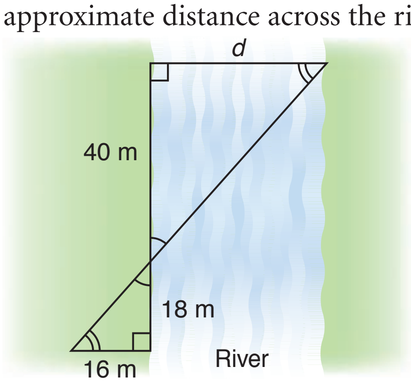


Calculate the length of: **a)** PN **b)** TS

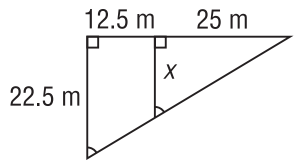
7.4 **9.** To determine the distance, *d*, across a pond, Ari uses this diagram. What is the distance across the pond?



10. This scale diagram shows a surveyor's measurements taken to determine the distance across a river. What is the approximate distance across the river?



11. How can you use similar triangles to calculate the distance x in this scale diagram?



7.5 **12.** Which of these traffic signs have line symmetry? How many lines of symmetry in each case?

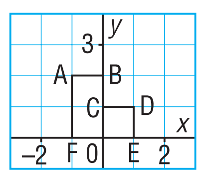
a)

b)

c)

d)

13. Hexagon ABCDEF is a part of a larger shape. Copy the hexagon on a grid.



a) Complete the shape by reflecting the hexagon:

i) in the y-axis

ii) in the x-axis

iii) in the line through $(-2, -1)$ and $(2, 3)$

b) Complete the shape with a translation R2.

c) List the ordered pairs of the vertices of each completed shape.

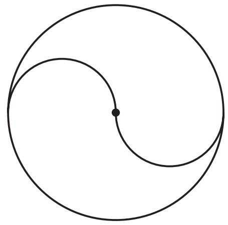
d) State whether each completed shape has line symmetry.

7.6

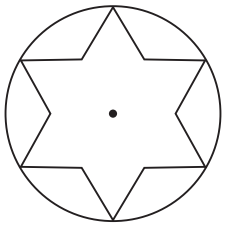
14. What is the order of rotational symmetry of each shape? How do you know?

a)

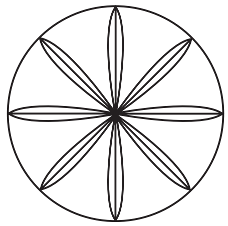
b)



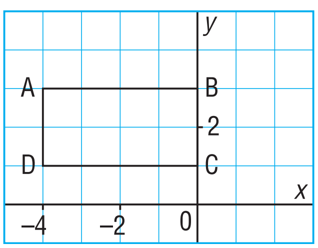
c)



d)



15. Rectangle ABCD is part of a larger shape that is to be completed by a transformation image.



a) Rotate rectangle ABCD as indicated, then draw and label each image.

i) 90° counterclockwise about the point $(-4, 2)$

ii) 180° about vertex B

iii) 270° counterclockwise about the point $(-2, 2)$

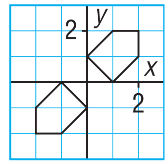
b) Which diagrams in part a have rotational symmetry? How do you know?

7.7

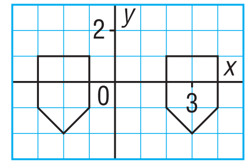
16. Look at the diagrams in question 15. Which diagrams have line symmetry? How do you know?

17. For each diagram, determine whether the two pentagons are related by any symmetry. Describe each type of symmetry.

a)



b)

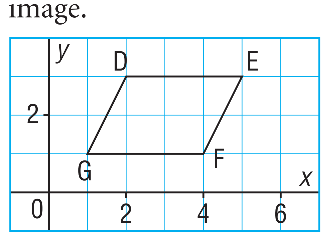


18. Identify and describe the types of symmetry in each piece of artwork.

a)

b)

19. a) Translate quadrilateral DEFG as indicated, then draw and label each image.

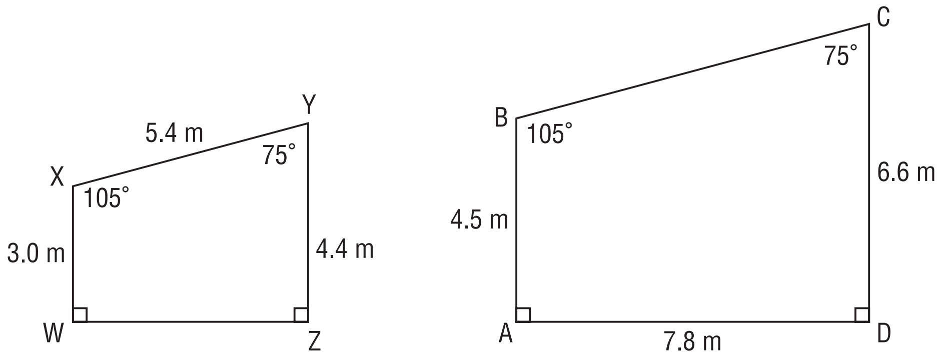


i) L4, D2 **ii)** R1, U2

b) Does each translation result in line symmetry or rotational symmetry? If your answer is yes, describe the symmetry. If your answer is no, explain why there is no symmetry.

Practice Test

1. These two quadrilaterals are similar.



a) Calculate the length of BC.
b) Calculate the length of WZ.
c) Draw an enlargement of quadrilateral WXYZ with scale factor 2.
d) Draw a reduction of quadrilateral ABCD with scale factor $\frac{1}{3}$.

2. Scott wants to calculate the height of a tree. His friend measures Scott's shadow as 3.15 m. At the same time, the shadow of the tree is 6.30 m. Scott knows that he is 1.7 m tall.
a) Sketch two triangles Scott could use to calculate the height of the tree.
b) How do you know the triangles are similar?
c) What is the height of the tree?

3. Use isometric dot paper or grid paper.
a) Draw these shapes: equilateral triangle, square, rectangle, parallelogram, trapezoid, kite, and regular hexagon
b) For each shape in part a:
   i) Draw its lines of symmetry.
   ii) State the order and angle of rotation symmetry.
c) Draw a shape that has line symmetry but not rotational symmetry.
d) Draw a shape that has rotational symmetry but not line symmetry.

4. Plot these points on a grid: A(2, 1), B(1, 2), C(1, 4), D(2, 5), E(3, 4), F(3, 2)
For each transformation below:
   i) Draw the transformation image.
   ii) Record the coordinates of its vertices.
   iii) Describe the symmetry of the diagram formed by the original shape and its image.
a) a rotation of 90° clockwise about the point G(2, 3)
b) a translation R2
c) a reflection in the line $y = 2$