Name: Notes Key
Date:

CHAPTER 2 NOTES – Powers and Exponent Laws

	Date:
2.1 – What is a Power?	
2.2 – Powers of Ten and the Zero Exponent	
Scientific Notation	
2.3 – Order of Operations with Powers	
2.4 – Exponent Laws I	
2.5 – Exponent Laws II	
Negative Exponents	
Review:	
Test:	

What You'll Learn:

2.1 – Use powers to represent repeated multiplication
2.2 – Use patterns to understand a power with exponent 0
Scientific Notation – Learn how to put numbers into and take them out of scientific notation
2.3 – Solve problems and perform operations (BEDMAS) involving powers
2.4/2.5 – Explain and apply exponent laws
Negative Exponents - To understand and apply negative exponents in evaluating powers

When are powers needed in the 'real world'?

- scientific formulas
- construction + architecture formulas

Paper Folding

In the paper folding investigation, what happens each time you make a new fold?

The number of layers double

How can you express this mathematically?

$\times 2$ each time : $\underset{\text{start}}{1} \underset{\text{one fold}}{\times 2} \underset{\text{two folds}}{\times 2} \underset{\text{three folds}}{\times 2}$ etc...

How can you simplify this pattern?

if you keep multiplying by 2, can use an exponent instead.

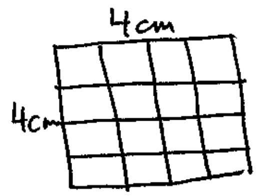
Number of layers $= 2^n$ where $n =$ number of folds.

2.1 – What is a Power?

Focus: Use powers to represent repeated multiplication

Main Ideas:

Warmup:
Suppose you have a square with length 4cm. Draw a picture and calculate the area with proper units.

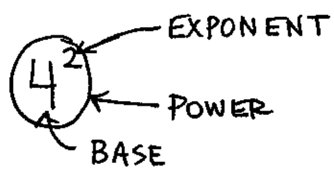


$A = 4 \times 4 = 16\,cm^2$

What's another way to write the calculation you did above?

$A = 4^2 = 16\,cm^2$

Name all of the parts of power form.

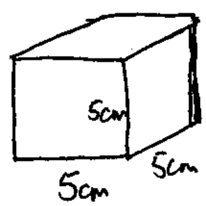


Give three ways to write 16.

$16,\ 4 \times 4,\ 4^2$

Ex1
Suppose you have a cube with side length 5cm. Draw the cube and give three ways to write the volume.

$V = 5 \times 5 \times 5 = 5^3 = 125\,cm^3$



What do you call a:
power with exponent 2?
power with exponent 3?

exponent 2 = 'square'

exponent 3 = 'cube'

Ex2
Write as a power and then in standard form:
a) 2 x 2 x 2 x 2 x 2

b) (3)(3)(3)

c) 7

(a) $2^5 = 32$ on calc: $\boxed{2}$ $\boxed{y^x}$ $\boxed{5}$ $\boxed{=}$

or $\boxed{x^y}$

or $\boxed{\wedge}$

(b) $3^3 = 27$

(c) $7^1 = 7$

Ex3
Write as a repeated multiplication & in standard form:
a) 4^6

b) 9^5

(a) $4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4096$

(b) $9 \times 9 \times 9 \times 9 \times 9 = 59049$

Ex4
Identify the base of each power, then evaluate the power
a) $(-3)^4$

b) -3^4

c) $-(-3)^4$

(a) $(-3)^4$ base $= -3$; $-3 \times -3 \times -3 \times -3 = 81$

(b) -3^4 base $= 3$; $-\ 3 \times 3 \times 3 \times 3 = -81$

(c) $-(-3)^4$ base $= -3$; $-\ -3 \times -3 \times -3 \times -3 = -81$

The base is whatever is inside the brackets. If no brackets, the base is the number immediately left of the exponent, and do not include the negative sign if there is one.

Reflection: How is $(-2)^4$ different from -2^4?

2.2 – Powers of Ten and the Zero Exponent

Focus: Explore patterns and powers of 10 to develop a meaning for the exponent 0.

Main Ideas:

Warmup:
Complete the table

Power	Standard Form	Words
10^6	1 000 000	one million
10^5	100 000	one hundred thousand
10^4	10 000	ten thousand
10^3	1000	one thousand
10^2	100	one hundred
10^1	10	ten

Using the pattern developed from the table, what is 10^0 ?

10^0 1 one

*The exponent in power form tells how many zeros in standard form

What is the Zero Exponent Law?

A power with any base (other than zero) that has a 0 exponent is equal to 1.

Ex1
Evaluate each expression:
a) 2^0

b) $(-2)^0$

c) -2^0

(a) $2^0 = 1$

(b) $(-2)^0 = 1$

(c) -2^0 base is 2 so $-2^0 = -1$

not part of base

Ex2
Write as a power of 10:
a) 500

b) 2 000 000

c) 4

(a) $5 \times 100 = 5 \times 10^2$

(b) $2 \times 1000000 = 2 \times 10^6$

(c) $4 \times 1 = 4 \times 10^0$

Reflection: Why is $(-4)^0$ equal to 1 but -4^0 equals -1?

Scientific Notation

Focus: Be able to put numbers into and take out of scientific notation.

Main Ideas:

Warmup:
Write the following as a power of ten:
a) 7000

b) 60

(a) $7 \times 1000 = 7 \times 10^3$

(b) $6 \times 10 = 6 \times 10^1$

What is Scientific Notation?

It's a way of expressing very large or very small numbers using base 10 powers so you don't have to write numbers in standard form.

Ex1
Put 80 000 into scientific notation

80 000

$8 \times 10\,000$

8×10^4 ← scientific notation

OR

80,000. 4 jumps

$= 8 \times 10^4$

Ex2
Put 2200 into scientific notation

2200. 3 jumps so 2.2×10^3

$2.2 \times 10^3 = 2.2 \times 1000 = 2200$

Ex3
Put into scientific notation:
a) 360 000

b) 525

(a) 360,000 5 jumps.

3.6×10^5

coefficient

(b) 525 2 jumps

5.25×10^2

Remember, scientific notation is just a different way to write the same number.

You always want the coefficient to be between 1 and 10, not including 10.

Ex4 Put 0.0000088 into scientific notation	0.0000088 6 jumps 8.8×10^{-6} When doing sci not for small numbers, decimal jump to the right and use a negative exponent
Ex5 Put into scientific notation: a) 0.00956 b) 0.000014	(a) 0.00956 (3) 9.56×10^{-3} (b) 0.000014 (5) 1.4×10^{-5}
How do you put numbers that are in scientific notation back into standard form?	If the exponent is positive, the number is a big number so jump right. If exponent is negative, the number is small, so jump decimal left.
Ex6 Put into standard form: a) 2.65×10^{-3} b) 7×10^{6} c) 8.3×10^{-5} d) 1×10^{0}	(a) 2.65×10^{-3} small number so jump left 002.65×10^{-3} 0.00265 (b) 7×10^{6} 7 000 000 (c) 008.3×10^{-5} 0.000083 (d) 1×10^{0} no jumps = 1
Ex7 Use your calculator: a) $(3.56 \times 10^{2})(7.4 \times 10^{-3})$ b) $6.7 \times 10^{4} + 2 \times 10^{3}$	To put in 3.56×10^{2} in calc: punch in 3.56, then $\boxed{EXP}$ or $\boxed{EE}$, then $\boxed{2}$ (a) 2.6 (b) 1.3×10^{8}

Reflection: Why is scientific notation even necessary?

2.3 – Order of Operations With Powers

Focus: Explain and apply the order of operations with exponents.

Main Ideas:

Warmup:
You win the big prize in the Thrifty's sweepstakes, but can only claim top prize if you get the skill testing question correct:
6 x (3 + 2) – 10 ÷ 2
What is the answer?

$$6 \times (3+2) - 10 \div 2$$
$$= 6 \times 5 - 10 \div 2$$
$$= 30 - 10 \div 2$$
$$= 30 - 5$$
$$= 25$$

What is the key word for order of operations?

BEDMAS

B: brackets
E: exponents
DM: divis/mult (whichever comes first)
AS: add/subtract (whichever comes first)

Ex1 – Evaluate
a) $3^3 + 2^3$

b) $3 - 2^3$

c) $(3 + 2)^3$

(a) $3^3 + 2^3$
$27 + 2^3$
$27 + 8$
$= \underline{\underline{35}}$

(b) $3 - 2^3$
$= 3 - 8$
$= \underline{\underline{-5}}$

(c) $(3+2)^3$
$= (5)^3$
$= \underline{\underline{125}}$

Ex2 – Evaluate
a) $[2 \times (-3)^3 – 6]^2$

b) $3 + 2^4 – 3 \times (2^2 – 1)$

c) $(18^2 + 5^0)^2 \div (-5)^3$

(a) $[2\times(-3)^3-6]^2$
$[2\times(-27)-6]^2$
$[-54-6]^2$
$[-60]^2$
$= \underline{3600}$

(b) $3+2^4-3\times(2^2-1)$
$3+2^4-3\times(4-1)$
$3+2^4-3\times 3$
$3+16-3\times 3$
$3+16-9$
$19-9$
$= \underline{\underline{10}}$

(c) $(18^2+5^0)^2 \div (-5)^3$
$(324+1)^2 \div (-5)^3$
$(325)^2 \div (-5)^3$
$105625 \div -125$
$= \underline{\underline{-845}}$

Ex3 – Evaluate to one decimal place
$$\frac{690}{2\times 4^2 + \pi \times 1^3}$$

$$\frac{690}{2\times 4^2 + \pi \times 1^3} = \frac{690}{2\times 16 + \pi\times 1} = \frac{690}{32+\pi} = \frac{690}{35.14} = \underline{\underline{19.6}}$$

when big division bar, do division last!

Reflection: How are mistakes most often made when using BEDMAS. How could you limit these mistakes?

2.4 – Exponent Laws I

Focus: Understand and apply the exponent laws for products and quotients of powers.

Main Ideas:

Warmup:
Complete the table and see if you can find a pattern:

Product of Powers	Product as Repeated Multiplication	Product as a Power
$5^4 \times 5^2$	$(5 \times 5 \times 5 \times 5) \times (5 \times 5)$	5^6
$3^5 \times 3^4$	$(3 \times 3 \times 3 \times 3 \times 3) \times (3 \times 3 \times 3 \times 3)$	3^9
$2^3 \times 2^3$	$(2 \times 2 \times 2) \times (2 \times 2 \times 2)$	2^6
$4^6 \times 4$	$(4 \times 4 \times 4 \times 4 \times 4 \times 4) \times 4$	4^7

Is there a pattern / shortcut that you see?

If multiplying powers, add exponents.

What is the Exponent Law for a Product of Powers?

To multiply powers with the same base, add the exponents (and keep the base the same).

Ex1
Write each expression as a power:
a) $3^5 \times 3^2$

b) $6 \times 6^3 \times 6^4$

(a) $3^5 \times 3^2 = 3^{5+2} = 3^7$

(b) $6^1 \times 6^3 \times 6^4 = 6^{1+3+4} = 6^8$

What is $8^7 \div 8^4$?

$$8^7 \div 8^4 = \frac{8^7}{8^4} = \frac{\cancel{8} \times \cancel{8} \times \cancel{8} \times \cancel{8} \times 8 \times 8 \times 8}{\cancel{8} \times \cancel{8} \times \cancel{8} \times \cancel{8}} = 8^3$$

So, $8^7 \div 8^4 = 8^3$

What is the Exponent Law for a Quotient of Powers?

To divide powers with the same base, subtract the exponents (and keep the base the same).

$8^7 \div 8^3 = 8^{7-3} = 8^4$

Ex2
Write each expression as a power:

a) $4^8 \div 4^3$

b) $\frac{(-5)^6}{(-5)^4}$

c) $3^2 \times 3^4 \div 3^3$

d) $\frac{2^3}{2^3}$

(a) $4^8 \div 4^3 = 4^{8-3} = 4^5$

(b) $\frac{(-5)^6}{(-5)^4} = (-5)^{6-4} = (-5)^2$

(c) $\underbrace{3^2 \times 3^4} \div 3^3$

$3^{2+4} \div 3^3$

$3^6 \div 3^3$

3^{6-3}

3^3

(d) $\frac{2^3}{2^3}$ OR $\frac{\cancel{2} \times \cancel{2} \times \cancel{2}}{\cancel{2} \times \cancel{2} \times \cancel{2}}$

$= 2^{3-3}$

$= 2^0$

$= 1$

everything cancels so $= 1$

this helps verify that any base (except 0) to the 0 exponent equals 1.

Ex3 – Evaluate

a) $2^3 \times 3^2$

b) $(-10)^4[(-10)^6 \div (-10)^4] - 10^7$

(a) $2^3 \times 3^2$

bases aren't the same so can't use shortcut
- use BEDMAS

$2^3 \times 3^2$

$= 8 \times 3^2$

$= 8 \times 9$

$= 72$

(b) $(-10)^4[(-10)^6 \div (-10)^4] - 10^7$

$(-10)^4[(-10)^2] - 10^7$

$(-10)^6 - 10^7$

$1000000 - 10\,000\,000$

$= -9000000$

Reflection: When can you use the exponent laws to evaluate an expression with powers? When can you *not* use these laws? Include examples.

2.5 – Exponent Laws II

Focus: Understand and apply exponent laws for powers of: products, quotients, and powers.

Main Ideas:

Warmup:
Complete the table and see if you can find a pattern:

Power	Repeated Multiplication	Expanded Form	Power
$(2^3)^2$	$2^3 \times 2^3$	2x2x2 x 2x2x2	2^6
$(4^2)^4$	$4^2 \times 4^2 \times 4^2 \times 4^2$	4×4×4×4×4×4×4×4	4^8
$(5^3)^3$	$5^3 \times 5^3 \times 5^3$	5×5×5 × 5×5×5 × 5×5×5	5^9
$[(-3)^2]^5$	$(-3)^2 \times (-3)^2 \times (-3)^2 \times (-3)^2 \times (-3)^2$	-3×-3×-3×-3×-3×-3×-3×-3×-3×-3	$(-3)^{10}$

Is there a pattern / shortcut that you see?

multiply exponents

What is the Exponent Law for a Power of a Power?

To raise a power to a power, multiply the exponents (and keep the base the same)

Ex1 – Simplify as a power:
a) $(9^5)^6$

b) $[(-1)^3]^4$

c) $-(3^7)^2$

(a) $(9^5)^6 = 9^{5\times6} = 9^{30}$

(b) $[(-1)^3]^4 = (-1)^{3\times4} = (-1)^{12}$

(c) $-(3^7)^2 = -3^{7\times2} = -3^{14}$

How can you simplify $(3 \times 4)^5$?

$$(3\times4)^5 = (3\times4)\times(3\times4)\times(3\times4)\times(3\times4)\times(3\times4)$$
$$= 3\times4\times3\times4\times3\times4\times3\times4\times3\times4$$
$$= 3\times3\times3\times3\times3\times4\times4\times4\times4\times4$$
$$= 3^5\times4^5$$

What is the Exponent Law for a Power of a Product?

$$(ab)^m = a^m b^m$$

ex. $(3\times4)^5 = 3^5\times4^5$

Ex2 – Simplify as a power:
a) $(2 \text{ x } 3)^6$

b) $[(-8) \text{ x } 4]^2$

c) $(2m)^3$

(a) $(2 \times 3)^6 = 2^6 \times 3^6$

(b) $[(-8) \times 4]^2 = (-8)^2 \times 4^2$

(c) $(2m)^3 = 2^3 m^3 = 8m^3$

How can you simplify $\left(\frac{2}{3}\right)^3$?

$$\left(\frac{2}{3}\right)^3 = \left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) = \frac{2 \times 2 \times 2}{3 \times 3 \times 3} = \frac{2^3}{3^3} = \frac{8}{27}$$

What is the Exponent Law for a Power of a Quotient?

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \quad \text{ex. } \left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3}$$

Ex3 – Simplify as a power:
a) $\left(\frac{5}{6}\right)^4$

$$\left(\frac{5}{6}\right)^4 = \frac{5^4}{6^4}$$

Ex4 – Simplify, then evaluate
a) $-(2 \text{ x } 3)^8 \div (3^3)^2$

b) $(3^2 \text{ x } 3^3)^3 - (4^3 \text{ x } 4^2)^2$

(a) $-(2 \times 3)^8 \div (3^3)^2$

$= -2^8 \times 3^8 \div 3^6$

$= -2^8 \times 3^2$

$= -256 \times 9$

$= \underline{-2304}$

(b) $(3^2 \times 3^3)^3 - (4^3 \times 4^2)^2$

$= (3^5)^3 - (4^5)^2$

$= 3^{15} - 4^{10}$

$= 14\,348\,907 - 1\,048\,576$

$= \underline{13\,300\,331}$

Reflection: Why do you add the exponents to simplify $3^2 \text{ x } 3^4$ but multiply exponents to simplify the expression $(3^2)^4$?

Negative Exponents

Focus: To understand and apply negative exponents in evaluating powers

Main Ideas:

Warmup:
Simplify as a power:

$$\frac{2^3}{2^5}$$

What do you notice?

$$\frac{2^3}{2^5} = 2^{3-5} = 2^{-2}$$ negative exponent

Now, expand as a repeated multiplication, cancel, and evaluate.

$$\frac{\cancel{2}\times\cancel{2}\times\cancel{2}}{\cancel{2}\times\cancel{2}\times\cancel{2}\times2\times2} = \frac{1}{2\times2} = \frac{1}{2^2} = \frac{1}{4}$$

So, $2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

What is the shortcut for negative exponents?

$$a^{-x} = \frac{1}{a^x}$$

① make a fraction $\frac{a^{-x}}{1}$

② flip the fraction and change the exponent to a positive $\frac{1}{a^x}$

Ex1: Simplify, then evaluate:
a) 4^{-2}

b) 2^{-5}

c) 76^{-1}

(a) $\frac{4^{-2}}{1} = \frac{1}{4^2} = \frac{1}{16}$

(b) $\frac{2^{-5}}{1} = \frac{1}{2^5} = \frac{1}{32}$

(c) $\frac{76^{-1}}{1} = \frac{1}{76^1} = \frac{1}{76}$

Ex2: Simplify, then evaluate:

$$\left(\frac{3}{-2}\right)^{-2}$$

if you have a negative exponent, and your base is a fraction, flip the fraction and change the exponent to a positive

$$\left(\frac{3}{-2}\right)^{-2} = \left(\frac{-2}{3}\right)^{2} = \frac{(-2)^2}{3^2} = \frac{4}{9}$$

Ex3 – Simplify:

a) $[(-2)^2]^{-3} \times (-2)^2$

b) $\left(\frac{1}{4}\right)^{-2} - \left(\frac{2^7 \times 2^{-5}}{2^3}\right)$

(a) $[(-2)^2]^{-3} \times (-2)^2$

$= (-2)^{-6} \times (-2)^2$

$= (-2)^{-4}$

$\frac{(-2)^{-4}}{1} = \frac{1}{(-2)^4} = \frac{1}{16}$

(b) $\left(\frac{1}{4}\right)^{-2} - \left(\frac{2^7 \times 2^{-5}}{2^3}\right)$

$\left(\frac{4}{1}\right)^2 - \left(\frac{2^2}{2^3}\right)$

$\frac{4^2}{1^2} - 2^{-1}$

$\frac{16}{1} - \frac{1}{2^1}$

$16 - \frac{1}{2}$

$15\frac{1}{2}$

Reflection: A common error when working with negative exponents is shown here in an example: $2^{-3} = -8$. Why is this wrong and what is 2^{-3} ?