Name: Key
Date:

CHAPTER 7 NOTES – Similarity and Transformations

	Date:
7.1 – Scale Diagrams and Enlargements	
7.2 – Scale Diagrams and Reductions	
7.3 – Similar Polygons	
7.4 – Similar Triangles	
7.5 – Reflections and Line Symmetry	
7.6 – Rotations and Rotational Symmetry	
7.7 – Identifying Types of Symmetry on the Cartesian Plane	
Review:	
Test:	

What You'll Learn

7.1/7.2 – Draw and interpret scale diagrams
7.3/7.4 – Apply properties of similar polygons
7.5/7.6/7.7 – Identify and describe line symmetry and rotational symmetry

What are some careers that require either the construction or analysis of scale diagrams?

- architects
- engineers
- designers
- surveyors
- tradespeople
- real estate developers

Read over the 'What Should I Recall' section on pgs. 316 & 317 of the text and note anything important below.

- if sides have the same hash marks, they're equal in length
- an isosceles triangle has two equal sides AND two equal angles
- sides are described with two capital letters ex AB
- angles are described with three capital letters ex ∠ABC
- if you know 2 sides of a right triangle, you can find the third side using Pythagoras.
- the 3 angles in a triangle always add to 180°

7.1 – Scale Diagrams and Enlargements

Focus: Draw and interpret scale diagrams that represent enlargements.

Main Ideas:

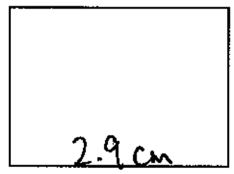
Warmup: Look at the digital camera drawings on the bottom of p. 318. a) How are the two drawings alike? b) How are they different? c) Using the grid, count each side length and for each set of sides, write the fraction: $\frac{length \cdot on \cdot enlargement}{length \cdot on \cdot actual}$ d) Write each fraction as a decimal. What do you notice about these numbers?	a) They have the same shape. b) They are different size. c) $\frac{2}{1}$; $\frac{6}{3}$; $\frac{10}{5}$; $\frac{8}{4}$; $\frac{8}{4}$ d) 2 ; 2 ; 2 ; 2 ; 2 They are all the same!
What is a scale diagram?	A diagram that is an enlargement or reduction of another diagram or an object.
What are corresponding lengths?	Lengths that are matching on the original diagram and the scale diagram.
What is scale factor?	the fraction $\frac{\text{length on scale diagram}}{\text{length on original diagram}}$
What is proportion?	for each set of matching lengths, the fraction above comes out equal, so the corresponding lengths are proportional.
What is an enlargement?	When the scale diagram is larger than the original diagram

What is a reduction?

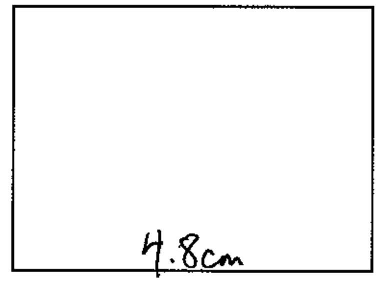
When the scale diagram is <u>smaller</u> than the original diagram.

Ex1
Use a ruler
to determine the scale
factor of the diagram.

*To calculate scale factor, the units you measure each diagram with must be the same.



Original diagram



Scale diagram

$$\frac{4.8}{2.9} = 1.66$$

$1.66 : 1$ or $5 : 3$

Ex2
A scale factor for an enlargement is $\frac{8}{3}$.
If the scale diagram has a length that is 60cm, what is the actual length of the original?

Scale factor : $\frac{\text{length of scale diagram}}{\text{length of original}}$

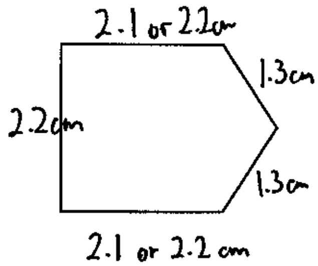
$$\frac{8}{3} = \frac{60}{x}$$

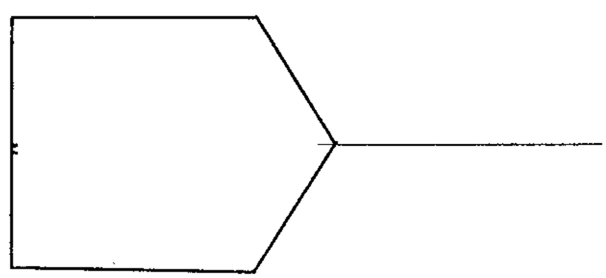
Cross-multiply
multiply the pairs,
divide the spare.

$8x = 60 \times 3$
$8x = 180$
$x = 180 \div 8$
$x = 22.5$

The actual length of the original is 22.5cm.

Ex3
Draw a scale diagram of the drawing. Use a scale factor of 1.5





$2.1 \times 1.5 = 3.2cm$
$2.2 \times 1.5 = 3.3cm$
$1.3 \times 1.5 = 1.95$ or $2cm$

Reflection: When you are given a scale factor, how do you know if it is an enlargement or a reduction. Use a couple examples to help explain.

7.2 – Scale Diagrams and Reductions

Focus: Draw and interpret scale diagrams that represent reductions.

Main Ideas:

Warmup:

What is a scale diagram reduction? Use the terms 'original diagram' and 'scale diagram' in your definition.

A reduction is when the scale diagram is <u>smaller</u> than the original diagram.

When are scale diagram reductions used in society?

- blueprints for buildings/rooms etc.
- maps

How can you tell by the scale factor that your scale diagram is a reduction?

the numerator (top) will be a smaller number than the denominator (bottom) ex. $\frac{1}{3}$

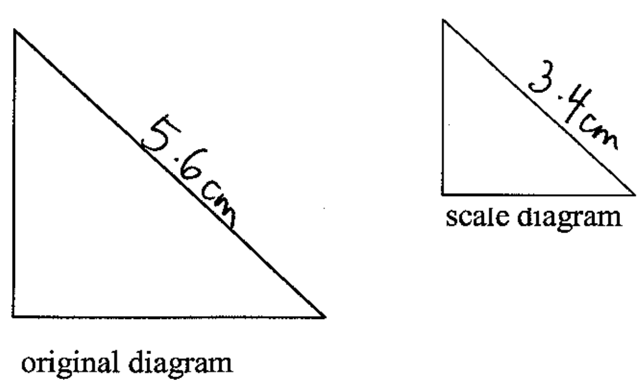
What's another way to write a scale factor?

like a ratio with two dots

scale diagram : original ex 1:3

Ex1

Use a ruler to find the scale factor.



0.61 : 1

$$\text{scale factor} = \frac{3.4}{5.6} = 0.61$$

or $\frac{0.61}{1}$

Ex2
A scale factor for a reduction of a chair is 1 : 8 (same as $\frac{1}{8}$).
If the height of the chair is 75cm, what is the height on the scale drawing?

$$\text{scale factor} : \frac{\text{scale diagram}}{\text{original}}$$

$$\frac{1}{8} = \frac{x}{75}$$

cross-multiply (multiply the pairs, divide the spare)

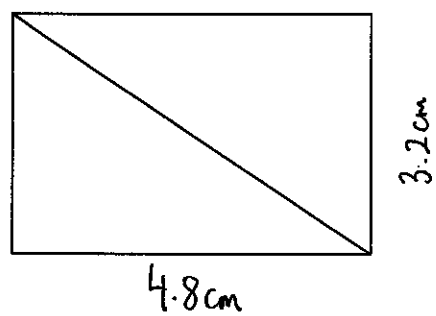
$$8x = 75 \times 1$$

$$x = \frac{75}{8}$$

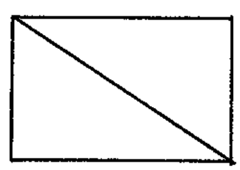
$$x = 9.375$$

The height on the scale drawing is 9.375cm.

Ex3
Draw a scale diagram of the drawing.
Use a scale factor of 0.6.



$4.8 \times 0.6 = 2.88$ or 2.9cm
$3.2 \times 0.6 = 1.92$ or 1.9cm



Reflection: What are two ways to write a scale factor? Explain using examples.

7.3 – Similar Polygons

Focus: Recognize and draw similar polygons, then use their properties to solve problems.

Main Ideas:

Warmup: Look at the right side of p.347. What is alike about the two triangles, and what is different? You should use the word 'angles' and the word 'sides' at some point during your answers.	There are matching sets of angles that are equal. There are matching sets of sides, but they are not equal. The two shapes are the same shape but different size.
What is a polygon?	a polygon is a closed shape that consists of line segments (not curved lines). ex triangle, hexagon etc.
What does similar mean?	when one shape is an enlargement or reduction of another i.e. if two shapes are the same shape but different size
What is a similar Polygon?	Two polygons that have the same shape but different size. (ex on bottom of p.335).
Ex1 Look at the similar polygons on the bottom of p.335. Read the 3 lines at the bottom of the page. a) What do you notice about corresponding angles? b) Can you find a common theme for the corresponding sides?	(a) Corresponding angles on similar polygons are equal. (b) If you make a fraction out of corresponding sides, the fraction is equal to fractions made from other corresponding sides. $\frac{2}{3} = \frac{3}{4.5} = \frac{2.5}{3.75} = \frac{2.5}{3.75} = \frac{1.5}{2.25} = 0.67$

What is another name for the common ratio produced by the corresponding sides?

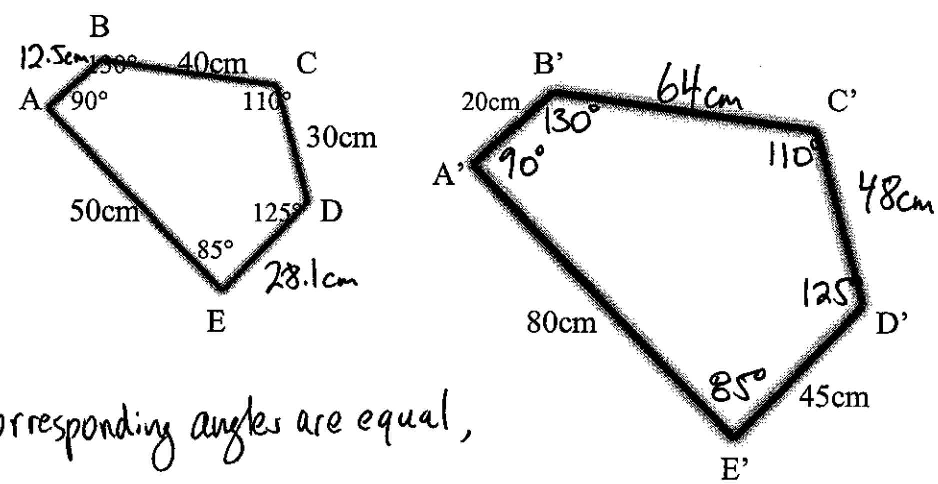
a scale factor (from 7.1/7.2)

What do all similar polygons have in common?

- the corresponding angles are equal
- pairs of corresponding sides have lengths that are proportional.

Ex1
Find the missing sides and angles

Keep the scale factor as a fraction and use cross-multiply.



Corresponding angles are equal,

so $\angle A = \angle A' = 90°$, $\angle B = \angle B' = 130°$

$\angle C = \angle C' = 110°$, $\angle D = \angle D' = 125°$

$\angle E = \angle E' = 85°$

scale factor : $\frac{50}{80}$ reduces to $\frac{5}{8}$

$$\frac{5}{8} = \frac{AB}{20} = \frac{40}{B'C'} = \frac{30}{C'D'} = \frac{ED}{45}$$

to find length AB:

$\frac{5}{8} = \frac{AB}{20}$

$8 \times AB = 5 \times 20$

$AB = \frac{100}{8} = 12.5$

B'C':

$\frac{5}{8} = \frac{40}{B'C'}$

$B'C' = \frac{40 \times 8}{5}$

$B'C' = 64$

C'D':

$\frac{5}{8} = \frac{30}{C'D'}$

$C'D' = \frac{8 \times 30}{5}$

$C'D' = 48$

ED:

$\frac{5}{8} = \frac{ED}{45}$

$ED = \frac{5 \times 45}{8}$

$ED = 28.1$

Scale factor: $\frac{5.4}{8.1}$

Ex2
Do Example 3 on p.339 of the text. Don't look at the solution until completed.

to get GH:

$\frac{5.4}{8.1} = \frac{GH}{32.4}$

$GH = \frac{32.4 \times 5.4}{8.1}$

$GH = \underline{21.6\,m}$

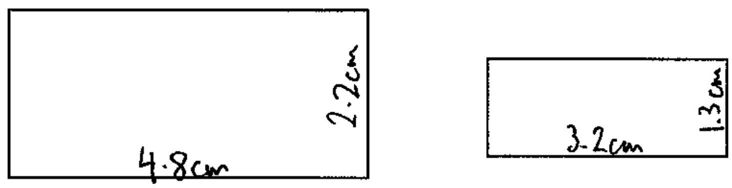
to get NP:

$\frac{5.4}{8.1} = \frac{27.0}{NP}$

$NP = \frac{8.1 \times 27.0}{5.4}$

$NP = \underline{40.5\,m}$

Ex3
Are the polygons similar?
Use your ruler to check so that you can be certain.



Is $\frac{4.8}{3.2}$ equal to $\frac{2.2}{1.3}$?

$\frac{4.8}{3.2} = 1.5$ $\qquad$ $\frac{2.2}{1.3} = 1.7$

No, the polygons are not similar as the ratios of corresponding sides are not equal.

Reflection: How do you know when two polygons are similar? Use proper vocabulary in your explanation.

7.4 – Similar Triangles

Focus: Use the properties of similar triangles to solve problems.

Main Ideas:

Warmup:
What are the characteristics of similar polygons from yesterday?

The measure of the corresponding angles are equal.

AND

Corresponding sides: The fractions created by pairs of corresponding sides are equal.

Both of the above two conditions must be true before we can call two polygons similar.

How is it different for similar triangles?

Triangles are polygons, but they are a special type of polygon. If you know that one of the two conditions is true for two triangles, you can say that the triangles are similar:

The measures of corresponding angles must be equal

OR

The ratios of the lengths of corresponding sides must be equal.

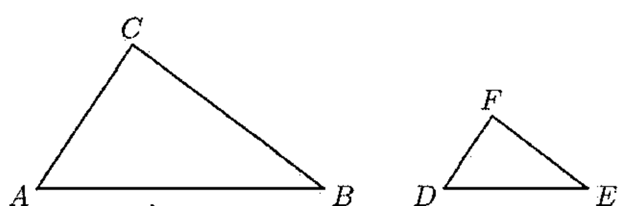
Read 'Connect' on p.344 of the text and note anything you feel is important.

if triangle ABC is similar to triangle DEF, it can be written as

$\triangle ABC \sim \triangle DEF$

'is similar to'

Ex1
The triangles are similar. Write math statements for corresponding angles, corresponding sides, and that the triangles are similar.



angles:

$\angle A = \angle D$

$\angle B = \angle E$

$\angle C = \angle F$

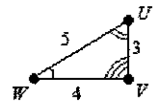
sides:

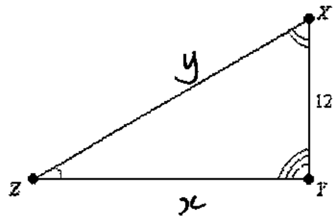
$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

$\triangle ABC \sim \triangle DEF$

- make sure corresponding angles are written in the same position in the statement above

Ex2
Find the missing sides.





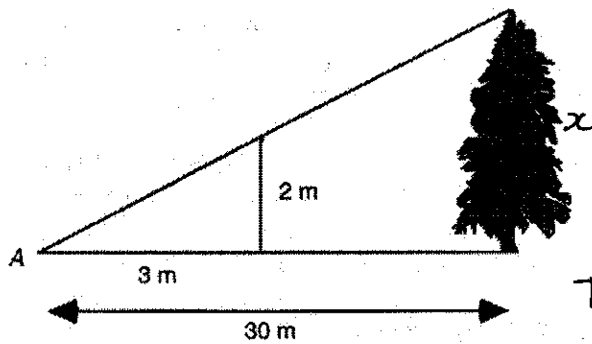
$$\frac{3}{12} = \frac{4}{x} = \frac{5}{y}$$

$$\frac{3}{12} = \frac{4}{x} \qquad \frac{3}{12} = \frac{5}{y}$$

$$x = \frac{12 \times 4}{3} \qquad y = \frac{12 \times 5}{3}$$

$$x = 16 \qquad y = 20$$

Ex3
The two triangles are similar. Find the height of the tree.



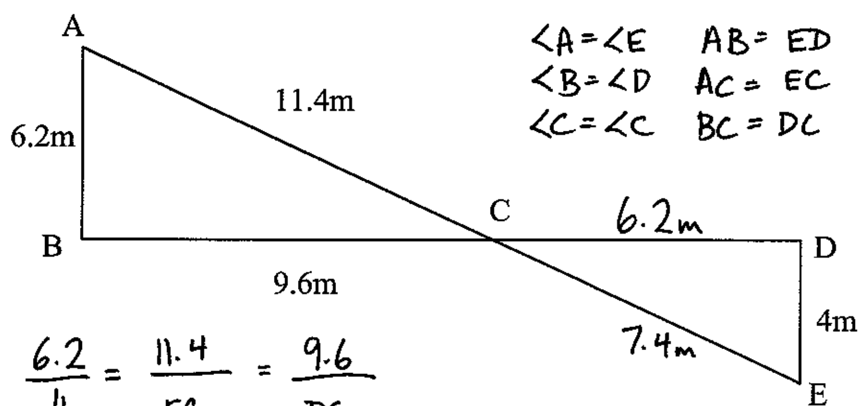
$$\frac{x}{2} = \frac{30}{3}$$

$$x = \frac{2 \times 30}{3}$$

$$x = 20\text{m}$$

The tree is 20m high.

Ex3
The two triangles are similar. Find the missing lengths.



$\angle A = \angle E$ AB = ED
$\angle B = \angle D$ AC = EC
$\angle C = \angle C$ BC = DC

$$\frac{6.2}{4} = \frac{11.4}{EC} = \frac{9.6}{DC}$$

$$\frac{6.2}{4} = \frac{11.4}{EC} \qquad \frac{6.2}{4} = \frac{9.6}{DC}$$

$$EC = \frac{4 \times 11.4}{6.2} = 7.4\text{m} \qquad DC = \frac{9.6 \times 4}{6.2} = 6.2\text{m}$$

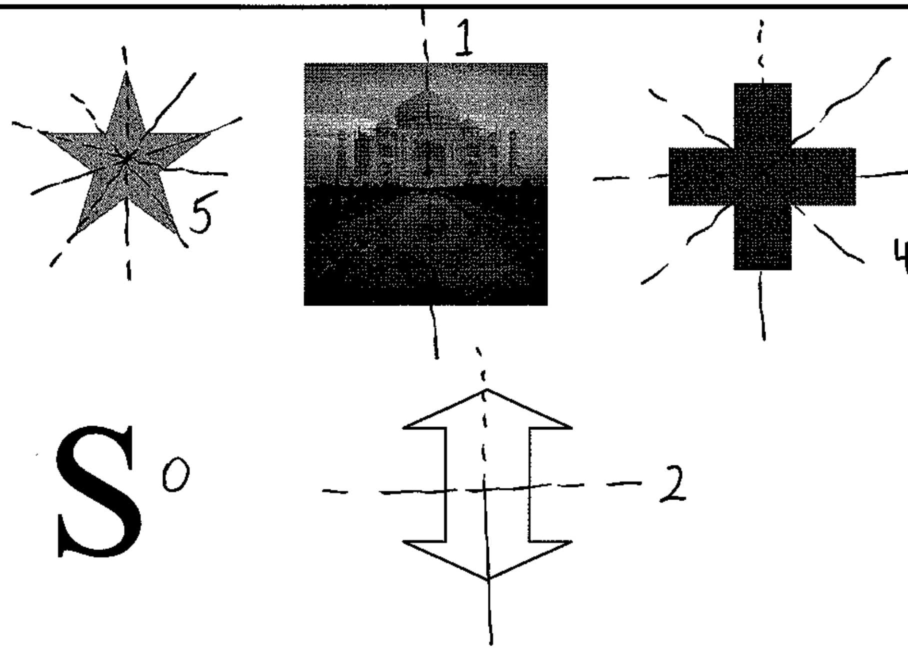
Reflection: How do you go about matching up the corresponding sides for similar triangles?

7.5 – Reflections and Line Symmetry

Focus: Draw and classify shapes with line symmetry

Main Ideas:

Warmup:
Draw any lines of symmetry for the following shapes. How many does each have?



What is another name for a line of symmetry?

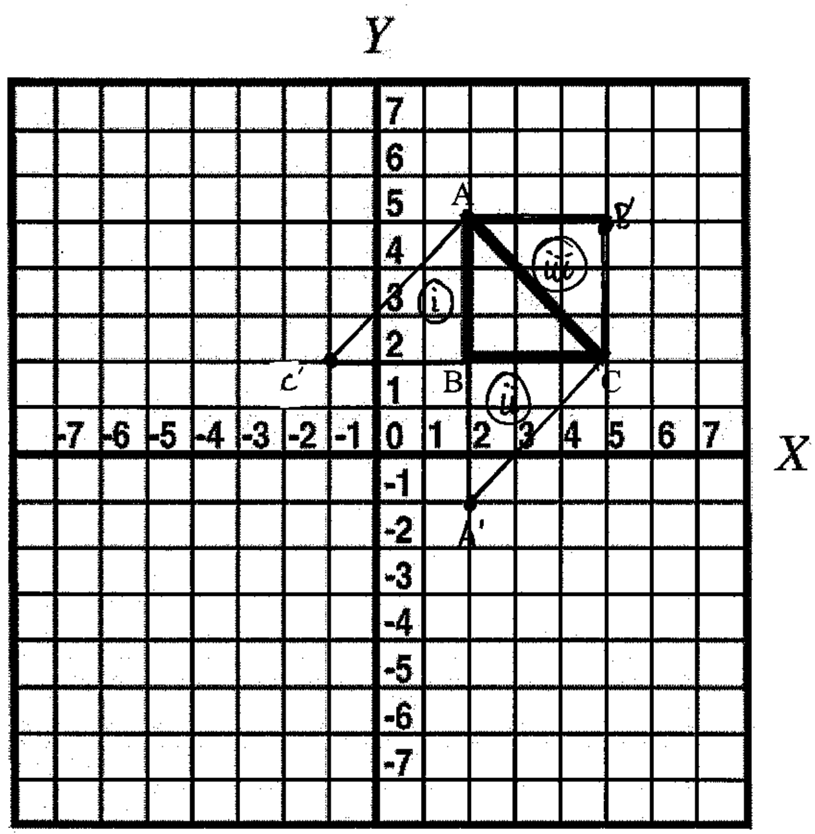
line of reflection or mirror line

Ex1
Go to p.355 in the text and see if you can answer example 2 without looking at the solution.

- A is related by a vertical line of symmetry at $x = 5$
- B is related by a horizontal line of symmetry at $y = 3$
- D is related by a diagonal line of symmetry

Ex2
Draw the reflection image of ΔABC through the following reflection lines:
i) vertical line through 2 on the x-axis
ii) horizontal line through 2 on the y-axis
iii) oblique line AC

Fill out the tables below



i)

Point	Image
A (2,5)	A'(2,5)
B (2,2)	B' (2,2)
C (5,2)	C' (-1,2)

ii)

Point	Image
A (2,5)	A' (2,-1)
B (2,2)	B' (2,2)
C (5,2)	C' (5,2)

iii)

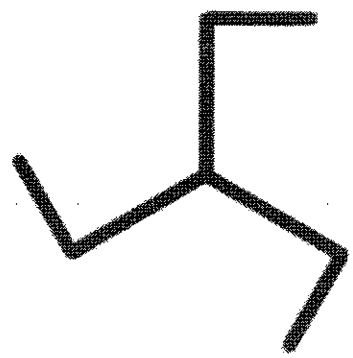
Point	Image
A (2,5)	A' (2,5)
B (2,2)	B' (5,5)
C (5,2)	C' (5,2)

Reflection: How do you identify whether a shape has a line of symmetry?

7.6 – Rotations and Rotational Symmetry

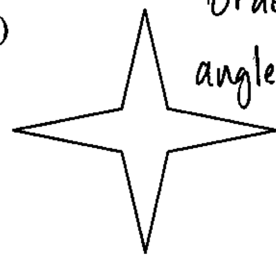
Focus: Draw and classify shapes with rotational symmetry.

Main Ideas:

Warmup: Put your pencil in the Centre of the shape. Rotate the tracing one complete turn about your pencil, counting the number of times the rotation coincides with the original shape, and write this number down.	 3
What is rotational symmetry?	A shape has 'rotational symmetry' when it coincides with itself after a rotation of less than 360° about its centre.
What is 'order of rotation'?	the number of times a shape coincides with itself after a rotation of 360°. For any shape, the order is atleast 1.
What is the 'angle of rotation symmetry'?	$\text{angle of rotation symmetry} = \dfrac{360°}{\text{order of rotation}}$
What shapes do not have rotational symmetry? Give an example.	a shape with order 1 does <u>not</u> have rotational symmetry. So to have rotational symmetry, a shape must coincide with itself at least once in a rotation of <u>less than</u> 360°.
What is a transformation?	a transformation is a translation, rotation, or reflection of an object translation: moving an object without turning or flipping it.

Ex1
Determine if the shape has rotational symmetry, and if it does, state the order of rotation and the angle of rotation symmetry.

a)

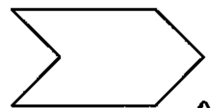


Order : 4
angle : $\frac{360}{4} = 90°$

b)

order: 2
angle : $\frac{360}{2} = 180°$

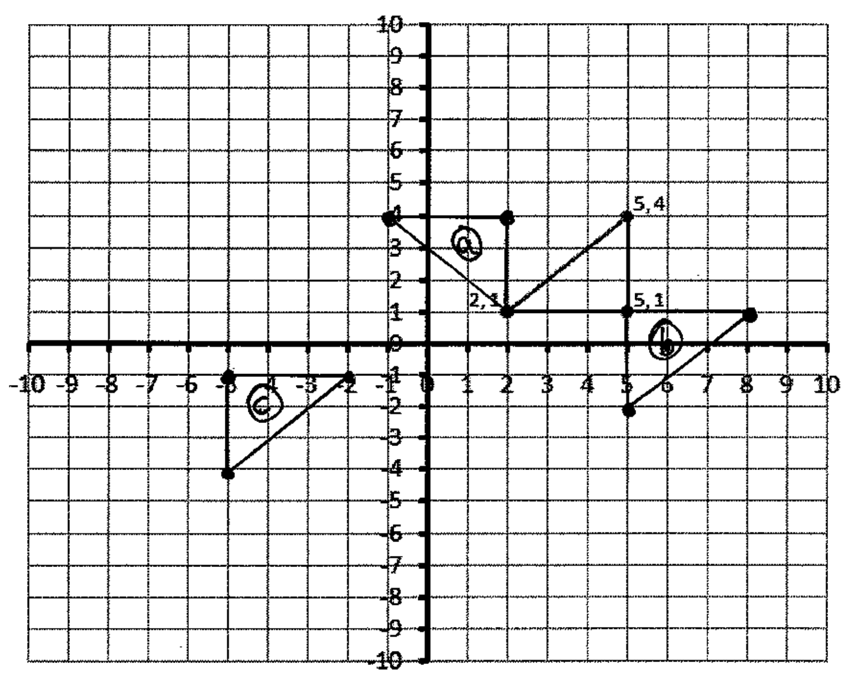
c)



order : 1
angle : $\frac{360}{1} = 360°$

Sometimes, the point of rotation is not the middle of the shape.

Ex2
a) Rotate the triangle 270° clockwise about the point (2, 1)
b) Rotate the triangle 180° clockwise about the point (5, 1)
c) 180° counter clockwise about the origin



Reflection: When you are being asked to rotate an image, what specifics must you first know?

7.7 – Identifying Types of Symmetry on the Cartesian Plane

Focus: Identify and classify line and rotational symmetry.

Main Ideas:

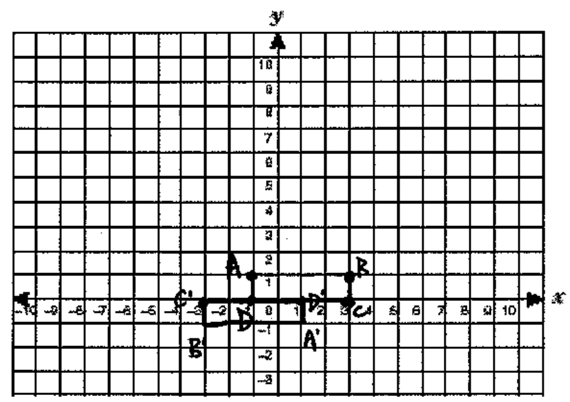
Warmup:
Look at Example 1 on p.369 (don't look at solutions).
For each of a, b, and c, determine whether they are related by line symmetry, rotational symmetry, or both.
Then give specifics about their symmetry.

a) rotational symmetry: 180° clockwise about the point (0, 3)

b) line symmetry: reflected through y = 0 line (x-axis)
rotational symmetry: 180° about the point (-2.5, 0)

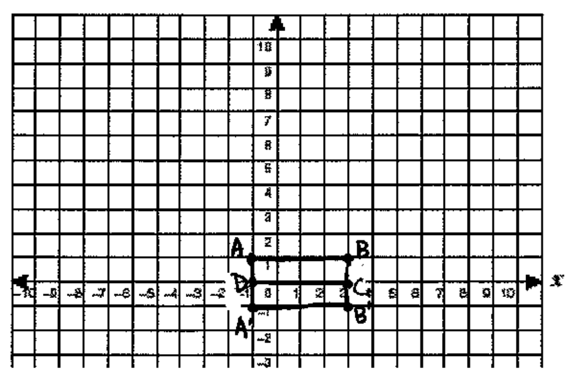
c) line symm: reflected through diagonal line
rotational symm: 90° clock or counterclock about the point (-5, 4)

Ex1
Cover p.371, and do example 2 on p.370 by first copying the image onto the grid.



a)

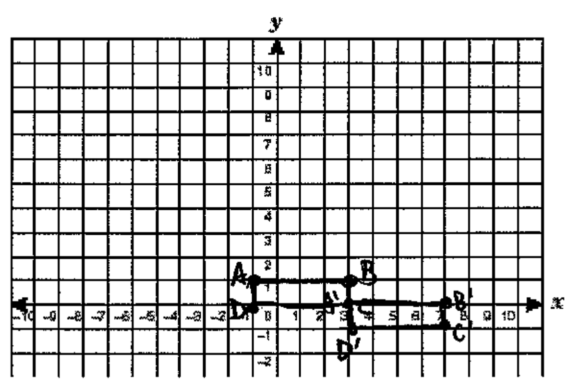
Point	Image
A(-1,1)	A' (1,-1)
B(3,1)	B' (-3,-1)
C(3,0)	C' (-3,0)
D(-1,0)	D' (1,0)



b)

Point	Image
A(-1,1)	A' (-1,-1)
B(3,1)	B' (3,-1)
C(3,0)	C' (3,0)
D(-1,0)	D' (-1,0)

For part c, a translation means to move each corner of the shape by what is indicated.



c)

Point	Image
A(-1,1)	A'(3,0)
B(3,1)	B'(7,0)
C(3,0)	C'(7,-1)
D(-1,0)	D'(3,-1)

For part c in the last example, what is another way to write the translation 4 units right and 1 unit down. What's another way to write 3 units left and 5 units up?

R4, D1

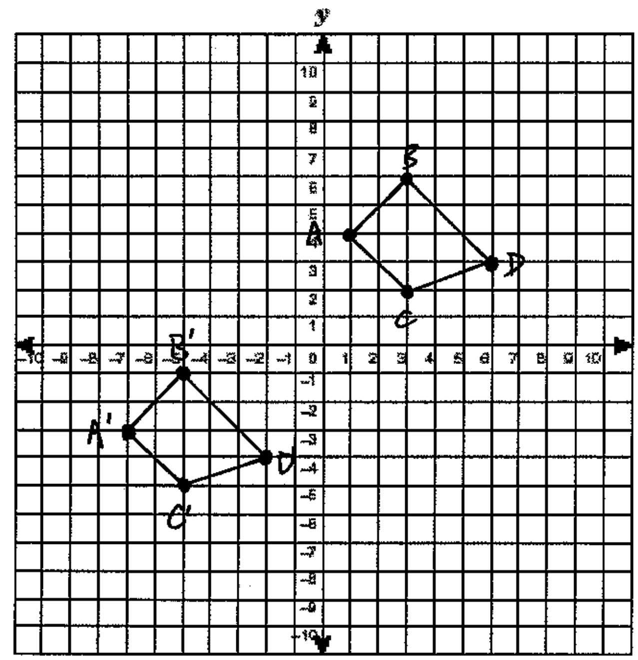
L3, U1

Ex3

a) Draw the following shape on the grid: A(1, 4), B(3, 6), C(3, 2), D(6, 3)

b) Translate the shape by L8, D7 and redraw and complete the table

c) Is there any line or rotational symmetry in the completed diagram?



Point	Image
A(1,4)	A' (-7,3)
B(3,6)	B' (-5,-1)
C(3,2)	C' (-5,-5)
D(6,3)	D' (-2,-4)

(c) no – neither.

Reflection: What do you have to practice most about transformations (translations, reflections, and rotations)? Explain.