Name: Notes Key
Date:

CHAPTER 4 NOTES – Linear Relations

What You'll Learn:

4.1 – Use expressions and equations to generalize patterns
4.1 – Verify a pattern by using substitution
4.2/4.3/4.4 – Graph and analyze linear relations
4.5 – Interpolate and extrapolate to solve problems

One definition of math is 'the study of patterns'. How can finding and studying patterns be important in our world?

- helps us to make predictions, as 'trends' are simply patterns.
- helps us to hypothesize in science

What word is inside the word 'linear'? So what may a linear relation be? Can you think of a real world example of a linear relation?

- line
- a straight line relation
- real world example: the increase in cost as you buy more + more tickets to a concert.

4.1 – Writing Equations to Describe Patterns

Focus: Use equations to describe and solve problems involving patterns.

Main Ideas:

Warmup:
Do the 'Investigate' on p.154. Can you describe the pattern:
a) in words
b) a table
c) an equation where
t = number of tables &
p = number of people
d) How many people at 25 tables?

(b)

Tables	People
1	4
2	6
3	8
4	10
5	12

(a) everytime a table is added, two more people are added

(c) $p = 2t + 2$

two people per table — 2 people on the ends.

(d) $p = 2t + 2$
$p = 2(25) + 2$
$p = 50 + 2 = \underline{\underline{52}}$.

Together, read through 'Connect' on p.155&156 and make any notes you think are important.

drawing a table can be very helpful in finding patterns.

Ex1
You go to the store to buy peanut butter.
a) Make notes on or around the table.
b) Write an equation that relates the cost to the number of jars of peanut butter.
c) What is the cost of 18 jars?

Jars	Cost ($)
0	0
1	5
2	10
3	15
4	20

+1 +1 +1 +1 ; +5 +5 +5 +5

As jars go up by 1, cost goes up by 5.

(b) Let j = jars Let C = cost
$C = 5j$

(c) $C = 5j$
$C = 5(18)$
$C = 90$

The cost of 18 jars is \$90.

Ex2
Find the equation using the pattern in the table

m	n
0	1
1	3
2	5
3	7

+1 +1 +1 ; +2 +2 +2

start with 1.

As m goes up by 1, n goes up by 2

$n = 2m + 1$

started with one.

Ex3
Find the equation using the pattern in the table

a	b
3	-1
6	0
9	1
12	2
0	-2

+3 +3 +3

+1 +1 +1

as a goes up by 3, b goes up by 1 but started at -1

$b = \frac{a}{3} - 2$

If a = 26, what is b?

$b = \frac{a}{3} - 2$ $\quad b = \frac{26}{3} - 2$ $\quad b = 8.\bar{6} - 2$

$b = \underline{6.\bar{6} \text{ or } 6.7}$

Ex4
Empress Cabs charges $3.75 plus $1.25 per km
a) if d = number of km, write an equation for the cost of Empress Cabs.
b) What is the fare for a 19 km ride?

d distance	c cost
0	3.75
1	5.00
2	6.25
3	7.50

start at 3.75

+1 +1 +1

+1.25 +1.25 +1.25

(a) $C = 1.25d + 3.75$

(b) $C = 1.25(19) + 3.75$

$C = 23.75 + 3.75$

$C = \underline{27.50}$ The fare for a 19 km ride is $27.50.

Reflection: What is the biggest challenge for you in this section and how do you think you can improve upon it?

4.2 – Linear Relations

Focus: Analyze the graph of a linear relation.

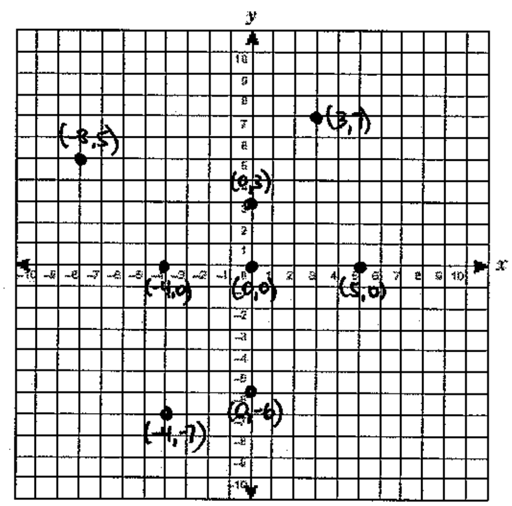
Main Ideas:

Warmup:
How do you plot points on a coordinate plane?

The x axis is horizontal and the y axis is vertical
The point will be given as, for example, (3, -5). The first number is **always** the x value and the second is the y value.
Start at (0, 0) on the graph (the middle). If x is positive, go right. If x is negative go left. From there, count your y. If y is positive, go up, and if negative, go down. Then plot your point.

Plot and label the points on the coordinate plane.

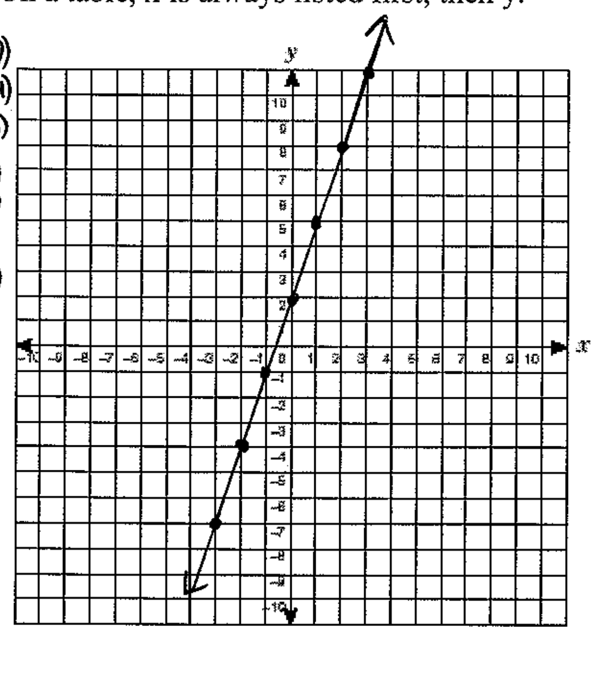
a) (0, 0) b) (5, 0)
c) (0, 3) d) (-4, 0)
e) (3, 7) f) (-8, 5)
g) (0, -6) h) (-4, -7)



Ex1
Suppose you were monitoring daily temperature. Three days ago, the temperature was -7°C. Everyday since, the temperature has/will increase by 3°C
a) Complete the table.
b) Graph the relation.
c) What kind of pattern and/or relationships do you notice in the table and/or graph?

On a table, x is always listed first, then y.

Day (x axis)	Temp (y)	
-3	-7	(-3,-7)
-2	-4	(-2,-4)
-1	-1	(-1,-1)
0	2	(0,2)
1	5	(1,5)
2	8	(2,8)
3	11	(3,11)

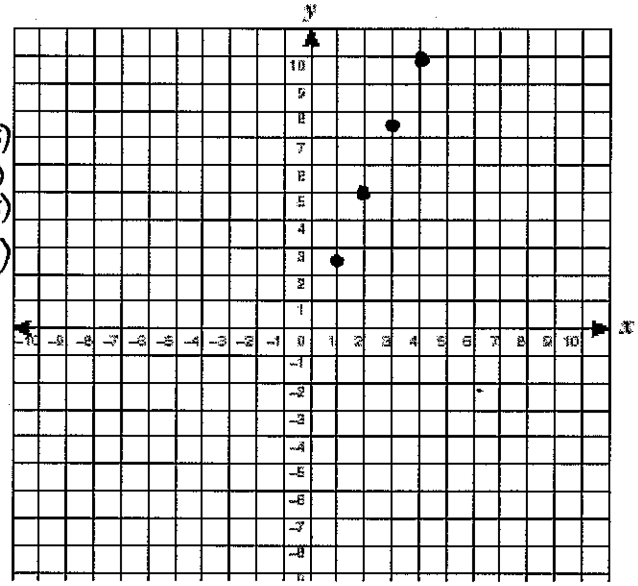


c) as x increases by 1, y increases by 3. in both the table and graph.

Ex2
The table shows the cost of renting DVDs at an online store.
a) Graph the points, but don't draw a line.
b) Use the table to describe the pattern in the rental costs. How is this pattern shown in the graph?
c) Why don't we draw a line?

DVDs	Cost ($)	
1	2.50	(1,2.5)
2	5.00	(2,5)
3	7.50	(3,7.5)
4	10.00	(4,10)

+1 +1 +1 +2.5 +2.5 +2.5



b) as DVDs go up by 1, cost goes up by $2.50

c) The points are discrete, meaning the space in between the points have no literal meaning. For example, the point (1.5, 3.75) has no meaning because you can't purchase 1.5 DVDs. Therefore, we don't draw a line.

Is the number of DVDs purchased related to the cost?

Yes. For every 1 DVD purchased, the cost increases by $2.50.

What is the equation for DVDs and cost in example 2? Use x for DVDs and y for cost.

$y = 2.50x$

What does the equation tell us?

Multiply the number of DVDs by $2.50 to find the total cost.

Does cost depend on the number of DVDs, or does the number of DVDs depend on cost?

The cost depends on the number of DVDs.

What is an independent variable, and what is a dependent variable?

The independent variable is the number of DVDs, because you can purchase any number of them. The independent variable is always the x value.
The dependent variable is the cost, because the cost depends on the number of DVDs you decide to buy. The dependent variable is always the y value.

What is a linear relation?

When two variable are related by a straight line, you have a linear relation.

A relation has the equation $y = 5 - 2x$

a) Create a table of values for values of x from -2 to 4. Find y for each.

b) Graph the relation. Should you join the points with a line?

c) What patterns do you see in the table and graph?

d) Is the relation linear?

x	y
-2	9
-1	7
0	5
1	3
2	1
3	-1
4	-3

$y = 5 - 2x$

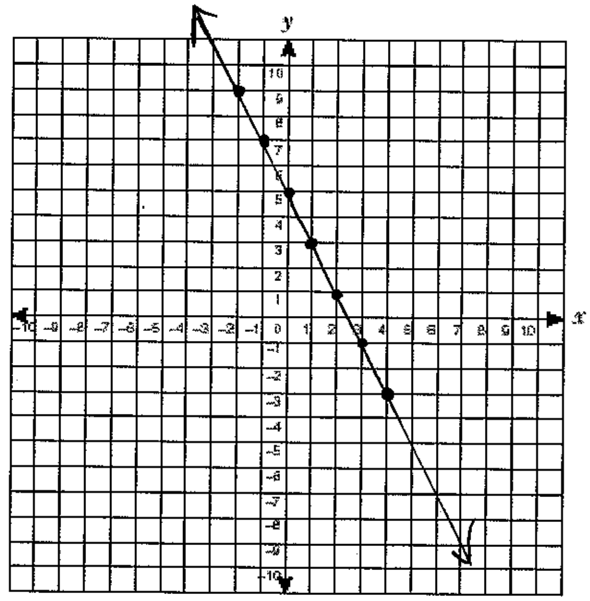
$y = 5 - 2(-2)$

$y = 5 - -4$

$y = 5 + 4$

$y = 9$

do this for $x = -1, 0, 1, 2, 3, 4$



(b) The points should be joined, as we don't really know if the data is discrete.

(c) As x increases by 1, y decreases by 2.

(d) The relation is linear, as x and y are related by a straight line.

If you know a relation is linear, how many points do you need to plot the line?

3 points.

2 is the bare minimum, but 3 will include a check to make sure you have the correct line.

Reflection: If you were to plot a linear relation between number of km driven vs. cost of gas, which of the two would be the independent variable and which would be the dependent variable. **Explain**.

4.3 – Another Form of the Equation for a Linear Relation

Focus: Recognize the equations of horizontal, vertical, and oblique lines, and graph them.

Main Ideas:

Warmup:

Suppose you have a piece of licorice 10cm long.

a) How many different ways could you cut it into two pieces?

b) In words, how are the lengths of the two pieces related?

c) If x = the length of the first piece, and y = the length of the second piece, write an equation for the relation.

d) How is your equation different from the equations we worked with in 4.2?

e) Make a table of values.

f) Graph the equation.

g) Is the relation linear?

(a) there are many ways to cut it into two pieces, such as 3.46 and 6.54cm. To keep it simple, let's work with whole numbers: 1cm + 9cm, 2cm + 8cm, 3cm + 7cm, 4cm + 6cm, 5cm + 5cm

(b) The two pieces will always add up to 10cm

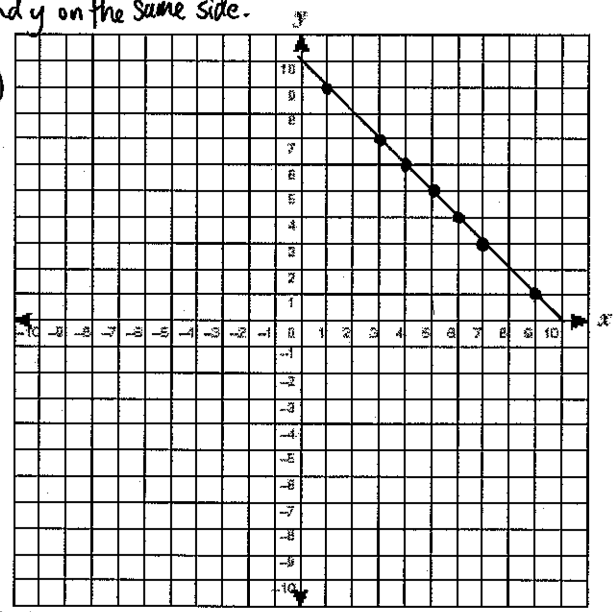
(c) $x + y = 10$

(d) The equations in 4.2 had y on one side of the 'equals' sign, and x on the other. This equation has x and y on the same side.

(e)

Piece #1 (x)	Piece #2 (y)
1	9
3	7
4	6
5	5
6	4
7	3
9	1

(f)



(g) The relation is linear as a straight line resulted.

We connect the dots with a line as the lengths of each licorice piece could be decimals as well

Let's look at 'Connect' on top half of p.175.

Ex1
For the equation
$3x - 2y = 6$:
a) Make a table of values for $x = -4, 0, 4$
b) Graph the equation.
c) What is another name for a 'slanted' line?

(a)

x	y
-4	-9
0	-3
4	3

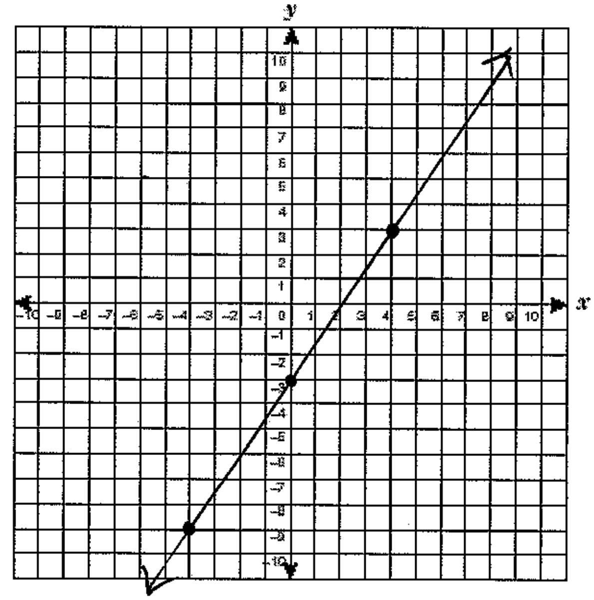
$3x - 2y = 6$

$3(-4) - 2y = 6$
$-12 - 2y = 6$
$+12 \qquad +12$
$-2y = 18$
$y = -9$

$3(0) - 2y = 6$
$0 - 2y = 6$
$-2y = 6$
$y = -3$

$3(4) - 2y = 6$
$12 - 2y = 6$
$-12 \qquad -12$
$-2y = -6$
$y = 3$

(b)



(c) Another name for a slanted line is an 'oblique' line.

Sometimes, only one variable appears in an equation, for example, $x = 2$, or, $y = -5$

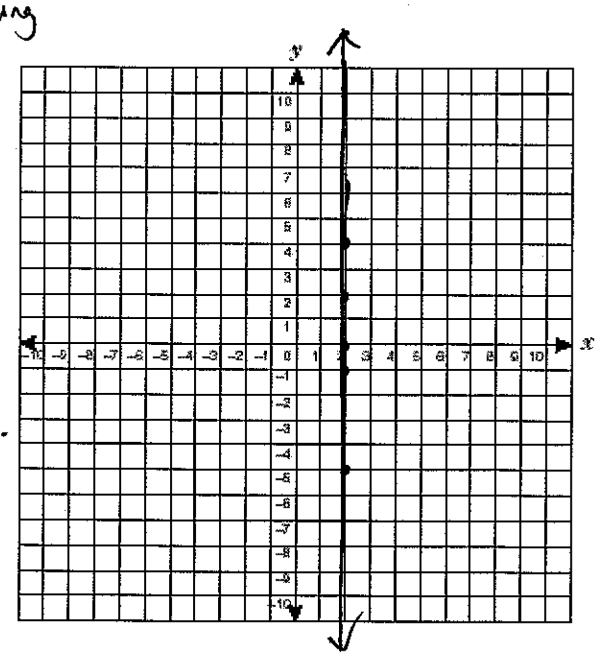
Ex2
Graph $x = 2$
Hint: The only requirement for a point on the graph is that the x value must be 2. So y can be anything, as long as x is 2.

use a table of values

x	y
2	-5
2	-1
2	0
2	2
2	4

x must always be 2

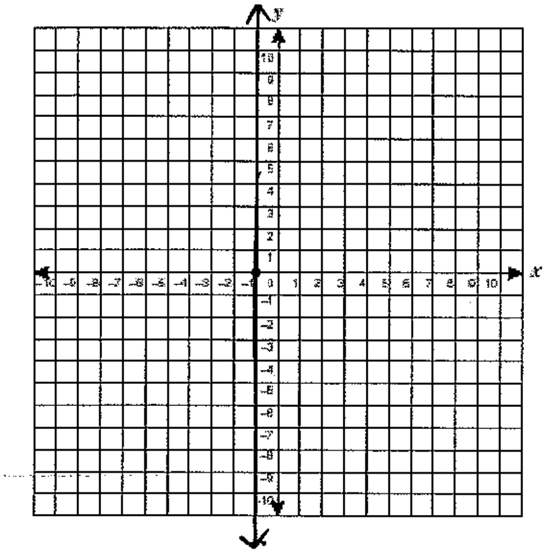
y can be anything



What kind of line is produced?

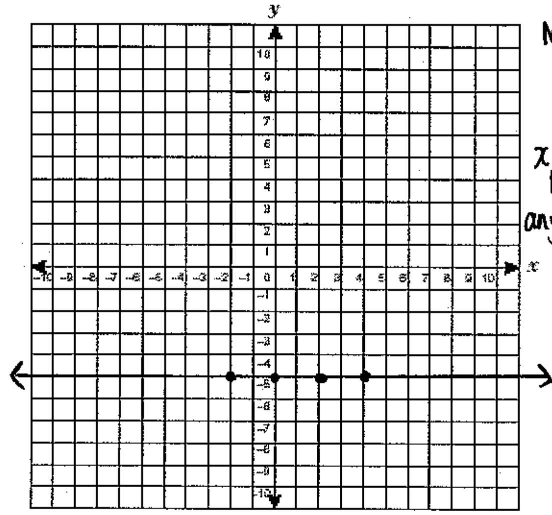
A vertical line where x is equal to 2.

Ex3
Graph $x = -1$



$x = -1$
is a vertical line
where x is equal to -1

Ex4
Graph $y = -5$
The only requirement is that for each point, the y value must be -5.



Make a table of values

x	y
-2	-5
0	-5
2	-5
4	-5

x can be anything

y must always be -5

What kind of line is produced?

$y = -5$ is a horizontal line where y is equal to -5.

What kind of equation Produces a:
a) oblique line?
b) vertical line?
c) horizontal line?

(a) an equation with x and y in it.
(b) an equation with just x in it
(c) an equation with just y in it

Reflection: Give an example of an equation that produces a: (i) horizontal line (ii) oblique line (iii) vertical line

4.4 – Matching Equations and Graphs

Focus: Match equations and graphs of linear relations.

Main Ideas:

Warmup: Do the investigate on p.183. Show any work etc. on the right.	Sari's graph has the equation $m = 4d$ Monica's graph has the equation $m = 2d + 3$ Bruce's graph has the equation $m = d + 5$ I put 0 in for d in each equation, calculated m, and matched it on the graphs. Sari's plan: \$4 per km Monica's: \$3 up front plus \$2 per km Bruce's: \$5 up front plus \$1 per km.
Look over the 'Connect' on p. 184. What is the best way to match an equation with its graph? How do you choose x values to test?	Use the equation, pick an x value, and substitute it in to find y. Then you have a point (x, y) to match on a graph. Repeat if necessary. Choose x values that are visible on the graphs
Ex1 Cover the solution of Example 1 on p.186 and try the question. Why is testing x = 0 unhelpful?	Test $x = 1$ in all 3 equations: ① $y = x$, $y = 1$, so has point $(1,1)$ GRAPH C ② $y = 2x$, $y = 2(1)$, $y = 2$, has point $(1,2)$ GRAPH A ③ $y = -3x$, $y = -3(1)$, $y = -3$, has point $(1,-3)$ GRAPH B
The solution of example 1 on p.186 gives another method for matching equations and graphs. Look it over to see if it's a method you may use.	

Ex2 Cover the solution of example 2 on p.187 and try the question (bottom of p.186).	$y = 3x - 4$ Test $x = 0$ to see what y will be ... $y = 3(0) - 4$ $y = 0 - 4$ $y = -4$ The point $(0,-4)$ is on the graph so line (iii)
When all lines cross the *y* axis at a different point, why is it smart to test x = 0 into the equation(s)?	Because any point where $x=0$ $(0, \_)$ will always end up on the y axis. So if all lines cross the y axis at a different point, it's very easy to identify your line quickly.

Reflection: What strategy will you used to match an equation with its graph. Explain your strategy in detail.

4.5 – Using Graphs to Estimate Values

Focus: Use interpolation and extrapolation to estimate values on a graph.

Main Ideas:

Warmup: Do the 'Investigate' on p.191.	(a) find 150 000 people on the x axis and estimate what that will be on the y axis : 1125 ML (b) find 1400ML on the y axis and estimate what that will be on the x axis : 187 000 people (c) possibly use the table for assistance : 1875 ML
What is interpolation?	When you estimate values that lie <u>between</u> 2 data points on a graph.
When did you use interpolation in the Investigate?	for the first two estimations
Look at the graph in the centre of p.192 to get a visual of interpolation.	use the linear relationship of the graph to Make estimations.
What is extrapolation?	When you predict values that lay beyond the data points on a graph, relying on the linear relation.
When did you use extrapolation in the Investigate?	for the last estimation.
Look at the graph at the top of p.193 to get a visual of extrapolation.	you can extend a graph to extrapolate

Ex1
Do example 1 on p.193 (don't look at the solution on p.194).

Are you interpolating or extrapolating? Explain.

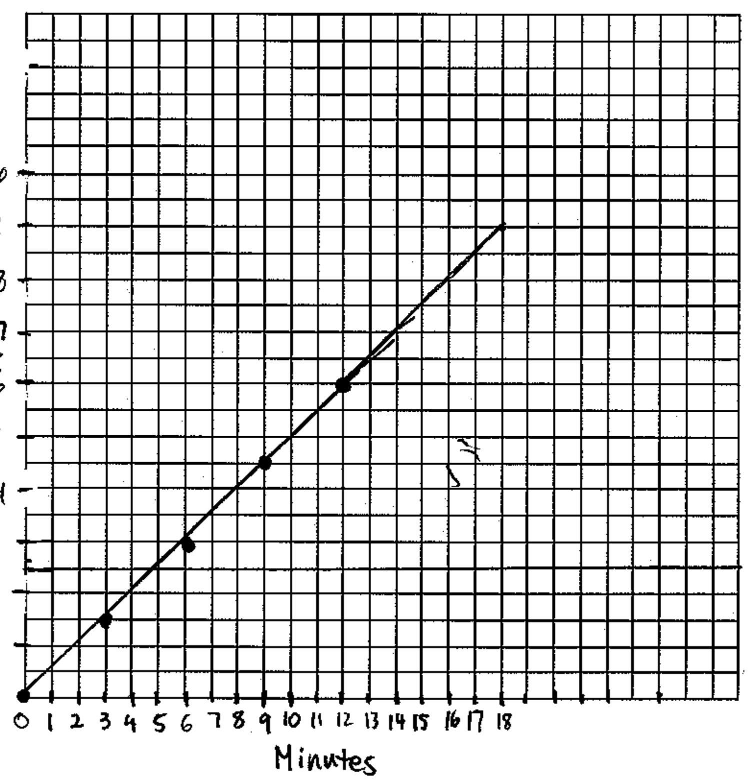
(a) $200
(b) $140
(c) 5 weeks

In example 1, we are using interpolation as all of our estimates were within the graph

Ex2
Corey goes biking. Every 3 minutes (x), Corey travels 1.5km (y).
a) Draw a graph for the first 12 minutes of biking, but leave room at the end of the graph.
b) How far has Corey biked after 7 minutes? Is this interpolation or extrapolation?
c) How far will Corey have biked after 17 minutes. Extend your graph to 18 minutes to assist you. Is this interpolation or extrapolation?

x	y
0	0
3	1.5
6	3.0
9	4.5
12	6.0

(a)

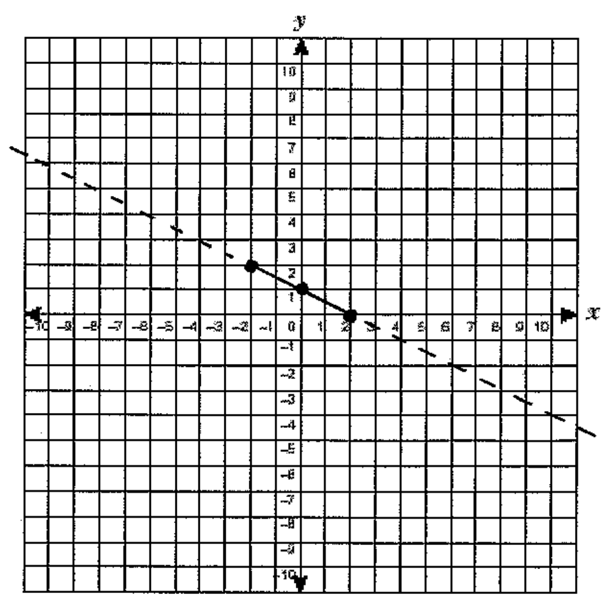


(b) After 7 mins, Corey has biked 3.5 km. This is interpolation
(c) After 17 mins, Corey has biked 8.5km. This is extrapolation.

Ex3
Do #6 on p.196 by first re-drawing and extending the graph. Do part a (*i*) & (*iii*) and part b (*i*) & (*iii*)

(a) i) when y=6, x=-10
ii) when y=-4, x=10

b) i) when x=-6, y=4
ii) when x=6, y=-2



Reflection: Give one real life example of extrapolation that could have a significant impact on our world.