## ADLC <br> Mathematics 9-4 KEY



Therne 2a P@ttrins, Formules, and Problem Solving

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## Patterns, Formulas, and Problem Solving

In this theme, you will learn how to identify patterns and then make predictions and draw conclusions based on those patterns. As part of this process, you will also learn how to create formulas for patterns. You will then turn your attention to word problems and learn about and practise strategies for solving these.

This theme includes four lessons:


- Lesson 7: Patterns, Predictions, and Conclusions
- Lesson 8: Using Formulas
- Lesson 9: Strategies for Word Problems
- Lesson 10: More Strategies to Solve Word Problems


## Lesson 7: Patterns, Predictions, and Conclusions

In this lesson, you will

- learn how to identify a pattern and make predictions and conclusions based on the pattern
- create formulas to represent a pattern



## Lesson 7: Activity 1 - Identifying Patterns



Have a look at the picture on the left.
Each season follows a pattern.
It shows the same tree in each of the four seasons. Each year, the tree may look similar in each of the seasons. We could say that the seasons follow a pattern.

Patterns exist in nature and in everyday life.
Patterns occur over and over again. Patterns show relationships.

Can you think of some examples of patterns that occur in everyday life?
Remember, it must be something that happens over and over again. (You may recall some examples of patterns that were given in the video you watched at the beginning of this course.)

Some examples may include

- construction materials, everything from walls to roadways; in walls, roofs, and doors
- days of the week or months of the year
- growth of plants

These are just a few examples; you may have thought of different ones.

You will find a lot of patterns in math as well. When you have a number pattern in math, it means that the numbers in a group follow a specific rule that allows one number to work with the next number. This may sound complicated, but it isn't.

If you have the numbers $2,4,6,8$, you have a pattern. This is a pattern because you add 2 to each number to get the next number. Knowing that, you can figure out what the next number is. All you have to do is add 2 to the last number.
$8+2=10$
The next number in the pattern is 10 .
What would be the next number in the pattern? Again, add 2 to the last number. (The next number would be 12.)

It is important to note that not all groups of numbers follow a pattern. For example, the numbers $1,3,9,10,11$ follow no pattern. However, it is important to look and see if you can find a pattern in a group of numbers.

It may involve adding, subtracting, multiplying, or dividing to get the next number in the pattern. You have to try each method before you can decide whether you have a pattern or not.


Watch this video on different types of patterns. Please type this link in your browser:

## http://quick.adlc.ca/algebc564

There are three things that can help you to identify the type of pattern that was used.

1. See if you can subtract each pair of consecutive numbers and get the same answer. If you can, you have a simple addition pattern. This is also known as an arithmetic sequence.

Example: If you have the pattern $5,10,15$, 20 and want to know the next number in the pattern, you check like this:
$10-5=5$
$15-10=5$
$20-15=5$


Notice that the difference between each group of numbers is 5 . That makes this pattern an arithmetic sequence. To find the next number you simply add $5+20=\mathbf{2 5}$.
2. If you do not have a consistent difference between the numbers, see if there is a pattern to the differences.

Example: If you have the pattern 2, 4, 7, 11 and want to know the next number in the pattern, you check like this:
$4-2=2$
$7-4=3$
$11-7=4$

Notice that the difference between each group of numbers increases by 1. This means that the next number has to be 5 more than the last number. $11+5=16$
3. If neither of the above patterns is visible, try dividing each group of numbers. If you come up with a constant answer, you know you have a geometric sequence.
geometric sequence:

- a sequence made by multiplying by the same value each time (e.g., 2, 4, 8, 16, 32, 64, 128each number is two times the number before it)

Example: If you have the pattern 2, 4, 8, 16 and want to know the next number in the pattern, you check like this:
$4 \div 2=2$
$8 \div 4=2$
$16 \div 8=2$
Notice that the answer is 2 each time. This means that to get the next number in the pattern, you have to multiply the last number by 2.
$16 \times 2=32$

## Lesson 7: Activity 1 - Self-Check



In this self-check activity, you will identify patterns by looking at a student's school schedule.

| Period | Day 01 | Day 02 | Day 03 | Day 04 | Day 05 | Day 06 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | Math | Social | Math |  | Math | Social |
| 2 | Lang. Arts | Lang. Arts | Science | Science | Lang. Arts |  |
| 3 | Science |  | Lang. Arts |  | Science | Science |
| 4 | Foods | Drama |  | Drama |  |  |
| 5 |  | Foods | Drama | Foods | Drama | Foods |

1. Look at the Period 1 classes for each day. Which subject is missing from Day 4 ? (Math and Social form a pattern.)
A. Math
B. Social
C. Science
2. Look at the Period 2 classes for each day. Which class is missing on Day 6 ?
A. Math
B. Science
C. Lang. Arts
3. Look at the Period 3 classes for each day. Which subject is missing from Day 2 ?
A. Math
B. Science
C. Lang. Arts
4. Look at the Period 4 classes for each day. Which subject is missing from Day 3 and Day 5?
A. Foods
B. Drama
C. Social
5. Look at the Period 5 classes for each day. Which subject is missing from Day 1?
A. Math
B. Drama
C. Foods
g 'G
$\forall$ 't
g ' $\varepsilon$
О 'Z
g 'b

## Lesson 7: Activity 1 - Assignment

In this assignment, identify the next number in the arithmetic and geometric sequences shown.

For the first two questions (arithmetic sequences), identify the next number in the pattern.
A 1. 6, 12, 18, 24, 30,
A. 36
B. 34
C. 38

B 2. $12,15,18,21,24$, $\qquad$
A. 26
B. 27
C. 30

For the next two questions (geometric sequences), identify the next number in the pattern.
$\qquad$ 3. $3,6,12,24$, $\qquad$
A. 48
B. 50
C. 30
$\qquad$ 4. $1,3,9,27$, $\qquad$
A. 81
B. 80
C. 36


Marking Guide: This assignment is worth 4 marks.

## Lesson 7: Activity 2 - Making Predictions and Drawing Conclusions from Patterns

Now that you have practised solving patterns that involve numbers that come at the end of patterns, you will look at numbers missing within patterns.


Here is an arithmetic sequence like the ones you solved in Activity 1.
Sometimes you will have only part of a pattern and be missing some of the middle terms of the pattern. You have to use the information you have been given to figure out the pattern and then use that information to fill in the blanks.


To find the missing term in either an arithmetic sequence or a geometric sequence, you have to find the pattern. Once you have done that, you can use the pattern to find the missing value.

Example 1: If you had the pattern 3, 6, 9, _, 15, 18 and wanted to know the missing number, you would have to follow the steps from the previous lesson to find out what kind of a pattern you had. Start by subtracting to see if you have a common difference.
$6-3=3$
$9-6=3$
$18-15=3$
You have a common difference of 3 , so you have an arithmetic sequence. To find the missing number, all you have to do is add 3 to 9 . You can also find the missing number by subtracting 3 from 15.
$9+3=12$
$15-3=12$
The missing number in the pattern is 12.

Example 2: What about the pattern 2, 4, 8, 16, _, 64?
$4-2=2$
$8-4=4$
$16-8=8$
There is no common difference between the numbers. Nor is there an increase in the difference that is constant. Try dividing the numbers to see if the answers are the same.
$4 \div 2=2$
$8 \div 4=2$
$16 \div 8=2$
You get the same answer when you divide consecutive numbers, so you have a geometric sequence. In order to find the missing number, simply multiply 16 by 2. You can also find the missing number by dividing 2 into 64.
$16 \times 2=32$
$64 \div 2=32$
The missing number is 32 .

Example 3: Find the missing number in this pattern:
3, 6, $\qquad$ , 24, 48
$6 \div 3=2$
$48 \div 24=2$
$6 \times 2=12$ or $24 \div 12=2$
The missing number is 12.

You can also make predictions using patterns. A prediction is a statement about something you don't know for sure. You are making an educated guess as to what will happen.

If you use the pattern from the second example above ( $2,4,8,16$, , 64) and want to figure out the next number in the pattern, you can predict that it will be 2 times as much as 64 .

You can make a prediction based on the pattern that you see.


What about if you were given the information that Person A travelled 100 kilometres in 1 hour and 200 kilometres in 2 hours and were asked to predict how far they would go in 6 hours?

The first thing you should notice is that they are travelling 100 kilometres per hour. In order to predict how far they would go in 6 hours, you would simply multiply 100 by 6.
$100 \times 6=600$

You would predict that they will travel 600 kilometres in 6 hours.
What about conclusions? A conclusion is a decision about what is occurring. You had to make a conclusion in the above example before you could answer the question. You had to conclude that Person A was travelling consistently at $100 \mathrm{~km} / \mathrm{h}$ before you could go on.


Conclusions can also be made using a whole set of numbers. For example, let's say that 1 person will eat 2 pieces of pizza; 2 people will eat 4 pieces of pizza, and 3 people will eat 6 pieces of pizza. From this information, you can conclude two things: that each person is eating 2 pieces of pizza and that each additional person will eat 2 pieces of pizza as well.

Taking that a step further, if you were asked how many pieces of pizza 10 people would eat, you could quickly multiply 10 people by 2 pieces of pizza.
$10 \times 2=20$

You can conclude that 10 people will eat 20 pieces of pizza.


## Lesson 7: Activity 2 - Self-Check

In this self-check, you will find the missing terms in patterns.
$\qquad$ 1. $1,2,4$, $\qquad$ , 11
A. 7
B. 6
C. 8
$\qquad$ 2. 11,22 , $\qquad$ 88
A. 43
b. 44
C. 34
$\qquad$ 3. 9,12 , $\qquad$ 18
A. 15
B. 16
C. 13
$\qquad$ 4. 72,36 , $\qquad$ 9
A. 27
B. 16
C. 18
5. $4,8,12,16$, $\qquad$ 24
A. 19
B. 20
C. 21
g 'G
$\forall \quad$ '

* ' $\varepsilon$
g 'Z
$\forall$ '


## Lesson 7: Activity 2 - Assignment

In this assignment, draw conclusions from the information presented and complete the patterns given.

1. One person will eat 3 cupcakes, two people will eat 6 cupcakes, three people will eat 9 cupcakes.
A. How many cupcakes does each person eat?

Each person eats 3 cupcakes.
B. How many cupcakes will each additional person eat?

Each additional person will eat 3 cupcakes each.
C. How many cupcakes will 15 people eat?
$15 \times 3=45$, so 15 people will eat a total of 45 cupcakes.
2. Determine the pattern and fill in the missing parts of the table, then describe what the pattern is.

| Money Earned | $\$ 1.00$ | $\$ 2.00$ | $\$ 3.00$ | $\$ 4.00$ | $\$ 5.00$ | $\$ 6.00$ | $\$ 7.00$ | $\$ 8.00$ | $\$ 9.00$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Money Saved | $\$ 0.25$ | $\$ 0.50$ | $\$ 0.75$ | $\$ 1.00$ | $\$ 1.25$ | $\$ 1.50$ | $\$ 1.75$ | $\$ 2.00$ | $\$ 2.25$ |

The pattern is an increase of 25 cents each time.
3. If the pattern from the table in the last question continued, what would be the missing amounts in the table below?

| Money <br> Earned | $\$ 12.00$ | $\$ 15.00$ | $\$ 18.00$ | $\$ 20.00$ | $\$ 25.00$ | $\$ 30.00$ | $\$ 35.00$ | $\$ 48.00$ | $\$ 50.00$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Money <br> Saved | $\$ 3.00$ | $\$ 3.75$ | $\$ 4.50$ | $\$ 5.00$ | $\$ 6.25$ | $\$ 7.50$ | $\$ 8.75$ | $\$ 12.00$ | $\$ 12.50$ |

Marking Guide: This assignment is worth 18 marks.

## Lesson 7: Activity 3 - Creating Formulas for Simple Patterns

So far in this lesson, you have looked at finding out what kind of pattern you have and at figuring out what you have to do to each number to get the next number in the pattern.

In this activity, you are going to take that a step further and create formulas for patterns.

Let's say you have the pattern $2,4,6,8$. What can you tell by looking at the numbers?

Find the Common Difference


- First off, you are adding 2 to each number to get the next number.
- Second, because you are adding, you know that you have an arithmetic sequence.

It would not be hard to figure out what the next term (next number) in the sequence is or for that matter, what the next three numbers are.

However, it gets a bit harder if you are asked for the hundredth term in the sequence. This is where a formula can be very helpful.

Each number in a sequence is given a term number.
In the pattern we've just looked at, 2 is term one ( $t 1$ ), 4 is term two ( $t 2$ ), 6 is term three ( $t 3$ ), and 8 is term four (t4). Any unknown term is written as $t_{n}$. If you wish, you could write it like this:
$2,4,6,8$
$t_{1}, t_{2}, t_{3}, t_{4}$
If you wanted to make a formula to find the hundredth term in the pattern, you could do it like the following:
$t_{100}=2 n$
$t_{100}=2 \times 100=200$
Notice that the $n$ in the formula is replaced by the term number. The hundredth term in the pattern is 200.

Watch a video that will help you practise using terms in formulas. Please type this link in your browser:

## Lesson 7: Activity 3 - Assignment

$\square$ $\square$

In this assignment, create a formula you could use to solve each of the patterns in the following questions. NOTE: Do not leave any spaces in the formulas you write.

1. $10,20,30,40 \quad t_{n}=10 n$
2. $27,28,29,30 t_{n}=n+26$
3. $6,12,18,24 \quad t_{n}=6 n$

Marking Guide: This assignment is worth 3 marks.

## Lesson 8: Using Formulas

In this lesson, you will learn how to use formulas, graphs, and charts to

- find missing data
- to solve problems



## Lesson 8: Activity 1 - Using Formulas to Find Missing Information

In the last activity, you worked on creating formulas for patterns.
Example 1: Each term is multiplied by 25 . This could be written as $t_{n}=25 n$.
The eighth term is found by using this formula:
$t_{8}=25 n$
$t_{8}=25 \times 8$
$t_{8}=200$
For a review, find the 12th term.
$t_{12}=25 n$
$t_{12}=25 \times 12$
$t_{12}=300$

In this activity, you will use formulas to find missing information.
Example 2: Let's say you are hosting a party and know it is going to cost $\$ 9$ per person. You are not sure whether you want to invite 10 or 12 of your friends. The main problem is that you only have a budget of $\$ 100$ for the party. You could create a formula to help figure out how many friends you could invite.

Let's use the formula Total Cost $(T)=9 \times$ Number of Friends Invited $(f)$ :
For 10 friends: For 12 friends:
$\begin{array}{ll}T=9 \times f & \\ T=9 \times 10 \times f \\ T=\$ 90 & T=9 \times 12 \\ & T=\$ 108\end{array}$


So it will cost $\$ 90$ to invite 10 friends and $\$ 108$ to invite 12 friends. Since you only have $\$ 100$ to spend, you can only invite 10 friends to your party. You can use a formula to plan a party.

Example 3: Let's go back to your party. A couple of your friends want the party to be bigger and are willing to help pay for the party. They want to invite 43 people in total. How much will the party cost now?

Use the formula you already have:
$T=9 \times f$
$T=9 \times 43$
$T=\$ 387$
The party will now cost $\$ 387$.


It is much easier to use a formula to solve for many missing pieces of information in one problem than to solve for each piece individually.

Example 4: What about if you were bringing cupcakes to school for your classmates? You want to bring two more cupcakes than the number of classmates you have in case the class has a visitor you didn't know about. There are 12 students in your shop class and 8 in your science class.

You can use a formula for this problem too.
Let's use the formula Cupcakes $(C)=$ Number of Students in Your Class $(n)+2$.
For the shop class:
$C=n+2$
$C=12+2$
$C=14$ cupcakes
For the science class:
$C=n+2$
$C=8+2$
$C=10$ cupcakes


You need to bring 14 cupcakes to your shop class and 10 cupcakes to your science class. The nice thing about using a formula is that once you have it created, you can use it to solve for any missing number in a problem.

Example 5: Write a formula and solve the following problem. What if you were asked to bring cupcakes for the entire school? There are 451 people in the school. Remember that you want to bring 2 extra just in case somebody is visiting.
$C=n+2$
$C=451+2$
$C=453$ cupcakes
You will have to bring 453 cupcakes.

## Lesson 8: Activity 1 - Assignment

In this assignment, solve the following problems by using formulas.

1. You are bringing cupcakes for your class. There are 16 students, but sometimes, 2 students come into your room for extra help.

Use the formula below to calculate how many cupcakes you need to bring.
Number of Cupcakes $(C)=$ Number of Students $(n)+2$
$C=n+2$
$C=n+2$
$C=16+2$
$C=18$
2. It is going to cost $\$ 200$ to go on a raft trip.

Use the given formula to calculate how much each person needs to pay to cover their costs in the two groups below.

What Each Has to Pay $(P)=200 \div$ Number of People ( $n$ )
$P=200 \div n$
A. 10 people
$P=\frac{200}{n}$
$P=\frac{200}{10}$
$P=\$ 20$
B. 12 people
$P=\frac{200}{n}$
$P=\frac{200}{12}$
$P=\$ 16.67$
3. You are buying rope to make swings. Each swing is 2.5 metres in length and uses two lengths of rope.

Use the given formula to find out how much rope you need to make each of the sets of swings below.

Length $(L)=2 s$ (Length of Rope) times $n$ (Number of Swings)
$L=2$ sn
A. 3 swings
$L=2 s \times 3$
$L=2(2.5) \times 3$
$L=5 \times 3$
$L=15 \mathrm{~m}$
B. 5 swings
$L=2 s \times 5$
$L=2(2.5) \times 5$
$L=5 \times 5$
$L=25 \mathrm{~m}$

## Lesson 8: Activity 2 - Using a Chart/Graph to Find Missing Information

Another way to solve a problem is by using a graph or a chart. You can do either one of two things once you have a graph.

## Interpolation:

This is where you estimate a value between two known values.

For example, if you know 2 students will eat 4 hot dogs and 4 students will eat 8 hot dogs, you can interpolate (estimate) that 3 students will eat 6 hot dogs.


## Extrapolation:

This is where you estimate a value beyond known values. It usually involves extending a graph beyond where it ends to solve for a missing piece of information.

If you use the example above, you could extrapolate that 5 students will eat 10 hotdogs. You have gone beyond the available information, and using the information you have, you made a logical choice to answer the question.

Watch a video on using graphs and charts. Please type this link in your browser:
http://quick.adlc.ca/mathcf943


## Lesson 8: Activity 2 - Assignment

In this assignment, use this graph to answer the questions that follow.

1. If you spend 1.5 hours studying, what would you estimate your mark would be?
about 20\%
$\qquad$
2. If you spend 5.5 hours studying, what would you estimate your mark would be?
about 85\%

3. How many hours would you need to study to get a mark of 60\%?

## 4 hours

$\qquad$
$\qquad$

> Time (Hours)
4. Based on your calculations, what conclusion can you draw about the relationship between study time and test scores?

The more time spent studying, the higher the test score.
$\qquad$
$\qquad$
$\qquad$
$\qquad$

Marking Guide: This assignment is worth 4 marks.

## Lesson 8: Activity 3 - Drawing Conclusions from Everyday Contexts

Graphs are a part of life. They don't disappear when math ends. They are on television news, in newspapers, in magazines; they are everywhere. They report on everything from how many vehicles are in a particular city to the price of gas to how one team does against another.

See the example of a graph on the right. Find an example of a graph in a newspaper, a magazine, or a book. Describe what information the graph is explaining.

It is an important skill to be able to read graphs and understand what they are trying to communicate.

A graph that appears a lot in the media is a graph of the value of the Canadian dollar compared to the value of the American dollar. An example of this graph is below. It shows the value of the Canadian dollar in American dollars for 30 days ending October 11, 2015.


Days of the Week


Notice that even though the value of the Canadian dollar goes up and down across the graph, a logical conclusion is that after September 28, the value of the Canadian dollar was increasing compared to the American dollar.

Another type of graph you may look at is a census graph that tells you about the population of an area (or the population change in an area).

A census is a regularly occurring and official count of a particular population. The Census Program provides a statistical portrait of the country every five years. (The last census taken in Canada was in 2016.)

## Lesson 8: Activity 3 - Assignment

In this assignment, you will use the graphs provided to interpret information.
Use the following graph to answer the first four questions. This graph shows the median ages in Canada and Alberta during each census year between 1921 and 2011.
$\square$ Canada Alberta


1. In 1971, the median age in Canada was 26 years. What was the median age in Alberta?

## 25

$\qquad$
2. In 1981, was the median age in Canada higher or lower than in Alberta?
higher
$\qquad$
3. Find the median age of Albertans in 1941 and 2001. How much of a difference was there between these years?

$$
35-26=9
$$

4. After viewing this graph, what conclusion can you come to regarding the ages of Albertans?

Students should conclude that Alberta's population is aging.
$\qquad$

The following graph shows the percentage of households in Canada with only one person and those households with five or more persons. Answer the three questions that follow.

5. What percentage of households had five or more persons in 1981?

15
6. In 2006, what percentage of households had only one person living in them?

25\%
7. What do you notice about households with five or more persons as the graph moves from 1961 to 2011?

Students should notice that the number of households with five or more persons declined each year, with the exception of 1996 where the number of households remained the same.

Marking Guide: This assignment is worth 7 marks.

## Lesson 9: Strategies to Solve Word Problems

In this lesson, you will learn about and practise using different strategies to solve word problems.

## Lesson 9: Activity 1 - The 4-Step Method

The remaining lessons in this theme focus on solving math word problems. You will learn about and practise using strategies to work through problems you will come across in math work.

You may have come across some of these strategies in previous courses, while some of them may be new to you.

Think of a time when you experienced a problem that you had to solve. It might not be a math problem but a problem at school or outside of school.


How did you go about solving it?
Did you have a plan of action?
Did it involve following steps?
Did having a plan make it easier to solve the problem?
Sometimes you might come across a math word problem and have no clue how to even start it. It is important to have a plan for approaching problems. We call such a plan the 4-Step Method. This is a step-by-step process for solving math problems; devised by mathematician George Polya.

## ${ }^{1} 2_{3}$ 4-Step Method

## Step 1: Understand the Problem

To understand a word problem, you need to be able to answer some questions:

- What information is given?
- What are you asked to find?
- Does the question need an exact or approximate answer?

You can use one or more of the following strategies to help you answer these questions. You might

- interpret visuals such as pictures, charts, tables, or graphs
- identify the keywords in the problem
- identify which information is important and which is unimportant


## Step 2: Think of a Plan

The keywords in a problem and the important information given will help you develop a plan for solving problems. You may decide to

- look for patterns
- draw a picture or use manipulatives
- make the problem simpler
- practise using guess and check
- work backwards
- organize information


## Step 3: Carry Out the Plan

You can solve the problem using a pencil and paper, manipulatives, a calculator, a computer, or by using another strategy that you identified in your plan.

Remember to always state the solution to the problem in a statement.

## Step 4: Look Back and Verify

You can choose one or more of these strategies to check your answer for reasonableness:

- Use the information and/or numbers in the problem to estimate a solution. Then compare your answer to this estimate to help you determine if your answer is reasonable.
- Check that decimals are placed correctly.
- Use a calculator.
- Use the opposite operation (subtract if you added, divide if you multiplied, etc.).
- Compare your answer with others.

A further thing to do when looking back is to

- think of another strategy that might have worked just as well or better than the one you used
- think about what other kinds of problems you can solve using the strategies you employed


## Using the 4-Step Method

Look at this problem:

Sage was hosting a birthday party. She invited 14 girls and 12 boys. She made 1 dozen pink cupcakes and 2 dozen blue cupcakes. When the party was over, there were 4 cupcakes remaining.

How many cupcakes were eaten?

## Step 1: Understand the Problem

In this problem, we are given quite a bit of information. Let's identify which information is most important and what you are asked to find:

Sage was hosting a birthday party. She invited 14 girls and 12 boys. She made 1 dozen pink cupcakes and 2 dozen blue cupcakes. When the party was over, there were 4 cupcakes remaining.

How many cupcakes were eaten?

The most important information and what you are asked to find has been made bold above. It is interesting how many people Sage invited, but it is not important information. So now we understand the problem.


## Step 2: Think of a Plan

There are many strategies you can use in this step. For this problem, it would be best to make it simpler. That is, we must first figure out the total number of cupcakes made and then determine how many were eaten.

We could write an equation to do this: (1 Dozen +2 Dozen) - $4=$ Number of Cupcakes Eaten

## Step 3: Carry Out the Plan

Now we will begin to solve our equation: (1 Dozen +2 Dozen) - $4=$ Cupcakes Eaten

First, we must remember that a dozen $=12$.
Our new equation would look like this:
$(1 \times 12+2 \times 12)-4$
$(12+24)-4=$ $36-4=32$

This means that the number of cupcakes eaten was 32.

## Step 4: Look Back and Verify

This final step is very important. Does the answer make sense? In the problem, we wanted to know how many cupcakes were eaten. Our answer was 32, which is less than the total number made (36), so that answer makes sense.

We can check our answer further by using the opposite operation $(32+4=36)$, so our answer checks out!

## Lesson 9: Activity 1 - Assignment

In this assignment, you will complete steps 1 and 2 of the 4-Step Method of the following problem. (You do not have to solve the problem at this time.)

Amy had 75 cents after purchasing a ticket for the movies.
She found 25 cents at the movie theatre.
She bought a bag of salt and vinegar chips to eat at the film for 55 cents. How much money did Amy have left?

Step 1: Understand the Problem
(1). What information is given?

Amy has 75 cents and she found 25 cents more. She spent 55 cents on chips.
(2) 2. Identify important and unimportant information.

The amounts of money are important. Where she is going and what she bought is unimportant.
(1) 3. What are you asked to find?

How much money Amy has left.
(1) 4. Does the question need an exact or approximate answer?

The question does not ask for an estimate so requires an exact answer.

Step 2: Think of a Plan
(2) 5. Which strategy or strategies will you use to solve this problem?

Students may use a sub-goal to use an equation to determine the amount of money she has in total before she spend any. Students may also choose other strategies such as drawing pictures or using manipulatives.

Marking Guide: This assignment is worth 7 marks.

## Lesson 9: Activity 2 - Interpreting Visuals and Identifying Keywords

As you learned in the previous activity, the first step to solving a math word problem is to understand the problem.

Two strategies that can be used to do this are interpreting visuals and identifying keywords. You will look more closely at these two strategies in this activity.


## Interpreting Visuals

Pictures, charts, tables, and graphs are visual clues that can be used to help solve word problems.

To use these clues,

- read the title of the visual
- read axis titles, column headings, and subheadings
- read the print under, beside, or above visuals
- compare and analyze pictures, bars, lines, or markers
- find trends or patterns
- discuss the information with classmates or others
- ask questions
- reread to connect the information in word problems to the information in visuals


## Examples of visuals:

## Pictures



## Charts

| Flower <br> Pot <br> Colour | Number <br> of Flowers <br> Growing |
| :--- | :--- |
| Red | 8 |
| Green | 3 |
| Yellow | 6 |
| Purple | 5 |

Graphs


## Identifying Keywords

Keywords are also called clue words and offer valuable hints to help solve word problems.

To use keywords,

- read over the problem several times
- identify and underline the keywords
- determine their meanings

Keywords often provide clues about which math operation to use. When you are solving word problems, use the chart below to help you figure out the math operation to use.

## Chart of Keywords

| Addition Clue Words | Subtraction Clue Words | Multiplication Clue Words | Division Clue Words |
| :---: | :---: | :---: | :---: |
| Add | Subtract | Times | Quotient of |
| Sum | Difference | Product | Divided by |
| Total | Take away | Doubled | Half (or a fraction) |
| Plus | Less than | Multiplied by | Split |
| In all | Remain | Twice as much | Separated |
| Both | Decreased by | $A$ times as much as $B$ | Cut up |
| Together | Have left | By (dimension) | Parts |
| How many in all | Change (money problems) | Older than | Share |
| Increased by | How much more |  | Ratio |
| All together | How much less |  |  |
| Older than | Fewer |  |  |
| Larger than | Reduce |  |  |
|  | Younger than |  |  |
|  | Smaller than |  |  |
|  | Left over |  |  |

## Lesson 9: Activity 2 - Self-Check

In this self-check, you will match keywords with the correct math operation.
A. Addition
B. Subtraction
C. Multiplication
D. Division
$\qquad$ 1. left over
2. twice as much
3. quotient of
$\qquad$ 4. how many in all
5. increased by
6. difference
7. divided by
8. product

○ '8
a 'L
8 '9
$\forall$ 'G
$\forall \quad$ '

- ' $\varepsilon$

○ 'Z
( ${ }^{\prime}$ '

## Lesson 9: Activity 2 - Assignment

In this assignment, in the chart provided, write the math operation and symbol from the box below beside the description of the operation. Underline or highlight the keywords in each description. The first one has been completed for you.

|  |  | Math Operations and Symbols | Description of Math Operations in Words |
| :---: | :---: | :---: | :---: |
| A | 1. | $2+6$ | A. two increased by six |
| H | 2. | $8 \times 9$ | B. thirty-six less fifteen |
| F | 3. | $100 \div 25$ | C. two-thirds plus seven |
| G | 4. | $(6-2)+9$ | D. the quotient of nine into fifty-four |
| B | 5. | 36-15 | E. one-tenth times two |
| D | 6. | $54 \div 9$ | F. one hundred cut into 25 parts |
| E | 7. | $\frac{1}{10} \times 2$ | G. six minus two increased by nine |
| C | 8. | $\frac{2}{3}+7$ | H. 8 multiplied by nine |

Marking Guide: This assignment is worth 7 marks.

# Lesson 9: Activity 3 - Identifying Important and Unimportant Information 

Understand the Problem
A third strategy that you should use when trying to understand a math word problem is looking for important information and disregarding information that may seem interesting but does not help you solve the problem. This activity will let you practise this strategy.

## Looking for Information

Important information will include keywords and other details necessary to solve the problem. Unimportant information includes details that add to the "story" of the problem, but these details can be discarded as you investigate the problem further.

To determine what is important and what is not, ask yourself the following questions:

- What information has been given?
- Has enough information been provided?
- Has too much information been provided?
- What is the question that needs to be answered?

It is a good idea to read the problem several times. You can underline or highlight important information. You may also find it useful to cross out unimportant information.

Here's a word problem with the important information bolded and some unimportant information crossed out.

Example 1: Mara has $\mathbf{\$ 1 4 . 0 0}$. Lexie has $\mathbf{\$ 2 1 . 0 0}$. Mara and Lexie are saving their money to attend a concert this fall.

How much money do they have in all?
$\$ 14.00+\$ 21.00=\$ 35.00$
The girls have $\$ 35.00$ in all.


Try this problem regarding video games. Practise underlining or highlighting the important information and crossing out the unimportant information.

Example 2: Evan has 7 video games.
Sunita has 3 times as many as Evan. She enjoys playing video games and loves playing with others online.

How many video games does Sunita have?


Evan has-7video games.
Sunita-has-3-times as many-as Evan. She enjoys playing video games and lovesplaying with others online.

How many video games does Sunita have? $7 \times 3=21$
Sonita has 21 video games.

## Lesson 9: Activity 3 - Self-Check

In this self-check, you will practise underlining or highlighting important information and crossing out unimportant information.

1. Jen bought 4 t-shirts, one for each of her four brothers, for $\$ 9.95$ each. The cashier charged her an additional $\$ 2.38$ in sales tax. She left the store with $\$ 7.28$. How much money did Jen start with?
2. An animal care society tested 442 pet animals. They found 271 were infected by diseases. How many were healthy pet animals?
3. Mark took a book out of the school library that contained 649 pages. He read 495 pages in the last 2 weeks. How many pages does he have left to read?
4. Dean had collected 40 toy whistles. He gave 2 whistles to each of his 6 friends at camp. How many whistles did Dean have left?
5. Ann has 92 cents in her pocket, taken from her piggy bank at home. Sam has twice as much money. How much money does Sam have?








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## Lesson 9: Activity 3 - Assignment

In this assignment, each of the following problems. Highlight (or underline) the important information and cross out the unimportant information in each problem. Then, solve each problem by writing an equation and a concluding statement.

1. Rain had 29 crunchies in her cat bowl. They were salmon-flavoured. She has eaten 14 of them. How many treats are left?

Rain had 29 erunchies in her cat bowl. They were-salmon-flavoured. She has eaten 14 of them. How many treats are left?
$29-14=15$

Rain has 15 treats left.
2. Zack did 45 push-ups in gym class today. Luke did 15 more push-ups than Zack. How many push-ups did Luke do?

Zack did 45 -push-ups in gym class today. Luke did-15-more push-ups than Zack. How many-push-ups-did Luke do?
$45+15=60$
Luke did 60 push-ups.
3. Josh and Brent have $\$ 25.00$ together from collecting bottles. They gave their friend, Shawn, $\$ 3.00$ to buy some snacks. Then, they divided the rest of the money into 2 equal parts. How much money do they each have now?

Josh and Brent have \$25.00together from collecting bottles. They gave their friend, Shawn, $\$ 3.00$ to buy some snacks. Then, they divided the rest of the money into 2 equal parts. How much money do they each have now?
$(\$ 25.00-\$ 3.00) \div 2=\$ 11.00$
They each have \$11.00 now.

Marking Guide: This assignment is worth 6 marks.

## Lesson 10: More Strategies to Solve Word Problems

In this lesson, you will

- learn about and practise using different strategies as you develop a plan for solving math word problems
- carry out the plan to solve the problem
- think about what to do when you have completed a problem



## Lesson 10: Activity 1 - Patterns, Pictures, and Simplify

## Think of a Plan and Carry Out the Plan

As you learned in the previous lesson, the second step to solving a math word problem is to think of a plan or a strategy. Strategies that can be used as you do this include looking for patterns, drawing pictures, and making the problem simpler. You will look closely at these strategies in this activity.

As you practise these strategies, you will also carry out the plan and solve the problem.

## Patterns

Remember that patterns occur when something is repeated several times. They may be used to predict what may come next.

Example 1: Problem with a pattern.
Seven friends are planning a surprise party for a mutual friend. They estimate that, on average, each person will eat 2 bags of chips and drink 3 cans of pop. How many bags of chips and cans of pop do they need for the party?


What is the pattern?

For each person, the number of bags of chips increases by 2 and the number of cans of pop increases by 3 .

| Number <br> of <br> People | Bags <br> of <br> Chips | Cans <br> of <br> Pop |
| :---: | :---: | :---: |
| 1 | 2 | 3 |
| 2 | 4 | 6 |
| 3 | 6 | 9 |
| 4 | 8 | 12 |
| 5 | 10 | 15 |
| 6 | 12 | 18 |
| 7 | 14 | 21 |
| 8 | 16 | 24 |

They will need 16 bags of chips and 24 cans of pop.

Example 2: Solve the problem.
Sierra bought 1 pair of shoes, 2 pairs of jeans, and 3 shirts.
How many new outfits does Sierra have?

| Shoes | Jeans | Shirts | Number of Outfits |
| :---: | :---: | :---: | :---: |
| 1 | 1st | 1 |  |
|  | pair | 2 | 3 |
|  | 3 |  |  |
| 1 | 2nd | 1 |  |
|  | pair | 2 | 3 |
|  | 3 |  |  |

Sierra has 6 new outfits.


## Drawing Pictures

You can also draw pictures or use objects to represent a word problem．

| Shoes | Jeans | Shirts |
| :---: | :---: | :---: |
| N | ${ }^{5-2}$ | ？ |
| N | 5－2 | $\cdots$ |
| N | ${ }^{5-2}$ | $\cdots$ |
| N゙ひ | TV | $\square$ |
| N゙気 | TV |  |
| N゙ひ | $\square$ | $\cdots$ |



## Make It Simpler

Some of the problems you will be asked to solve may have several parts．Solving one part of the problem at a time may make it easier to solve．

Example：Here＇s a two－part problem．You＇ll notice the important information has been bolded．

Sam bought 6 packs of red bouncy balls， 9 packs of yellow bouncy balls，and 7 packs of blue bouncy balls．There were 20 bouncy balls in each package．How many bouncy balls did Sam buy in all？

1．Identify what you need to know．
How many of each colour of bouncy balls are there？
How many did Sam buy in all？
2．Solve each question．
Red： $6 \times 20=120$
Yellow： $9 \times 20=180$
Blue： $7 \times 20=140$
$120+180+140=440$
Sam bought 440 bouncy balls in all．

## Lesson 10: Activity 1 - Assignment

In this assignment, solve the following problems. Look for important information, including keywords, as you first read the problems. (You may highlight important information and cross out unimportant information as you did previously.)

Solve the first problem by looking for patterns and solve the next two problems by making them simpler. If you'd like, you may draw pictures or use manipulatives for the problems. Remember to show your work and write a concluding statement when done.

1. A sporting goods store asked a well-known hockey player to sign autographs on a Saturday. The manager decided to control the number of people entering the store. At 9:00 a.m., 6 people were allowed to enter the store; at 9:05 a.m., 10 people were in the store; and at 9:10 a.m., 14 people were in the store. What is the pattern?

Complete this table to show that if people continued to enter the store at this rate, and not leave, how many will be in the store at 10:00 a.m.

| Time | $9: 00$ | $9: 05$ | $9: 10$ | $9: 15$ | $9: 20$ | $9: 25$ | $9: 30$ | $9: 35$ | $9: 40$ | $9: 45$ | $9: 50$ | $9: 55$ | $10: 00$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Customers <br> in the Store | 6 | 10 | 14 | 18 | 22 | 26 | 30 | 34 | 38 | 42 | 46 | 50 | 54 |

How many were in the store at 10:00 a.m.? Write a concluding statement.
The pattern is 4 people are added into the store at each 5 -minute interval.
At 10:00 a.m., there would be 54 people in the store.
2. At a cafe that Geri and Nick had breakfast, muffins cost $\$ 1$ each and fruit cups cost $\$ 4$ each. Geri had 1 muffin and 1 fruit cup. Nick had 2 muffins and 2 fruit cups. How much did their breakfast cost?

Identify what you need to know.
How much did Geri spend? $\$ 1.00+\$ 4.00=\$ 5.00$
How much did Nick spend?

$$
(2 \times \$ 1.00=\$ 2.00)+(2 \times \$ 4.00=\$ 8.00)=\$ 10.00
$$

How much is their breakfast in total? $\$ 5.00+\$ 8.00=\$ 15.00$
Write a concluding statement.
Their breakfast in total cost $\$ 15.00$.
3. There are 9 red pens in Liz's desk drawer. There are 7 more black pens than red pens. There are also 10 more blue pens than red pens. How many pens are there in all?

Identify what you need to know:
How many black pens are there? $9+7=16$
How many blue pens are there? $10+9=19$
How many pens in all? 9 red pens +16 black pens +19 blue pens $=44$ pens

Write a concluding statement.
There were 44 pens in all.

# Lesson 10: Activity 2 - Guess and Check and Work Backwards 

Think of a Plan and Carry Out the Plan
There are other strategies that can be used as you think through a plan for solving problems. They include guess and check and working backwards. You will look closely at these strategies in this activity.

As you practise these strategies, you will also carry out the plan and solve the problem.

## ${ }^{1} 2_{3}$ Guess and Check

Step 1: Guess a solution or way to solve a problem.

Step 2: Test the solution to see if it is correct. Use strategies such as calculating, researching, and asking questions.

Step 3: If the guess is incorrect, you can guess again.
Example: Use the above strategy to answer the following problem.
Three friends have been hired to help set up tables for a flea market.
Tui worked twice as many minutes as Joey.
Tara worked 10 minutes more than both Tui and Joey together.
In total, they worked 64 minutes.
How many minutes did each person work?

## Important information:

Tui worked twice as many minutes as Joey.
Tara worked 10 minutes more than both Tui and Joey together.
They worked 64 minutes in total.

| Guesses | Joey | Tui | Tara | Total | Is the total 64 minutes? |
| :---: | :---: | :---: | :---: | :---: | :---: |
| First | 5 | 10 | 25 | 40 | Too low |
| Second | 6 | 12 | 28 | 46 | Too low |
| Third | 8 | 16 | 34 | 58 | Too low |
| Fourth | 10 | 20 | 40 | 70 | Too high |
| Fifth | 9 | 18 | 37 | 64 | YES! |

Joey worked 9 minutes, Tui worked 18 minutes, and Tara worked 37 minutes.

You can check the answer by using information from the question.
Tui worked twice as many minutes as Joey. $9 \times 2=18$
Tara worked 10 minutes more than both Joey and Tui together. $9+18=27+10=37$
The total time worked was 64 minutes. $9+18+37=64$

## ${ }^{1} 2_{3}$ Work Backwards

Some problems provide part of an end result and ask you to find other information.
To solve these kinds of problems, you can do the following:
Step 1: Begin with the information given in the problem.
Step 2: Work backwards to get an answer.
Step 3: Solve the problem.
Example: Use the above strategy to answer the following problem:
Sasha got on the school bus. At the stop after Sasha's, 5 students got on. Six students got on the bus at the next stop. At the last stop before the school, 9 students got on. When the bus arrived at school, 38 students got off. How many students were already on the bus when Sasha got on?

Step 1: 38 students got off the bus at school.
Step 2: Work backwards:

$$
\begin{aligned}
& 38-9=29 \\
& 29-6=23 \\
& 23-5=18 \\
& 18-1 \text { (Sasha) }=17
\end{aligned}
$$

Step 3: There were 17 students already on the bus when Sasha got on.

See the strategies of finding patterns. Please type this link in your browser:
http://quick.adlc.ca/workb4a62
Review the strategies of finding patterns, drawing diagrams, making charts and tables, and guess and check. Please type this link in your browser:
http://quick.adlc.ca/math60a68

## Lesson 10: Activity 2 - Assignment

In this assignment, solve the following problems. Look for important information, including keywords, as you first read the problems. (You may highlight important information and cross out unimportant information as you did previously.)

Solve the first problem by guessing and checking and solve the next two problems by working backwards. If you'd like, you may draw pictures or use manipulatives for the problems. Remember to show your work and write a concluding statement when done.
(9) 1. Mary reads 8 more books than John in one year. If they read a total of 32 books in one year, how many books does each read? You may want to start with a guess of 10. Write a concluding statement.

| Guesses | John's <br> Books | Mary's <br> Books | Total Books Read (32) |
| :---: | :---: | :---: | :---: |
| First | 10 | $8+10=18$ | $10+18=28$ |
| Second | 11 | $8+11=19$ | $11+19=30$ |
| Third | 12 | $8+12=20$ | $12+20=32$ |

(2) 2. Brett gave 10 stamps from his collection to both Sammy and Rand. Then he gave 14 stamps to Kat and 6 stamps to Geri. He still has 275 stamps left. How many stamps were in Brett's collection to begin with? Write a concluding statement. (Hint: Begin with 275 and work backwards.)
$275+6($ Geri $)=281+14($ Kat $)=295+10($ Rand $)=305+10($ Sammy $)=315$
Brett had 315 stamps to begin with.
(2) 3. Pat went shopping at her local drugstore. She bought 2 CDs on sale for $\$ 8.95$ each, a notebook for $\$ 2.58$, and shampoo for $\$ 5.29$. When Pat paid for her purchases, the cashier gave her $\$ 4.23$ in change. How much money did Pat give the cashier? Write a concluding statement. (Hint: Begin with $\$ 4.23$ and work backwards.)
$\$ 4.23+\$ 5.29($ shampoo $)=\$ 9.52+\$ 2.58($ notebook $)=\$ 12.10+\$ 8.95$ (1st CD) $+\$ 8.95$ $(2 n d C D)=\$ 30.00$

Pat gave the cashier \$30.00.
Marking Guide: This assignment is worth 13 marks.

## Lesson 10: Activity 3 - Look Back and Verify

When you have found the answer to the math problem and written a concluding statement, you're still not done!

It's now the time to check your answer for reasonableness and to verify for correctness. In this activity, you will look more closely at strategies to do this: estimating and using the opposite operation.

## Reasonableness and Estimation



You have done quite a lot of estimating for reasonableness already in this course. You know that you can estimate when an exact answer isn't needed and you can estimate to help you judge whether your answer is reasonable.

Example: Sean was figuring out his earnings for a week of after-school work. He kept a record of his earnings for 5 days. They were as follows:

Sean calculated his earnings and was quite happy when he got a sum of $\$ 393.00$. However, he checked for reasonableness and realized that he

| Day | Hours <br> Worked | Amount <br> Earned |
| :---: | :---: | :---: |
| 1 | 2 | $\$ 12.00$ |
| 2 | 3 | $\$ 18.00$ |
| 3 | 3.5 | $\$ 21.00$ |
| 4 | 2 | $\$ 12.00$ |
| 5 | 5 | $\$ 30.00$ | must have made an error in his calculations. In the past, his pay cheques for a week's work have never been over \$100.00.

He looked back at the number of hours worked and estimated that he had worked about 15 hours. At $\$ 6.00$ an hour, he should have received about $\$ 90.00$. Sean rechecked his first calculations. He had placed a decimal in the wrong place: $\$ 30.00$ had become \$300.00.


He recalculated and got a sum of \$93.00. Although Sean would have been happy with the extra money, he was also happy that he rechecked, because he may have spent money that he did not have.

## Opposite Operation

A way to check for the accuracy of your answer is to perform the opposite operation.
This chart shows the opposite operation for each math operation:

| If the mathematical operation <br> used to solve the problem was |  | use the opposite operation to <br> check the accuracy of the answer. |
| :---: | :--- | :---: |
| Addition | $\longrightarrow$ | Subtraction |
| Subtraction | $\longrightarrow$ | Addition |
| Multiplication | $\longrightarrow$ | Division |
| Division | $\longrightarrow$ | Multiplication |

Here are mores examples using the opposite operation:

| If the mathematical operation <br> used to solve the problem was |  | use the opposite operation to <br> check the accuracy of the answer. |
| :---: | :--- | :---: |
| $23+14=37$ |  | $37-14=23$ |
| $65-40=25$ |  | 25 |
| $12 \times 4=48$ |  |  |
| $27 \div 3=9$ |  | $48 \div 4=12$ |

## Lesson 10: Activity 3 - Assignment

In this assignment, solve each problem using all of the steps in the 4-Step Method.
Step 1: Understand the Problem
Step 2: Think of a Plan
Step 3: Carry Out the Plan
Step 4: Look Back and Verify

1. Step 1: Amy had 75 eents. She found 25 eents on the sidewalk. She decided to go to a movie. She bought a bag of salt and vinegar chips to eat at the film for 55 eents. How much money did Amy have left?

Step 2: Students may choose a strategy such as make it simpler, draw a picture, or use manipulatives.

Step 3: $\$ 0.75+\$ 0.25=\$ 1.00$
$\$ 1.00-\$ 0.55=\$ 0.45$ Amy had $\$ 0.45$ left.
Step 4: Estimate: Students may estimate having half the money left, approximately \$0.50.
Opposite Operation: $0.45+0.55=\$ 1.00-0.25=0.75$
2. Step 1: Rain had 29 erunchies in her cat bow. They were salmon-flavoured. She has eaten 14 of them. How many treats are left?

Step 2: Students may choose to draw a picture or use manipulatives.
Step 3: $29-14=15$ There were 15 treats left.
Step 4: Estimate: $30-15=15$
Opposite Operation: $15+14=29$

## Theme 2: Review



In Theme 2, you

- learned how to identify a pattern and made predictions and conclusions based on the pattern
- created formulas to represent patterns
- learned how to use formulas and charts and graphs to find missing data and to solve problems
- learned about the 4-Step Method to solve problems and practised how to understand a word problem
- learned about and practised using different strategies as you developed a plan for solving
 problems
- carried out a plan to solve problems
- learned what to do after completing a word problem


## Theme 2: Show What You Have Learned

Complete the following activities to show what you have learned in the lessons for this theme.

1. Find the missing numbers in the following patterns:
A. $9,11,14,18,23,29$
B. $3,6,9,12,15$
C. $3,6,12,24,48$
D. $7,14,21,28,35$
2. For every hour worked, Tim makes $\$ 8.50$. Determine the pattern and fill in the missing parts in the chart.

| Hours <br> Worked | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Wage <br> Earned | $\$ 8.50$ | $\$ 17.00$ | $\$ 25.50$ | $\$ 34.00$ | $\$ 42.50$ | $\$ 51.00$ |

3. Using $t$ for the term and $n$ for the number, write a formula to find the 9th number in the following sequence.
$5,10,15,20,25,30$
Formula: $t_{9}=5 n$

Solution: $t_{9}=5 \times 9$
$t_{9}=45$
4. Choose one of the problems below to solve. Each problem involves the use of one of the strategies you learned about in this theme. Use the 4-Step Method to solve.

## A. Strategy: Working Backwards

Allen's Mom baked some cookies for the school bake sale. Dakota bought 3 of the cookies and Chandra bought 2. Mr. Walker, the science teacher, bought 1 dozen of the cookies. Matt and Starr each bought 6 cookies. That left only 7 cookies for Scott to buy. How many cookies in total did Allen's Mom bake for the sale?
Allen's Mom baked some cookies for the school bake sale. Dakota bought 3 of theeookies and Chandra bought 2. Mr. Walker, the science teacher, bought 1 dozen of the cookies. Matt and Starr each bought 6 eookies. That left only 7 eookies for Scott to buy. How many cookies in total did Allen's Mom bake for the sate?

7 (left over) +6 (Starr) +6 (Matt) +12 ( 1 dozen Mr Walker) $+2+3=36$
Allen's Mom baked 36 cookies for the sale.
Estimate answer = approximately 30
Opposite operation: $36-3-2-12-6-6=7$
B. Strategy: Make It Simpler

Dan bought an old bike for $\$ 10$. He wanted to fix it up, so he spent $\$ 4.95$ on paint and $\$ 7.39$ for a new chain. He was able to sell the bike for $\$ 50$. How much profit did he make?
Dan bought-an old bike for $\$ 10$. He wanted to fix it up so spent $\$ 4.95$ on paint, and $\$ 7.39$ for a new chain. He was able to sell the bike for $\$ 50$. How much profit-did hemake?
$\$ 10+\$ 4.95+\$ 7.39=\$ 22.34$
\$50 - \$22.34 = \$27.66
Matt made $\$ 27.66$ in profit.
Estimate answer: $\$ 22$ paid out, approximately $\$ 30$ in profit
Opposite operation: $\$ 22.34-\$ 7.39-\$ 4.95=\$ 10.00 \quad \$ 27.66+\$ 22.34=\$ 50.00$
C. Strategy: Look for a Pattern

Your math teacher let you have a 2-minute break during class on Day 1, a 4-minute break during class on Day 2, a 6-minute break on Day 3, and so on. How many minutes would you get on Day 5 and Day 7?

| Day 1 | Day 2 | Day 3 | Day 4 | Day 5 | Day 6 | Day 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 minutes | 4 minutes | 6 minutes | 8 minutes | 10 minutes | 12 minutes | 14 minutes |

Marking Guide: This assignment is worth 18 marks.

## Theme 2: Reflection

Now that you have completed Theme 2, reflect on the following questions. Answer these questions in complete sentences.

1. What did you do well in this theme on patterns, formulas, and problem solving?
$\qquad$
$\qquad$
2. What did you enjoy learning about the most in this theme?
$\qquad$
$\qquad$
3. What challenges or difficulties did you experience while working through this theme?
$\qquad$
$\qquad$
4. Thinking about problem-solving strategies, which ones did you use most often? Why did you choose these particular strategies?
$\qquad$
$\qquad$

## Theme 2: Glossary

3-D object: an object that has height, width, and depth

4-Step Method: the 4 steps to solve a math word problem

- understand the problem
- think of a plan
- carry out the plan
- look back and verify
acute angle: an angle that is less than 90 degrees
acute triangle: a triangle with all angles less than 90 degrees
area: the amount of space inside the boundary of a flat (two-dimensional) object such as a triangle
arithmetic sequence: a sequence made by adding the same value each time (e.g.,1, 4, 7, 10, $13,16,19,22,25$-each number is three larger than the number before it)
bar graph: a graphical display of data using bars of different heights (sometimes called a bar chart)

Canadian gallon: an imperial unit of capacity that equals 4.54 litres
capacity: the amount of weight a container can hold
cell: each small rectangle in a spreadsheet
cell address: the name given to the cell's location in a spreadsheet
Celsius: a metric scale of temperature in which water freezes at $0^{\circ}$ and boils at $100^{\circ}$ under standard conditions
centimetre: a metric unit of length that equals a hundredth of a metre
circumference: the distance around a circle
column: the vertical divisions in a spreadsheet
complementary angles: angles that add up to 90 degrees (a right angle)
composite number: a number with at least one factor other than 1 and the number itself
conversion: the process of changing from one unit to another
conversion factor: a number used for converting a quantity in one set of units (e.g., metric units) into an equivalent set of units (e.g., imperial units)
coordinates: a set of values that show an exact position; on maps and graphs, it is common to have a pair of numbers to show where a point is-the first number shows the distance along and the second number shows the distance up or down
coordinate plane: the plane determined by a horizontal number line, called the $x$-axis, and a vertical number line, called the $y$-axis, intersecting at a point called the origin; each point in the coordinate plane can be specified by an ordered pair of numbers
cup: an imperial unit of capacity that is equal to 8 fluid ounces
denominator: the number below the line of a fraction that tells the number of parts the whole has been divided into
diameter: a straight line passing from side to side through the centre of a circle
dividend: the quantity or amount to be divided
division: the act of separating something into parts or groups
divisor: the number you divide by
enlargement: when a document or image is made bigger or larger
equivalent fractions: different fractions that name the same number, such as $\frac{1}{2}$ and $\frac{2}{4}$
equivalent measurements: two numbers that are equal in value measurements
estimating: the act of roughly calculating the value, number, or quantity of something
extrapolation: the act of estimating a value beyond known values
factors: numbers that are multiplied to get an answer (the product)
Fahrenheit: an imperial scale of temperature in which water freezes at $32^{\circ}$ and boils at $212^{\circ}$ under standard conditions
fluid ounce: unit of capacity equal to one sixteenth of a US pint
foot: an imperial unit of length that equals 12 inches
fraction: a part of a group, a number, or a whole
frequency distribution table: a table that lists a set of scores and their frequency (how many times each one occurs)
gallon: a unit of volume for liquid measure
geometric sequence: a sequence made by multiplying by some value each time (e.g., 2, 4, 8, $16,32,64,128,256$-each number is two times the number before it)
gram: the base unit of mass in the metric system
greatest common factor (GCF): the greatest factor (number) that can divide evenly into two or more numbers
height: the measurement of an object from the base to the top
hexagon: a six-sided polygon
hundredths: the position of the second digit to the right of the decimal point (e.g., the 5 in 1.25)
imperial system: the system used in the United States for measurement
improper fraction: a fraction in which the numerator is greater than the denominator (e.g., $\frac{5}{4}$ )
inch: an imperial unit of length that is equal to one twelfth of a foot
income: money received, especially on a regular basis, for work
integers: positive and negative numbers
interpolation: the act of estimating a value between two known values
kilogram: a metric unit of mass equal to 1000 grams
kilometre: a metric unit of length that is equal to 1000 metres
label: the words at the top of a column
line graph: a graph that shows information that is connected in some way (such as change over time)
line of reflection: the line that is reflected over in a reflection; also called the mirror line
line of symmetry: the imaginary line where you could fold the image and have both halves match exactly
litre: the basic unit of capacity in the metric system
long ton: a measure of mass in the imperial system that equals 2240 pounds
lowest common multiple (LCM): the smallest number that two or more whole numbers can divide into evenly
lowest terms: when the greatest common factor (GCF) of the numerator and denominator is 1
mass: the measurement of the amount of matter an object has
mean: the average of a set of numbers; to calculate-add up all the numbers, then divide by how many numbers there are
median: the middle number in a sorted list of numbers
metre: the basic unit of length in the metric system; equal to 100 centimetres or 1000 millimetres
metric system: a system of measuring length, weight, and volume with one basic unit of measure; the main system used in Canada
mile: an imperial unit of length that is equal to 1760 yards
milligram: a metric unit of mass that is equal to one thousandth of a gram
millilitre: a metric unit of capacity that is equal to one thousandth of a litre
millimetre: a metric unit of length that is equal to one thousandth of a metre
mirror image: an image or object that is identical in form to another but with the structure reversed, as in a mirror
mixed number: a number consisting of a whole number and a proper fraction (e.g., $4 \frac{1}{2}$ ) mode: the number that appears most often in a set of numbers
multiple: the product of a known number and a whole number
numerator: the number above the line in a fraction that tells how many parts of the whole obtuse angle: an angle that measures more than 90 degrees but less than 180 degrees obtuse triangle: a triangle that has an angle greater than 90 degrees
ordered pair: two numbers written in a certain order, usually like this: $(4,5)$; can be used to show the position on a graph where the " $x$ " (horizontal) value is first and the " $y$ " (vertical) value is second
origin point: the point where two axes meet
ounce: the smallest imperial unit of mass that equals 30 grams
path: a route or course to follow to accurately reach a destination
percent: a number written in parts per 100 (e.g., $95 \%$ is 95 parts per 100 parts)
perimeter: the distance all the way around the outside of a two-dimensional shape
pictograph: a graph that uses pictures or symbols to show the value of data pint: an imperial unit of capacity equal to 2 cups
place value: the value of a number based on the position in that number
polygon: any closed 2-D shape that has three or more straight sides
pound: an imperial unit of mass that equals 16 ounces
powers of 10: a way of writing large or small numbers
prediction: an educated guess on a problem
prime factorization: the prime factors that multiply to make up a number
prime number: a number with only two factors: 1 and the number itself
probability: the chance that something will happen; how likely it is that some event will happen
product: the answer to a multiplication problem
proper fraction: a fraction that is less than one, with the numerator less than the denominator (e.g., $\frac{4}{5}$ )
proportion: when two ratios (or fractions) are equal
protractor: an instrument for measuring angles, typically in the form of a flat semicircle marked with degrees along the curved edge
quadrant: any of the four areas made when we divide up a plane by $x$ and $y$ axes
quadrilateral: a flat shape with four straight sides
quart: an imperial unit of capacity equal to 2 pints or 4 cups
quotient: the answer to a division problem
radius: a straight line from the centre to the outer edge of a circle
range: the difference between the highest and lowest values
rates: comparisons of two different measurements with different units
ratio: a term describing how much of one thing there is compared to another thing
ray: a line with a start point but no end point (it goes to infinity)
reflection: when a shape is given a mirror image based on a line of reflection; also called a flip
reflex angle: an angle that is more than 180 degrees but less than 360 degrees
relevant information: information that is given in the problem that is actually needed to solve the problem
remainder: the amount left over after division
right angle: an angle of 90 degrees, as in a corner of a square or at the intersection of two perpendicular straight lines
right-angled triangle: a triangle with a 90-degree angle
rotation: when a shape is turned about a pivot point (centre of rotation or rotation point); simply means "turning around a centre"; the distance from the centre to any point on the shape stays the same
rotation point: the central point around which a figure is rotated
rounding: the act of making a number simpler but keeping its value close to what it was; the result is less accurate but easier to use
row: each horizontal division in a spreadsheet
scalene triangle: a triangle with all sides of different lengths
scientific notation: a way to write a number as a decimal multiplied by a power of 10
short ton: a measure of mass in the imperial system; 1 ton $=2000$ pounds

SI: another name for the metric system; sometimes called the SI system, which means System International from the French Le Système International d'Unités, or more commonly, Standard International Units
spreadsheet: an electronic document in which data is arranged in the rows and columns of a grid and can be manipulated and used in calculations
straight angle: an angle of 180 degrees; looks like a straight line
supplementary angles: angles that add up to 180 degrees (a straight angle)
symmetrical: when a shape is located directly on the reflection line so that both sides of the shape match exactly
symmetry: when each part of a divided object is a mirror image of the other part
tenths: the position of the first digit to the right of the decimal point (e.g., the 2 in 1.25 )
thousandths: the position of the third digit to the right of the decimal point (e.g., the 5 in 1.235)
translation: the movement of a shape left/right (horizontally) or up/down (vertically); also known as a slide

US gallon: an imperial unit of capacity equal to 16 cups, 8 pints, or 4 quarts
value: the numbers entered in a cell in a spreadsheet
variable: a letter or symbol that is used to represent an unknown quantity
vertex: a point where two or more straight lines meet; a corner
weight: the measurement of how strongly gravity pulls an object toward Earth
whole numbers: numbers that are zero (0) or larger
$\boldsymbol{x}$-axis: the line on a graph that runs horizontally (left/right) through zero
$x$-coordinate: the left/right (horizontal) motion on a coordinate plane
$\boldsymbol{y}$-axis: the line on a graph that runs vertically (up/down) through zero
$\boldsymbol{y}$-coordinate: the up/down (vertical) motion on a coordinate plane
yard: an imperial unit of length that equals 3 feet

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