Name: Notes Key
Date: _____

CHAPTER 8 NOTES – Circle Geometry

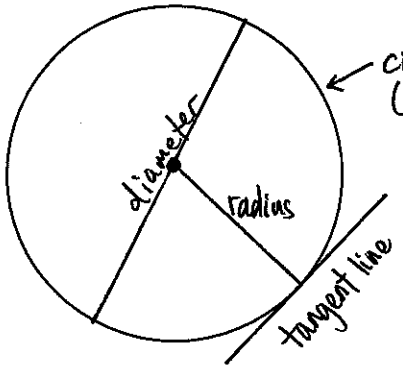
	Date:
Geometry Review – a review to help you with chapter 8	_____
8.1 – Properties of Tangents to a Circle	_____
8.2 – Properties of Chords in a Circle	_____
8.3 – Properties of Angles in a Circle	_____
8.4 – Using Circle Properties to Solve Problems	_____
Review:	_____
Test:	_____

What You'll Learn:

8.1 – Circle properties that relate a tangent to a circle and the radius of the circle
8.2 – Circle properties that relate a chord in a circle, its perpendicular bisector, and the centre of the circle
8.3 – Circle properties that relate the measures of angles in circles
8.4 – Problem solving using circle properties

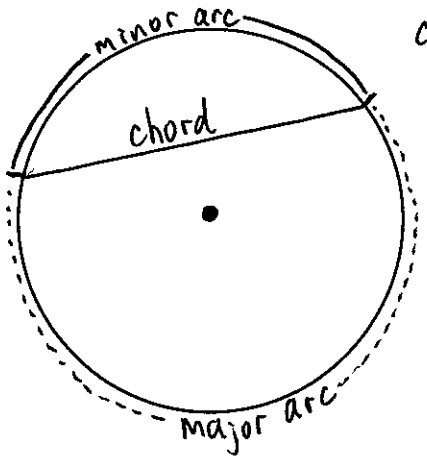
On the circle below, draw and label a circumference, radius, diameter, and tangent line.
What is a tangent line?

diameter = 2 x radius



A tangent line is a line that touches the circle only once.

On the circle below, draw and label a chord and describe what a chord is in words. Also, draw and label a minor arc and major arc.

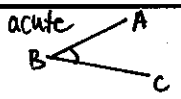
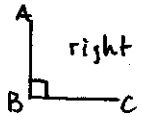
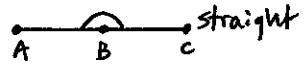
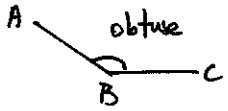
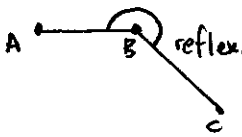
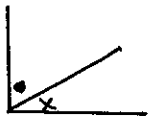
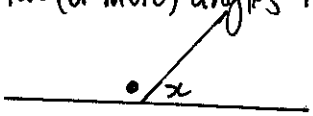
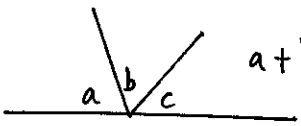
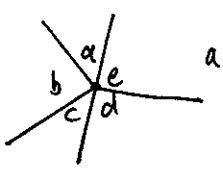


chord: a line segment that joins two points on a circle.

Geometry Review

Focus: Review basic geometry concepts and vocabulary from previous grades.

Main Ideas:

What is the difference between acute, right, obtuse, straight, and reflex angles? Draw each with capital letters and explain.	acute angle: less than 90°   right angle: 90° obtuse angle: greater than 90° but less than 180° straight angle: 180°   reflex angle: greater than 180° 
What are 'vertically opposite' angles and what is their relationship?	when two lines cross, vertically opposite angles are the angles opposite each other, and have equal measure. 
What does 'congruent' mean?	congruent means 'the same' or 'equal'
What are 'complementary' angles?	any two (or more) angles that add to 90° are complementary  • and x are complementary.
What are 'supplementary' angles?	any two (or more) angles that add to 180° are supplementary  • and x are supplementary.
What do 'angles on a line' add to?	'angles on a line' are supplementary (add to 180°)  a+b+c = 180°
What do 'angles at a point' add to?	'angles at a point' add to 360°  a+b+c+d+e = 360°

What do the 3 angles in any triangle add to?

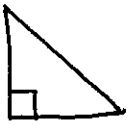
180°

What is an equilateral triangle?

a triangle that has 3 equal length sides and 3 equal angles. Each angle = $\frac{180}{3} = 60°$.

What is a right triangle, and if you know two sides, how do you calculate the 3rd side?

A right triangle is a triangle that has a right (90°) angle



If you know 2 sides, you can calculate the 3rd side using Pythagoras' Theorem

What is an isosceles triangle and what are its characteristics?

An isosceles triangle has two equal length sides and two equal angles.

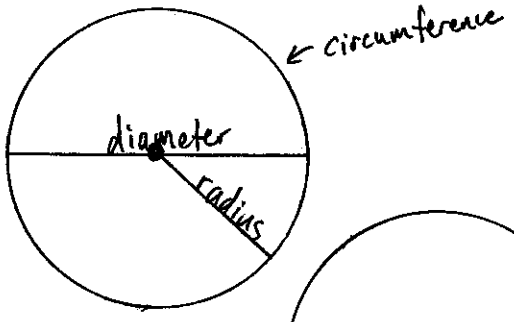
What is a name for any 4-sided object?

quadrilateral.

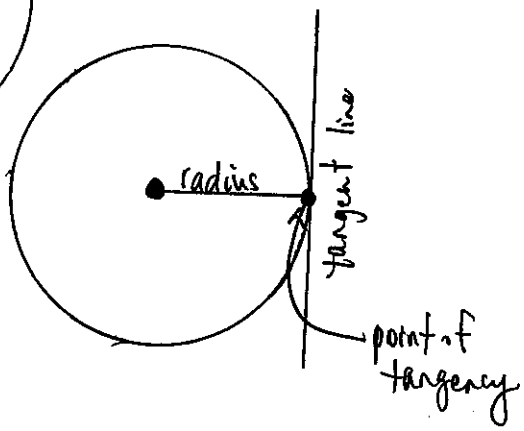
What is an important characteristic for any quadrilateral?

all angles in a quadrilateral add to 360°

Draw a circle and label the circumference, radius, and diameter



Draw a circle and a tangent line. Label the tangent line. Draw a radius to the tangent line and label the 'point of tangency'.



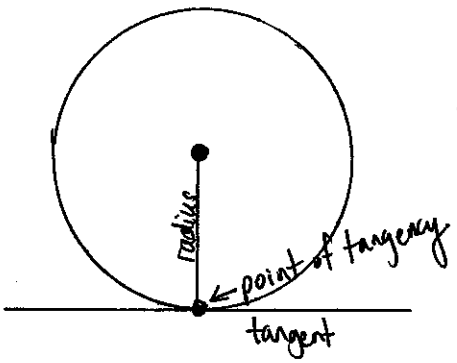
Reflection: What part of this review do you need to spend some extra time on and why?

8.1 – Properties of Tangents to a Circle

Focus: Discover the relationship between a tangent and a radius, then solve related problems.

Main Ideas:

Warmup:
Look at the picture at the top of p.384 and read the statements and question below the picture. Draw a diagram of the bike wheel and label the tangent, radius and point of tangency. Answer the question from the text as well.

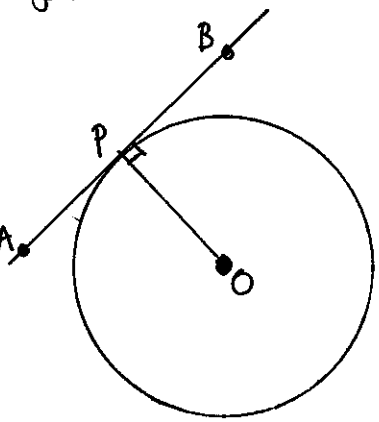


The red spoke appears to make a 90° angle with the ground.

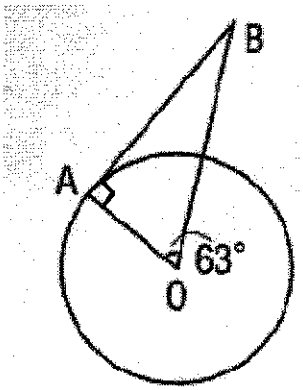
Read over 'Connect' on p.385 and note anything of importance.

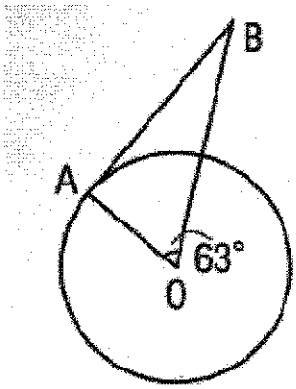
What is the Tangent-Radius Property?

A tangent to a circle is perpendicular to the radius at the point of tangency.
That is, $\angle APO = \angle BPO = 90°$



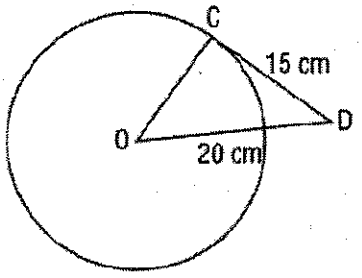
Ex1
Point O is the centre of a circle and AB is a tangent to the circle.
In ΔOAB, $\angle AOB = 63°$
Determine the measure of $\angle OBA$.

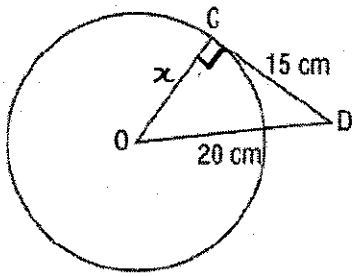




$\angle OAB = 90°$
as it's a point of tangency
So $\angle OBA = 180 - 90 - 63$
$= 27°$

Ex2
Point O is the centre of a circle and CD is a tangent to the circle.
CD = 15cm and
OD = 20cm
Determine the length of the radius OC. Give the answer to the nearest tenth.





$\angle OCD = 90°$ as it's a point of tangency

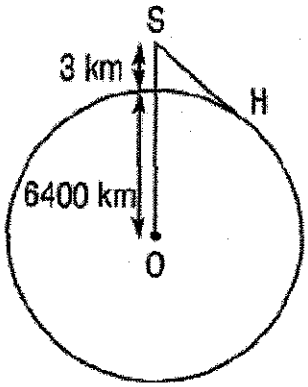
use Pythag. $x^2 + 15^2 = 20^2$

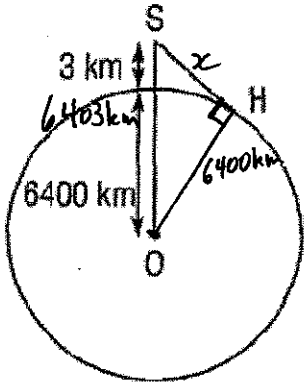
$x^2 + 225 = 400$

$x^2 = 175$

$x = \sqrt{175} = 13.2\text{cm}$

Ex3
A skydiver, S, jumps from a plane at an altitude of 3km. The radius of Earth is approx. 6400km. How far is the horizon, H, from the skydiver when she leaves the plane? Calculate this distance to the nearest kilometre.





use Pythag: $x^2 + 6400^2 = 6403^2$

$x^2 + 40\,960\,000 = 40\,998\,409$

$x^2 = 38\,409$

$x = \sqrt{38\,409} = 196\text{ km}$

The skydiver is 196 km from the horizon.

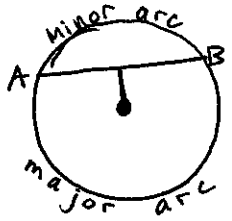
Reflection: The Pythagorean Theorem was used in examples 2 & 3. When is the Pythagorean Theorem useful for solving problems involving tangents?

8.2 – Properties of Chords in a Circle

Focus: Relate a chord, its perpendicular bisector, and the centre of the circle, then solve problems.

Main Ideas:

Warmup:
Draw a circle freehand. Label the centre O. Draw a chord AB in your circle. Label the minor arc and the major arc.
a) Draw a line from O to the halfway point of your chord.
b) What kind of angle do you make?



(b) a 90° angle

What is a chord?

a line that joins two points on a circle

What is a diameter?

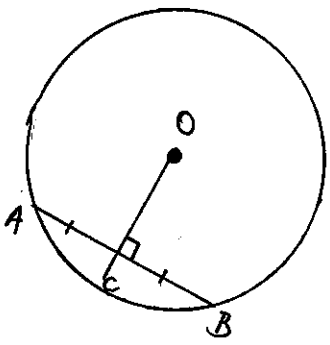
a chord that passes through the centre of the circle.

What is a perpendicular bisector?

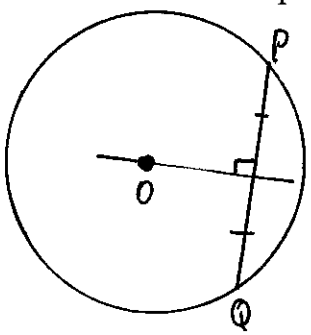
A line that intersects a chord at 90° and divides the chord into two equal parts.

What are the three Chord Properties?

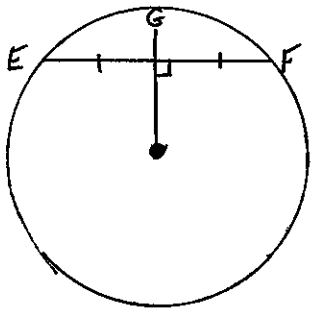
1. The perpendicular from the centre of a circle to a chord bisects the chord; that is, the perpendicular divides the chord into two equal parts.



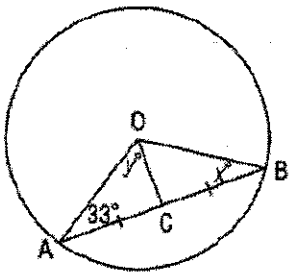
2. The perpendicular bisector of a chord in a circle passes through the centre of the circle.

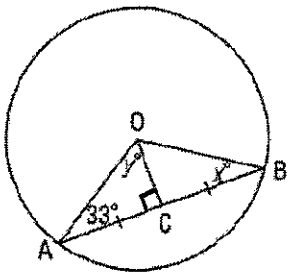


3. A line that joins the centre of a circle and the midpoint of a chord is perpendicular to the chord.



Ex1
Point O is the centre of a circle, and line segment OC bisects chord AB. $\angle OAC = 33°$
Determine the values of $x°$ and $y°$.





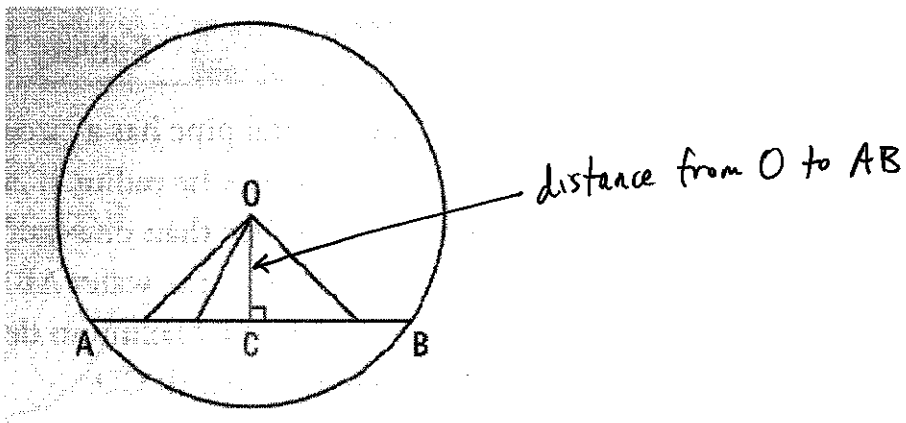
$\angle OCA = 90°$ due to Perpendicular to Chord Properties so

$y° = 180 - 90 - 33 = 57°$

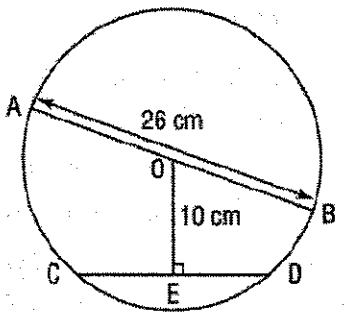
$\triangle OAC$ same as $\triangle OBC$ so $x° = 33°$

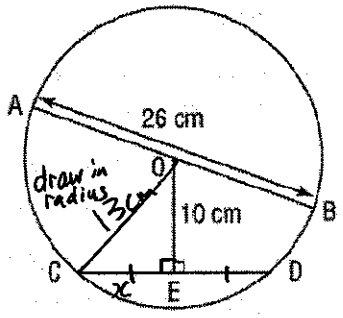
An important point to consider:

Many line segments can be drawn from O, the centre of a circle, to a chord AB. The *distance* from O to AB is defined as the shortest distance. This distance is the length of the perpendicular from O to AB; that is, the length of OC.



Ex2
Point O is the centre of the circle. AB is a diameter with length 26cm. CD is a chord that is 10cm from the centre of the circle. What is the length of chord CD? Give the answer to the nearest tenth.





- if diameter = 26cm, radius = 26 ÷ 2 = 13cm
- OC is a radius, so OC = 13cm
- $\angle OEC = 90°$
- use Pythag to find length CE, call it x: $x^2 + 10^2 = 13^2$

$x^2 + 100 = 169$

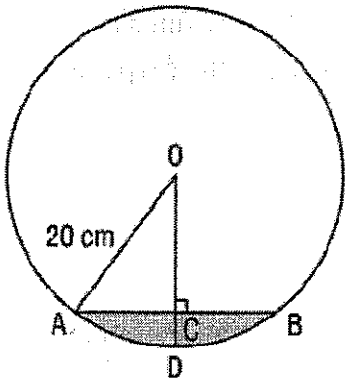
$x^2 = 69$, so $x = \sqrt{69}$

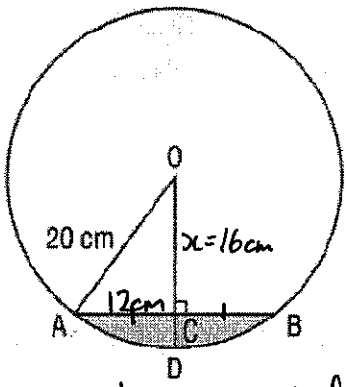
$x = 8.3$ cm

- CD is bisected by OE, so CD is exactly double CE,

so CD = 8.3 × 2 = 16.6 cm

Ex3
A horizontal pipe has a circular cross-section, with centre O. Its radius is 20cm. Water fills less than one-half of the pipe. The surface of the water AB is 24cm wide. Determine the maximum depth of the water, which is the depth CD.





if AB = 24cm, AC is half at 12cm. Use pythag to find OC, call it x: $x^2 + 12^2 = 20^2$

$x^2 + 144 = 400$

$x^2 = 256$

$x = \sqrt{256}$

$x = 16$ cm

OD is a radius = 20cm

OC = 16cm

So CD = 20 − 16 = 4cm

The water depth is 4cm.

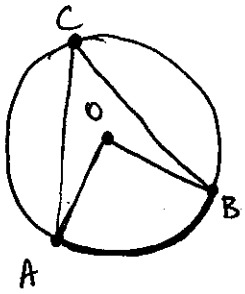
Reflection: What is the relationship between the centre of a circle, a chord, and the perpendicular bisector of the chord?

8.3 – Properties of Angles in a Circle

Focus: Discover the properties of inscribed angles and central angles, then solve related problems.

Main Ideas:

Warmup:
Draw a circle freehand with centre O. Label point A and point B on the circle.
a) Draw ∠AOB. Use the 'Connect' on p.405 to name this angle.
b) What arc is ∠AOB 'subtended off of? Make this arc thicker.
c) Label a point C on your circle so that is looks somewhat like the diagram on the bottom of p.404. Draw ∠ACB and name the angle (using 'Connect' for assistance).
d) What arc is ∠ACB subtended off of?



(a) ∠AOB is a central angle
(b) minor arc AB
(c) ∠ACB is an inscribed angle
(d) minor arc AB.

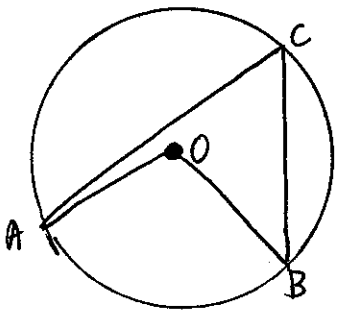
Central angle: the angle formed by joining the endpoints of an arc to the centre of the circle.

Inscribed angle: the angle formed by joining the endpoints of an arc to a point on the circle

Make accurate statements for central angle and inscribed angle using the warmup above.

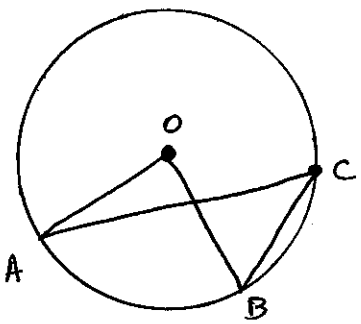
The 'Central Angle and Inscribed Angle Property':

In a circle, the measure of a central angle subtended by an arc is twice the measure of an inscribed angle subtended by the same arc.



∠AOB = 2 × ∠ACB

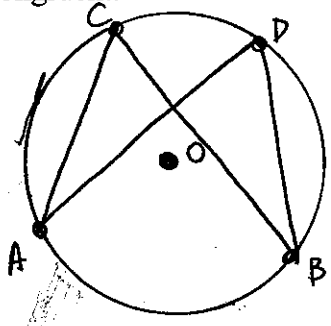
OR



∠AOB = 2 × ∠ACB

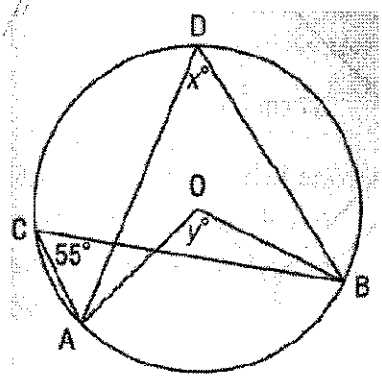
The 'Inscribed Angles Property' (bowtie):

In a circle, all inscribed angles subtended by the same arc are congruent.

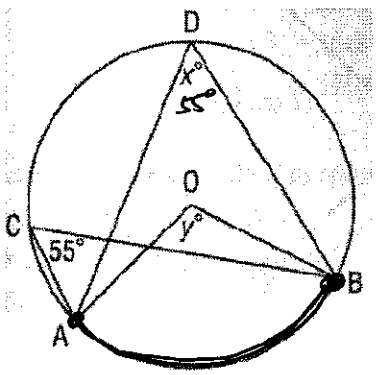


$\angle ACB = \angle ADB$

Ex1
Point O is the centre of a circle. Determine the values of $x°$ and $y°$.

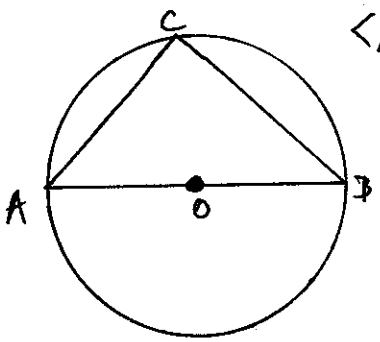


$\angle AOB = 2 \times \angle ACB$ due to 'Central Angle and Inscribed Angle Property', so $y = 110°$



$\angle ACB = \angle ADB$ due to 'Inscribed Angles Property' so $x° = 55°$

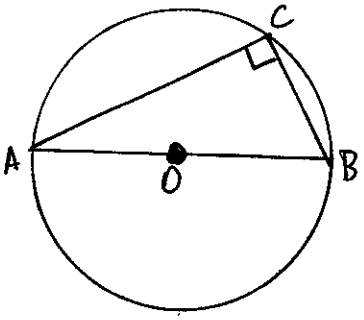
What if the central angle is 180°? Draw this freehand. Now draw an inscribed angle subtended off of the same arc and find its measure.



$\angle AOB = 2 \times \angle ACB$
if $\angle AOB = 180°$,
$\angle ACB = 90°$

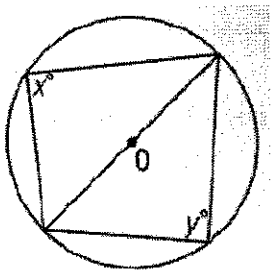
The 'Angles in a Semicircle Property':

All inscribed angles subtended by a semicircle are right angles. Therefore, the angle inscribed in a semicircle is a right angle.

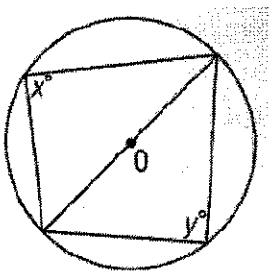


$\angle ACB = 90°$

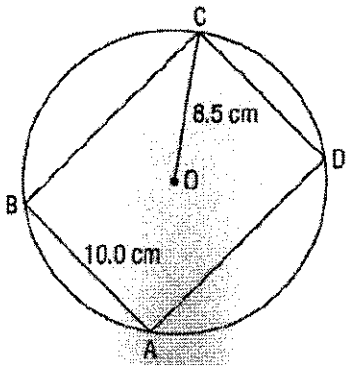
Ex2
Find $x°$ and $y°$

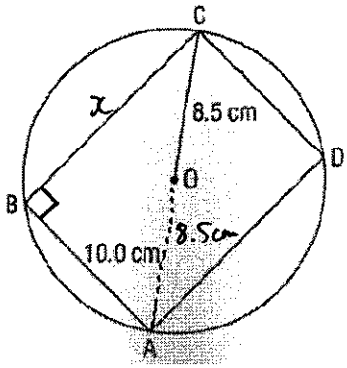


$x° = 90°$
$y° = 90°$



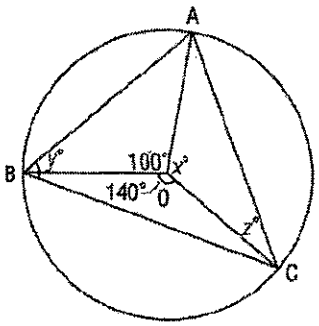
Ex3
Rectangle ABCD has its vertices on a circle with radius 8.5cm. The width of the rectangle is 10.0cm. What is its length to the nearest tenth?

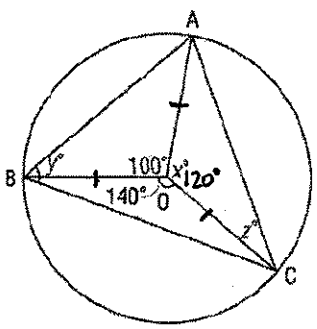




use pythag: $x^2 + 10^2 = 17^2$
$x^2 + 100 = 289$
$x^2 = 189$, $x = \sqrt{189}$
$x = 13.7\text{cm}$

Ex4
Determine the values of $x°$, $y°$, and $z°$.





$x° = 120°$ (angles at a point = 360)
$y° = 40°$ ($\triangle AOB$ is isosceles)
$z° = 30°$ ($\triangle AOC$ is isosceles)

Reflection: What do you need to work on most in this section? Explain.

8.4 – Using Circle Properties to Solve Problems

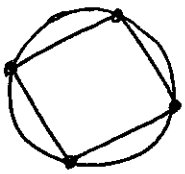
Focus: Solve problems involving applications of one or more of the circle properties.

Main Ideas:

Warmup:
Draw a quadrilateral. What do you know about its angles?

all angles add to 360°

Draw an 'inscribed' quadrilateral.

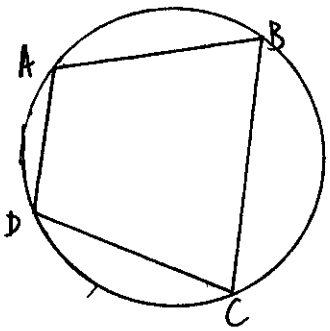


An 'inscribed' quadrilateral has all four corners on a circle.

Property for an Inscribed Quadrilateral:

The opposite angles of an inscribed quadrilateral add to 180°, which means they are supplementary.

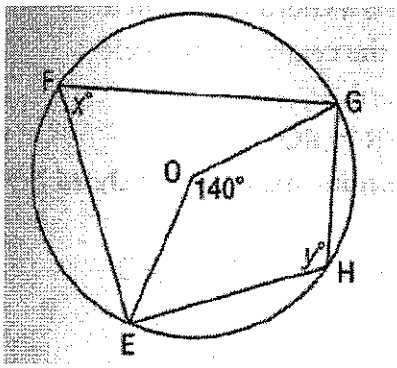
Draw a diagram to support the property:

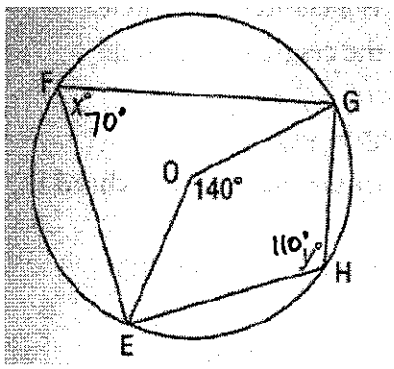


$\angle A + \angle C = 180°$

$\angle B + \angle D = 180°$

Ex1
Determine the values of $x°$ and $y°$.

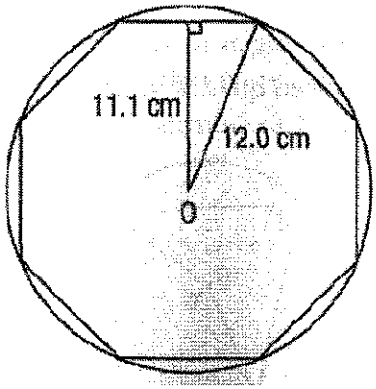


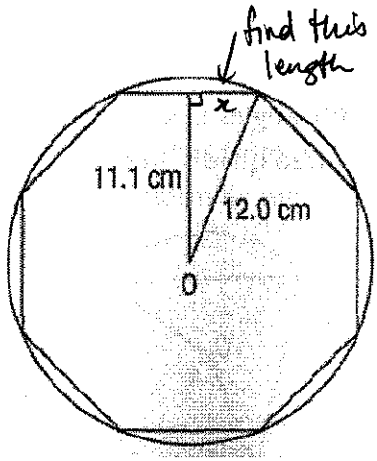


$x° =$ half of $140° = 70°$

$y = 180 - 70° = 110°$

Ex2
A regular octagon is inscribed in a circle with radius 12.0cm and centre O. What is the perimeter of the octagon to the nearest tenth?





use pythag: $x^2 + 11.1^2 = 12^2$

$x^2 + 123.21 = 144$

$x^2 = 20.79$

$x = \sqrt{20.79}$

$x = 4.56\text{cm}.$

An octagon's perimeter has 16 4.56cm lengths

$16 \times 4.56 = 73.0\text{ cm}$

The perimeter of the octagon is 73.0cm

Reflection: Sketch and label an inscribed quadrilateral. Explain what you know about its angles.