UNIT 6

Linear Equations and Inequalities

The Pep Club promotes school spirit at athletic events and school activities. The members of the club need new uniforms. They are thinking of selling healthy snacks at lunch time to raise the money needed. What information does the Pep Club need to gather? What math might the members use?

What You'll Learn

- Model and solve problems using linear equations.
- Explain and illustrate strategies to solve linear inequalities.

Why It's Important

Linear equations and inequalities occur in everyday situations involving ratios and rates, geometry formulas, scientific contexts, and financial applications. Using an equation or inequality to solve a problem is an important problem-solving strategy.

Key Words

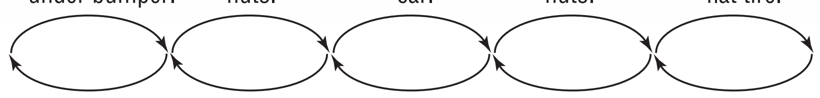
- inverse operations
- inequality

6.1 Solving Equations by Using Inverse Operations

FOCUS

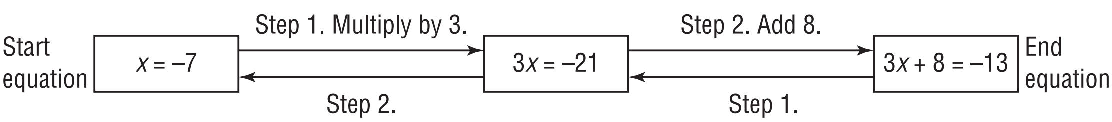
- Model a problem with a linear equation, use an arrow diagram to solve the equation pictorially, and record the process symbolically.

The top row of the arrow diagram shows the steps to remove a flat tire on a car. What steps are needed to put on a new tire?
How are these steps related to the steps to remove the flat tire?



Investigate

➤ This arrow diagram shows the operations applied to the start equation $x = -7$ to build the end equation $3x + 8 = -13$.



Copy and complete the diagram. What are Steps 1 and 2 in the bottom row?
What operations must be applied to the end equation to return to the start equation?

➤ Choose a rational number to complete your own start equation: $x = \square$
Multiply or divide each side of the equation by the same number.
Write the resulting equation.
Add or subtract the same number from each side of the equation.
Write the resulting equation. This is the end equation.
Trade end equations with your partner.
Determine your partner's start equation. Record the steps in your solution.

Reflect & Share

Share your end equations with another pair of classmates.
Determine each other's start equations. What strategies did you use?
How are the steps used to get from the start equation to the end equation related to the steps used to reverse the process?

Connect

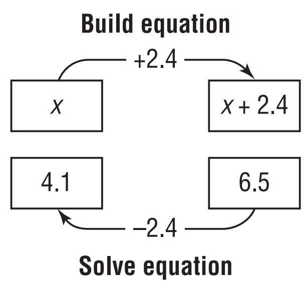
Inverse operations "undo" or reverse each other's results.
Addition and subtraction are inverse operations.
Multiplication and division are also inverse operations.

We can use inverse operations to solve many types of equations. To do this, we determine the operations that were applied to the variable to build the equation. We then use inverse operations to isolate the variable by "undoing" these operations.

For example, to solve $x + 2.4 = 6.5$:

➤ Start with x.
Identify the operation applied to x to produce the expression $x + 2.4$; that is, add 2.4 to get:
$x + 2.4$

➤ Since $x + 2.4$ is equal to 6.5, apply the inverse operation on 6.5 to isolate x; that is, subtract 2.4 to get:
$x + 2.4 - 2.4 = 6.5 - 2.4$
So, $x = 4.1$



Example 1 Writing Then Solving One-Step Equations

For each statement below, write then solve an equation to determine each number. Verify the solution.

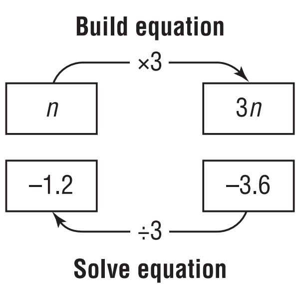
a) Three times a number is -3.6.

b) A number divided by 4 is 1.5.

A Solution

a) Let n represent the number. Then, 3 times n is -3.6.
The equation is: $3n = -3.6$

Inverse Operations



Algebraic Solution

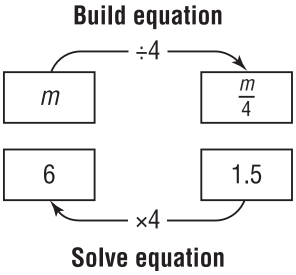
$3n = -3.6$
Undo the multiplication.
Divide each side by 3.

$$\frac{3n}{3} = \frac{-3.6}{3}$$
$$n = -1.2$$

Verify the solution: $3 \times (-1.2) = -3.6$, so the solution is correct.

b) Let m represent the number. Then, m divided by 4 is 1.5.
The equation is: $\frac{m}{4} = 1.5$

Inverse Operations



Algebraic Solution

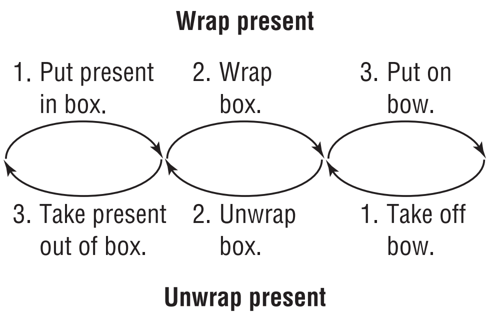
$$\frac{m}{4} = 1.5$$

Undo the division.
Multiply each side by 4.

$$4 \times \frac{m}{4} = 4 \times 1.5$$
$$m = 6$$

Verify the solution: $\frac{6}{4} = 1.5$, so the solution is correct.

To "undo" a sequence of operations, we perform the inverse operations in the reverse order. For example, compare the steps and operations to wrap a present with the steps and operations to unwrap the present.



Example 2 Solving a Two-Step Equation

Solve, then verify each equation.

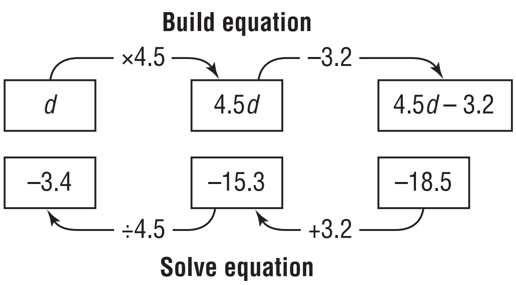
a) $4.5d - 3.2 = -18.5$

b) $\frac{r}{4} + 3 = 7.2$

A Solution

a) $4.5d - 3.2 = -18.5$

Inverse Operations



Algebraic Solution

$$4.5d - 3.2 = -18.5$$

Add 3.2 to each side.

$$4.5d - 3.2 + 3.2 = -18.5 + 3.2$$
$$4.5d = -15.3$$

Divide each side by 4.5.

$$\frac{4.5d}{4.5} = \frac{-15.3}{4.5}$$
$$d = -3.4$$

To verify the solution, substitute $d = -3.4$ into $4.5d - 3.2 = -18.5$.

Left side $= 4.5d - 3.2$ Right side $= -18.5$

$= 4.5 \times (-3.4) - 3.2$

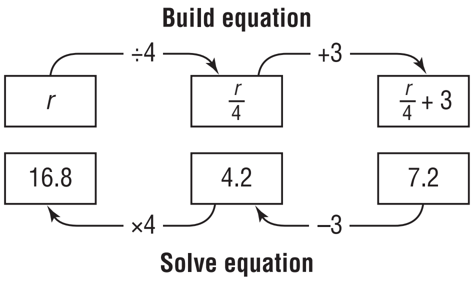
$= -15.3 - 3.2$

$= -18.5$

Since the left side equals the right side, $d = -3.4$ is correct.

b) $\frac{r}{4} + 3 = 7.2$

Inverse Operations



Algebraic Solution

$$\frac{r}{4} + 3 = 7.2$$

Subtract 3 from each side.

$$\frac{r}{4} + 3 - 3 = 7.2 - 3$$

$$\frac{r}{4} = 4.2$$

Multiply each side by 4.

$$4 \times \frac{r}{4} = 4 \times 4.2$$

$$r = 16.8$$

To verify the solution, substitute $r = 16.8$ into $\frac{r}{4} + 3 = 7.2$.

Left side $= \frac{r}{4} + 3$ Right side $= 7.2$

$= \frac{16.8}{4} + 3$

$= 4.2 + 3$

$= 7.2$

Since the left side equals the right side, $r = 16.8$ is correct.

We can use equations to model and solve problems. With practice, you can determine the inverse operations required to solve the equation mentally. In many situations, there may be more than one way to solve the equation.

Math Link

Science

When a freighter unloads its cargo, it replaces the mass of cargo with an equal mass of sea water. This mass of water will keep the ship stable. The volume of sea water added is measured in litres. To relate the volume of water to its mass, we use this formula for density, D:

$D = \frac{M}{V}$, where M = mass, and V = volume

Once a freighter has been unloaded, it is filled with 5 million litres of water. The density of sea water is 1.030 kg/L. What mass of water was added? Solve the equation $1.030 = \frac{M}{5\ 000\ 000}$ to find out.

Example 3 Using an Equation to Model and Solve a Problem

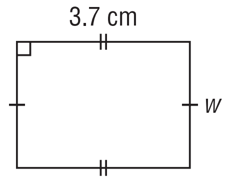
A rectangle has length 3.7 cm and perimeter 13.2 cm.

a) Write an equation that can be used to determine the width of the rectangle.

b) Solve the equation.

c) Verify the solution.

Solutions



a) Let w centimetres represent the width of the rectangle. The perimeter of a rectangle is twice the sum of the length and width. So, the equation is: $13.2 = 2(3.7 + w)$

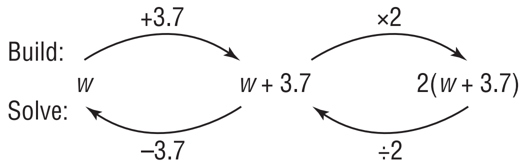
b) Solve the equation.

Method 1

Use inverse operations.

$13.2 = 2(3.7 + w)$

Think:



$$
\begin{aligned}
13.2 &= 2(3.7 + w) && \text{Divide each side by 2.}\\
\frac{13.2}{2} &= \frac{2(3.7 + w)}{2} \\
6.6 &= 3.7 + w && \text{Subtract 3.7 from each side.}\\
6.6 - 3.7 &= 3.7 + w - 3.7 \\
2.9 &= w
\end{aligned}
$$

Method 2

Use the distributive property, then inverse operations.

$$
\begin{aligned}
13.2 &= 2(3.7 + w) && \text{Use the distributive property to expand } 2(3.7 + w).\\
13.2 &= 2(3.7) + 2(w) \\
13.2 &= 7.4 + 2w \\
13.2 - 7.4 &= 7.4 + 2w - 7.4 && \text{Subtract 7.4 from each side.}\\
5.8 &= 2w && \text{Divide each side by 2.}\\
\frac{5.8}{2} &= \frac{2w}{2} \\
2.9 &= w
\end{aligned}
$$

c) Check: The perimeter of a rectangle with length 3.7 cm and width 2.9 cm is:

$$
\begin{aligned}
2(3.7\text{ cm} + 2.9\text{ cm}) &= 2(6.6\text{ cm}) \\
&= 13.2\text{ cm}
\end{aligned}
$$

The solution is correct. The width of the rectangle is 2.9 cm.

Example 4 Using an Equation to Solve a Percent Problem

Seven percent of a number is 56.7.

a) Write, then solve an equation to determine the number.

b) Check the solution.

A Solution

a) Let n represent the number. Then, 7% of the number is $7\% \times n$, or $0.07n$.

An equation is: $0.07n = 56.7$

$0.07n = 56.7$ Divide each side by 0.07.

$\frac{0.07n}{0.07} = \frac{56.7}{0.07}$ Use a calculator.

$n = 810$

The number is 810.

b) $7\% \text{ of } 810 = 0.07 \times 810$

$= 56.7$

So, the solution is correct.

Discuss the ideas

1. How are inverse operations used to solve an equation? How can you verify your solution of an equation?

2. When you build or solve an equation, why must you apply the operations or inverse operations to both sides of the equation?

3. When you verify the solution to an equation, why should you substitute the solution in the original equation?

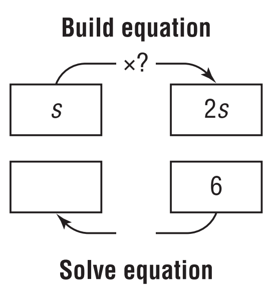
4. When you solve a two-step equation using inverse operations, how is the order in which you apply the inverse operations related to the order in which you would build the end equation?

Practice

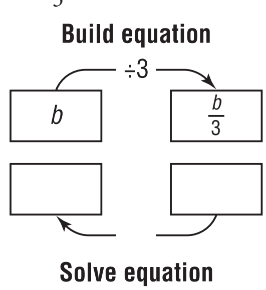
Check

5. Solve each equation by copying and completing the arrow diagram. How do you know that your solution is correct?

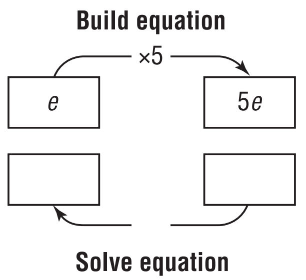
a) $2s = 6$



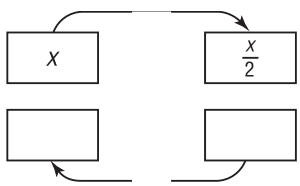
b) $\frac{b}{3} = 5$



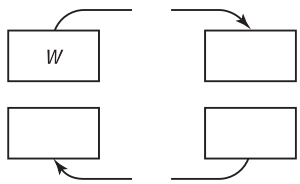
c) $5e = -35$



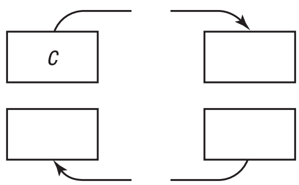
d) $\frac{x}{2} = -7$



e) $-9w = 2.7$

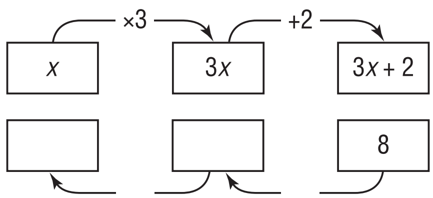


f) $\frac{c}{5} = -1.2$

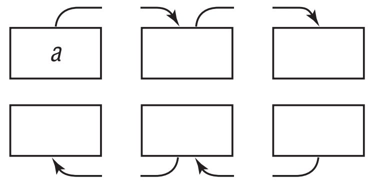


6. Solve each equation by copying and completing the arrow diagram. How do you know that your solution is correct?

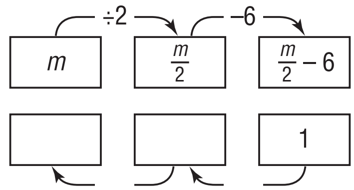
a) $3x + 2 = 8$



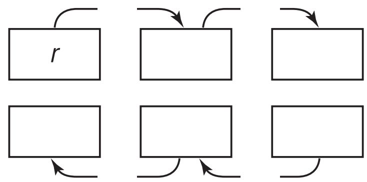
b) $-5a - 6 = 7$



c) $\frac{m}{2} - 6 = 1$



d) $\frac{r}{8} + 5.5 = 2$



7. A student tried to solve the equation $-5m = 15$ by adding 5 to each side. Explain what is wrong with the student's method.
Show the correct way to solve the equation.

Apply

8. Solve each equation.
Which strategy did you use?
Verify the solution.

a) $4x = 9.6$

b) $10 = 3b - 12.5$

c) $-5.25x = -210$

d) $-0.5 = -2x + 8.1$

e) $250 + 3.5n = 670$

f) $-22.5 = -2c - 30.5$

9. For each statement below, write then solve an equation to determine the number. Verify the solution.

a) Two times a number is -10.

b) Three times a number, plus 6.4, is 13.9.

c) Four times a number is -8.8.

d) Ten is equal to two times a number, plus 3.6.

10. Solve each equation. Verify the solution.

a) $\frac{c}{3} = 15$

b) $\frac{m}{6} - 1.5 = -7$

c) $-1.5 = \frac{n}{4}$

d) $5 = \frac{q}{-2} - 5$

e) $\frac{2c}{5} = 1.2$

f) $1.2 = \frac{2a}{3} + 5.1$

11. For each statement below, write then solve an equation to determine the number. Verify the solution.

a) A number divided by 4 is -7.

b) Three, plus a number divided by 5 is 6.

c) One-half of a number is 2.5.

d) One-third of a number, minus 4, is 2.

12. Jenna says that, to build the equation $-2b + 4 = -\frac{3}{4}$, she multiplied each side of the start equation by -2, then added 4 to each side. Can Jenna's partner solve this equation by dividing each side by -2, then subtracting 4 from each side? Explain why or why not.

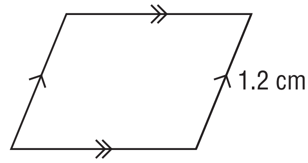
13. Erica is thinking of a number. If you divide her number by 3 then subtract 13.5, the result is 2.8.

a) Let b represent Erica's number. Write an equation to determine this number.

b) Solve the equation.

c) Verify the solution.

14. A parallelogram has one shorter side of length 1.2 cm and perimeter 6.6 cm.



a) Write an equation that can be used to determine the length of the longer side.

b) Solve the equation.

c) Verify the solution.

15. Twelve percent of a number is 39.48.

a) Write, then solve an equation to determine the number.

b) Check the solution.

16. Stephanie has a job in sales. She earns a monthly salary of \$2500, plus a commission of 8% of her sales. One month, Stephanie earns a total of \$2780. This can be represented by the equation

$2780 = 2500 + 0.08s$,

where s is Stephanie's sales in dollars.

a) Solve the equation to determine Stephanie's sales for that month.

b) Verify the solution.

17. Steve works in a clothing store. He earns \$1925 a month, plus a commission of 10% of his sales. One month, Steve earned \$2725.

a) Choose a variable to represent Steve's sales in dollars, then write an equation to determine Steve's sales that month.

b) Solve the equation. What were Steve's sales?

18. Solve each equation. Verify the solution.

a) $5(x - 7) = -15$

b) $2(m + 4) = 11$

c) $-3(t - 2.7) = 1.8$

d) $7.6 = -2(-3 - y)$

e) $8.4 = -6(a + 2.4)$

19. **Assessment Focus** Vianne took 4 bottles of water and 6 bottles of juice to a family picnic. Each bottle of juice contained 0.5 L. The total volume of water and juice was 4.42 L. What was the volume of 1 bottle of water?

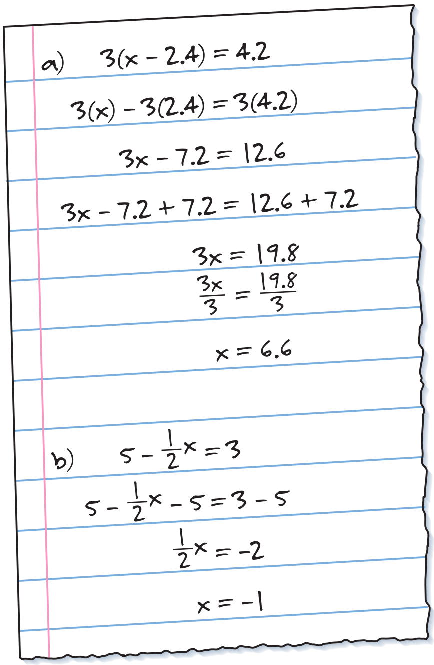
a) Choose a variable and write an equation for this situation.

b) Solve the equation.

c) Verify the solution.

Show your work.

20. On a test, a student solved these equations:



What mistakes did the student make?
Write a correct solution for each equation.

21. A large pizza with tomato sauce and cheese costs \$7.50, plus \$1.50 for each additional topping. A customer orders a large pizza and is charged \$16.50. How many toppings did the customer order?

a) Write an equation to solve the problem.

b) Solve the problem. Verify the solution.

22. An item increased in price by \$4.95. This is a 9% increase. What did the item cost before the price increase?

a) Write an equation to solve the problem.

b) Solve the equation. Verify your solution.

Take It Further

23. The expression $180(n - 2)$ represents the sum of the interior angles in a polygon with n sides. Suppose the sum of its interior angles is 1080°. How many sides does the polygon have?

a) Write an equation to solve the problem.

b) Kyler solves the equation after using the distributive property to simplify $180(n - 2)$. Show the steps in Kyler's solution.

c) Esta solves the equation by undoing the operations that were used to build the equation. Show the steps in Esta's solution.

d) Whose method do you prefer? Explain.

24. Solve each equation. Verify the solution.

a) $4x + \frac{37}{5} = -17$ **b)** $8m - \frac{6}{7} = \frac{176}{7}$

c) $\frac{3}{4} - 5p = \frac{67}{6}$ **d)** $\frac{22}{8} + 10g = \frac{62}{5}$

Reflect

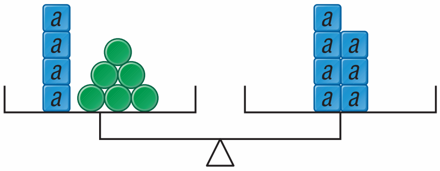
Choose a reversible routine from daily life. Explain why reversing the routine means undoing each step in the reverse order. Explain how this idea can be used to solve an equation. Include an example.

6.2 Solving Equations by Using Balance Strategies

FOCUS

- Model a problem with a linear equation, use balance strategies to solve the equation pictorially, and record the process symbolically.

Tracey has to solve the equation $4a + 6 = 7a$.
Could she use an arrow diagram to model this equation?
How could these balance scales help Tracey?



Each ● has a mass of 1 g.

Investigate

2

Use a model or strategy of your choice to solve this equation: $5a + 7 = 2a + 1$
Record the solution algebraically.
How do you know that your solution is correct?

Reflect & Share

Compare your strategies and solutions with those of another pair of classmates.
If you used different strategies, explain your strategy and choice of strategy.
What are the advantages and disadvantages of each strategy?
Which strategies did not work?

Connect

To solve an equation, we need to isolate the variable on one side of the equation.

In Lesson 6.1, we isolated the variable by reversing the operations acting on the variable. However, this strategy can only be used when the variable occurs once in the equation.

Another way to isolate the variable is to use a balance strategy modelled by balance scales. The scales remain balanced when we do the same thing to each side.

Example 1 Modelling Equations with Variables on Both Sides

a) Solve: $6x + 2 = 10 + 4x$

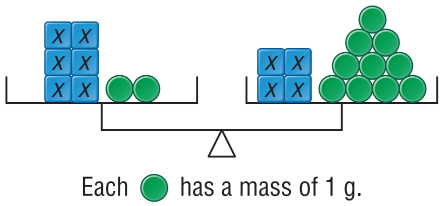
b) Verify the solution.

A Solution

a) Rearrange the equation so that both terms containing the variable are on the same side of the equation. Then isolate the variable to solve the equation.

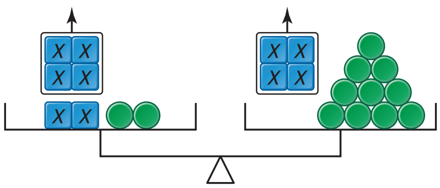
Pictorial Solution | **Algebraic Solution**

Draw balance scales. On the left pan, draw masses to represent $6x + 2$. On the right pan, draw masses to represent $10 + 4x$.



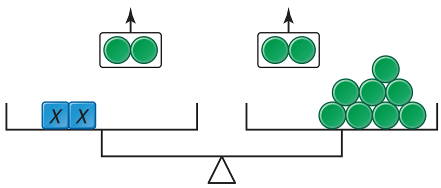
$$6x + 2 = 10 + 4x$$

Remove four x masses from each pan.



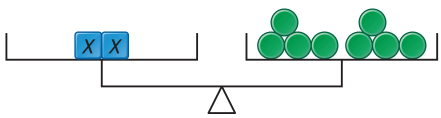
$$6x - 4x + 2 = 10 + 4x - 4x$$
$$2x + 2 = 10$$

Remove two 1-g masses from each pan.



$$2x + 2 - 2 = 10 - 2$$
$$2x = 8$$

Divide the masses in each pan into 2 equal groups. Each x-mass in the left pan corresponds to a group of 4 g in the right pan.



$$\frac{2x}{2} = \frac{8}{2}$$
$$x = 4$$

b) Check: Substitute $x = 4$ in each side of the equation.

Left side	$= 6x + 2$	Right side	$= 10 + 4x$
	$= 6(4) + 2$		$= 10 + 4(4)$
	$= 24 + 2$		$= 10 + 16$
	$= 26$		$= 26$

Since the left side equals the right side, $x = 4$ is correct.

We cannot easily use a balance scales model when any term in an equation is negative. But we can use algebra tiles to model and solve the equation. We use the principle of balance by adding the same tiles to each side or subtracting the same tiles from each side.

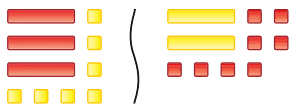
Example 2 Using Algebra Tiles to Solve an Equation

Solve: $-3c + 7 = 2c - 8$

A Solution

Algebra Tile Model

Model the equation with tiles.

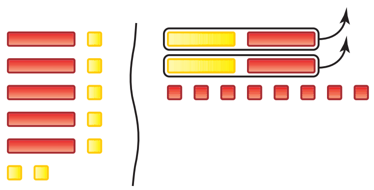


Algebraic Solution

$$-3c + 7 = 2c - 8$$

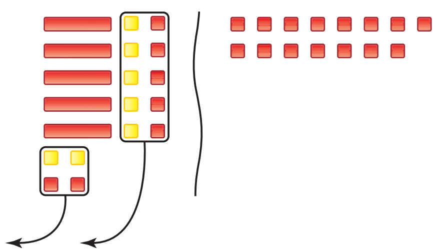
Add two $-c$-tiles to each side to get the terms containing c on the same side. Remove zero pairs.

$$-3c + 7 - 2c = 2c - 8 - 2c$$
$$-5c + 7 = -8$$

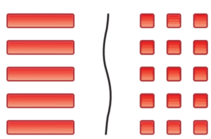


Add seven -1-tiles to each side to get the constant terms on the same side. Remove zero pairs.

$$-5c + 7 - 7 = -8 - 7$$
$$-5c = -15$$



Arrange the remaining tiles on each side into 5 groups.



One $-c$-tile is equal to -3.

$$-1c = -3$$

Flip the tiles. One c-tile is equal to 3.

$$c = 3$$

Equations with rational numbers in fraction or decimal form cannot be modelled easily with balance scales. However, we can solve these equations by doing the same thing to each side of the equation to isolate the variable. We may:

- Add the same quantity to each side.
- Subtract the same quantity from each side.
- Multiply or divide each side by the same non-zero quantity.

Example 3 Solving Equations with Rational Coefficients

Solve each equation, then verify the solution.

a) $\frac{122}{r} = 3, r \neq 0$ **b)** $\frac{2a}{3} = \frac{4a}{5} + 7$

A Solution

Create an equivalent equation without fractions.

a) To clear the fraction, multiply each side by the denominator.

$\frac{122}{r} = 3$ Multiply each side by r.

$r \times \frac{122}{r} = 3 \times r$ Think: $\frac{r}{1} \times \frac{122}{r} = \frac{122}{1}$

$122 = 3r$ Divide each side by 3.

$\frac{122}{3} = \frac{3r}{3}$

$\frac{122}{3} = r$

So, $r = \frac{122}{3}$, or $40\frac{2}{3}$, or $40.\overline{6}$

Check: Substitute $r = \frac{122}{3}$ in the original equation.

Left side $= \frac{122}{r}$ Right side $= 3$

$= \frac{122}{\frac{122}{3}}$

$= \frac{122}{1} \times \frac{3}{122}$

$= 3$

Since the left side equals the right side, $r = \frac{122}{3}$ is correct.

b) To clear the fractions, multiply each side by the common denominator.

$\frac{2a}{3} = \frac{4a}{5} + 7$ Multiply each side by the common denominator 15.

$15 \times \frac{2a}{3} = 15\left(\frac{4a}{5} + 7\right)$ Use the distributive property.

$\overset{5}{\cancel{15}} \times \frac{2a}{\cancel{3}_1} = \overset{3}{\cancel{15}} \times \frac{4a}{\cancel{5}_1} + 15 \times 7$

$10a = 12a + 105$ Subtract $12a$ from each side.

$10a - 12a = 12a + 105 - 12a$

$-2a = 105$ Divide each side by -2.

$$\frac{-2a}{-2} = \frac{105}{-2}$$

$$a = -52\frac{1}{2}, \text{ or } -52.5$$

Check: Substitute $a = -52.5$ in each side of the original equation.

Left side $= \frac{2a}{3}$

$= \frac{2(-52.5)}{3}$

$= -35$

Right side $= \frac{4a}{5} + 7$

$= \frac{4(-52.5)}{5} + 7$

$= -42 + 7$

$= -35$

Since the left side equals the right side, $a = -52.5$ is the correct solution.

Example 4 Using an Equation to Model and Solve a Problem

A cell phone company offers two plans.

Plan A: 120 free minutes, \$0.75 per additional minute

Plan B: 30 free minutes, \$0.25 per additional minute

Which time for calls will result in the same cost for both plans?

a) Model the problem with an equation.

b) Solve the problem.

c) Verify the solution.

A Solution

a) Let t minutes represent the time for calls.

For Plan A, you pay only for the time that is greater than 120 min.

So, the time you pay for is $(t - 120)$ min.

Each minute costs \$0.75, so the cost in dollars is: $0.75(t - 120)$

For Plan B, you pay only for the time that is greater than 30 min.

So, the time you pay for is $(t - 30)$ min.

Each minute costs \$0.25, so the cost in dollars is: $0.25(t - 30)$

When these two costs are equal, the equation is:

$0.75(t - 120) = 0.25(t - 30)$

b)

$0.75(t - 120) = 0.25(t - 30)$ Use the distributive property.

$0.75(t) + 0.75(-120) = 0.25(t) + 0.25(-30)$

$0.75t - 90 = 0.25t - 7.5$ Subtract $0.25t$ from each side.

$0.75t - 0.25t - 90 = 0.25t - 0.25t - 7.5$

$0.50t - 90 = -7.5$ Add 90 to each side.

$0.50t - 90 + 90 = -7.5 + 90$

$0.50t = 82.5$ Divide each side by 0.50.

$\frac{0.50t}{0.50} = \frac{82.5}{0.50}$

$t = 165$

The cost is the same for both plans when the time for calls is 165 min.

c) For Plan A, you pay for: 165 − 120, or 45 min
The cost is: $45 \times \$0.75 = \$33.75$
For Plan B, you pay for: 165 − 30, or 135 min
The cost is: $135 \times \$0.25 = \$33.75$
These costs are equal, so the solution is correct.

Discuss the ideas

1. Why can we not use an arrow diagram to solve an equation with a variable term on each side?

2. When you solve an equation with variables on both sides of the equation, does it matter if you isolate the variable on the left side or the right side of the equation? Explain.

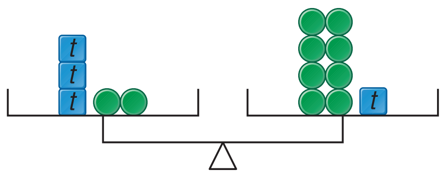
3. For an equation such as $\frac{122}{r} = 3$, why do we include the statement that $r \neq 0$?

Practice

Check

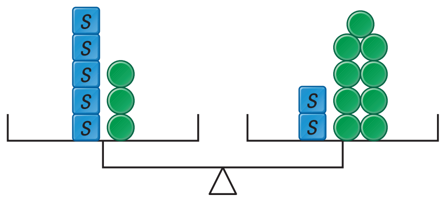
4. Write the equation represented by each picture. Solve the equation. Record the steps algebraically.

a)



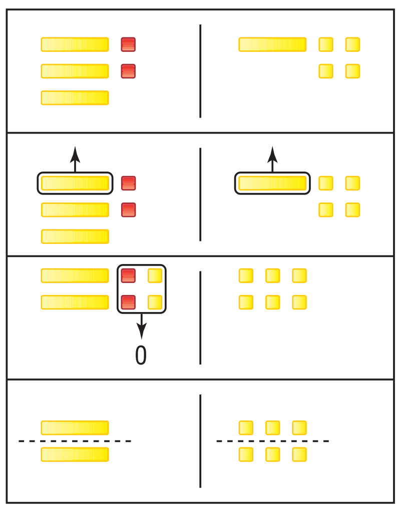
Each ● has a mass of 1 g.

b)



Each ● has a mass of 1 g.

5. Hammy uses algebra tiles to solve the equation $3f - 2 = f + 4$. These pictures show the steps in the solution:



a) Explain the action taken in each step.
b) Record each step algebraically.

6. Use algebra tiles to solve each equation. Record the steps.

a) $4g = 7 - 3g$

b) $4k + 4 = -2k - 8$

c) $-4a - 3 = 3 - a$

d) $3h - 5 = 7 - 3h$

Apply

7. a) Solve each equation.

i) $\frac{6}{h} = 2, h \neq 0$ ii) $\frac{-6}{h} = 2, h \neq 0$

iii) $-2 = \frac{6}{h}, h \neq 0$ iv) $\frac{6}{-h} = 2, h \neq 0$

v) $-2 = \frac{-6}{h}, h \neq 0$ vi) $\frac{6}{-h} = -2, h \neq 0$

b) Explain why there are only 2 solutions to all the equations in part a.

8. Solve each equation.
What strategy did you use?
Verify the solution.

a) $2.4 = \frac{4.8}{s}, s \neq 0$

b) $\frac{-5.4}{t} = 1.8, t \neq 0$

c) $-6.5 = \frac{-1.3}{w}, w \neq 0$

9. Ten divided by a number is -3. Write, then solve an equation to determine the number. Verify the solution.

10. Solve each equation.
What strategy did you use?
Verify the solution.

a) $-12a = 15 - 15a$

b) $10.6y = 2.1y - 27.2$

c) $-10.8 + 7z = 5z$

d) $6u - 11.34 = 4.2u$

e) $-20.5 - 2.2b = -7.2b$

f) $-5.3p = -9 - 8.9p$

11. Solve each equation. Verify the solution.

a) $2 - 3n = 2n + 7$

b) $13 - 3q = 4 - 2q$

c) $-2.4a + 3.7 = -16.1 + 3.1a$

d) $8.8v + 2.1 = 2.3v - 16.1$

e) $-2.5x - 2 = -5.7x + 6$

f) $6.4 - 9.3b = 25.3 - 3.9b$

12. Two rental halls are considered for a wedding.

Hall A costs \$50 per person.

Hall B costs \$2000, plus \$40 per person.

Determine the number of people for which the halls will cost the same to rent.

a) Model this problem with an equation.

b) Solve the problem.

c) Verify the solution.

13. Five subtract 3 times a number is equal to 3.5 times the same number, subtract 8. Write, then solve an equation to determine the number. Verify the solution.

14. A part-time sales clerk at a store is offered two methods of payment.

Plan A: \$1500 per month with a commission of 4% on his sales

Plan B: \$1700 per month with a commission of 2% on his sales

Let s represent the sales in dollars.

a) Write an expression to represent the total earnings under Plan A.

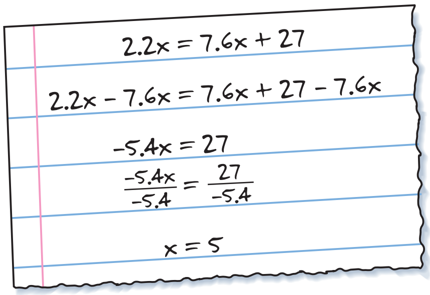
b) Write an expression to represent the total earnings under Plan B.

c) Write an equation to determine the sales that result in the same total earnings from both plans.

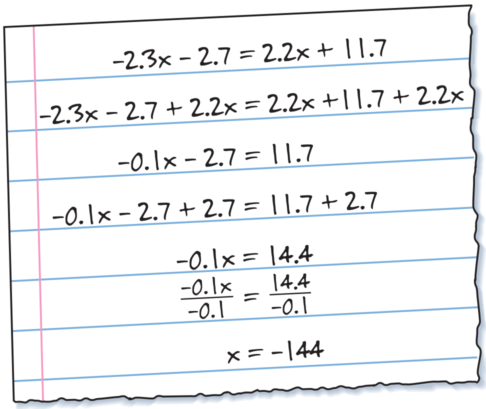
d) Solve the equation. Explain what the answer represents.

15. Verify each student's work.
If the solution is incorrect, write a correct and complete solution.

a) Student A:


$2.2x = 7.6x + 27$
$2.2x - 7.6x = 7.6x + 27 - 7.6x$
$-5.4x = 27$
$\frac{-5.4x}{-5.4} = \frac{27}{-5.4}$
$x = 5$

b) Student B:


$-2.3x - 2.7 = 2.2x + 11.7$
$-2.3x - 2.7 + 2.2x = 2.2x + 11.7 + 2.2x$
$-0.1x - 2.7 = 11.7$
$-0.1x - 2.7 + 2.7 = 11.7 + 2.7$
$-0.1x = 14.4$
$\frac{-0.1x}{-0.1} = \frac{14.4}{-0.1}$
$x = -144$

16. a) Solve each pair of equations.

i) $\frac{x}{27} = 3; \frac{27}{x} = 3, x \neq 0$

ii) $\frac{a}{36} = 12; \frac{36}{a} = 12, a \neq 0$

b) How are the steps to solve for a variable in the denominator of a fraction similar to the steps used to solve for a variable in the numerator? How are they different? Explain.

17. Solve each equation. Verify the solution.

a) $4(g + 5) = 5(g - 3)$

b) $3(4j + 5) = 2(-10 + 5j)$

c) $2.2(h - 5.3) = 0.2(-32.9 + h)$

d) $0.04(5 - s) = 0.05(6 - s)$

18. Assessment Focus Hendrik has a choice of 2 companies to rent a car.
Company A charges \$199 per week, plus \$0.20 per kilometre driven.
Company B charges \$149 per week, plus \$0.25 per kilometre driven.
Determine the distance that Hendrik must drive for the two rental costs to be the same.

a) Model this problem with an equation.

b) Solve the problem.

c) Verify the solution.

Show your work.

19. Solve each equation.

a) $\frac{7}{2}(m + 12) = \frac{5}{2}(20 + m)$

b) $\frac{1}{3}(5 - 3t) = \frac{5}{6}(t - 2)$

c) $\frac{3}{2}(1 + 3r) = \frac{2}{3}(2 - 3r)$

d) $\frac{2}{3}(6x + 5) = \frac{4}{5}(20x - 7)$

20. Both Dembe and Bianca solve the equation:
$\frac{x}{3} + \frac{x}{4} = x - \frac{1}{6}$
Dembe clears the fractions by multiplying each side by 12. Bianca clears the fractions by multiplying each side by 24.

a) Solve the equation using each student's method. Compare the solutions.

b) When you solve an equation involving fractions, why is it a good idea to multiply each side by the least common denominator?

21. Solve each equation. Verify the solution.

a) $\frac{x}{4} + \frac{7}{4} = \frac{5}{6}$

b) $\frac{5x}{16} - \frac{5}{4} = \frac{x}{4}$

c) $2 - \frac{x}{24} = \frac{5x}{24} + 1$

d) $\frac{25}{9} + \frac{x}{9} = \frac{7x}{6} - \frac{5}{2}$

Take It Further

22. In volleyball, statistics are kept about players. The equation $B = M + \frac{1}{2}A$ can be used to calculate the total blocks made by a player. In the equation, B is the total blocks, M is the number of solo blocks, and A is the number of assisted blocks. Marlene has 9 total blocks and 4 solo blocks. How many assisted blocks did Marlene make? How do you know that your answer is correct?

23. A cell phone company offers two different plans.

Plan A
Monthly fee of \$28
30 free minutes
\$0.45 per additional minute

Plan B
Monthly fee of \$40
No free minutes
\$0.25 per minute

a) Write an equation to determine the time in minutes that results in the same monthly cost for both plans.

b) Solve the equation.

c) Verify the solution.

Reflect

List some strategies for solving an equation. For each strategy, provide an example of an equation and its solution.

Math Link

Science

Ohm's Law relates the resistance, R ohms, in an electrical circuit to the voltage, V volts, across the circuit and the current, I amperes, through the circuit: $R = \frac{V}{I}$

For a light bulb, when the voltage is 120 V and the resistance is 192 Ω, the current in amperes can be determined by solving this equation: $192 = \frac{120}{I}$

Start Where You Are

How Can I Use My Problem-Solving Skills?

Suppose I have to solve this problem:

The sale price of a jacket is \$41.49.
The original price has been reduced by 17%.
What was the original price?

➤ What do I know?

- The sale price is \$41.49.
- This is 17% less than the original price.

➤ What strategy could I use to solve the problem?

- I could write, then solve an equation.
  Let d dollars represent the original price.
  17% of d is $0.17d$.
  A word equation is:
  (original price) − (17% of original price) is \$41.49
  An algebraic equation is:

$$1d - 0.17d = 41.49 \quad \text{Combine the terms in } d.$$
$$0.83d = 41.49 \quad \text{Divide each side by 0.83.}$$
$$\frac{0.83d}{0.83} = \frac{41.49}{0.83}$$
$$d \doteq 49.99$$

- I could write, then solve a proportion.
  Let d dollars represent the original price, which is 100%.
  Since the price has been reduced by 17%,
  the sale price is 100% − 17%, or 83% of the original price.
  So, the ratio of sale price to original price is equal
  to the ratio of 83% to 100%.
  As a proportion: $\frac{41.49}{d} = \frac{83}{100}$

$$\frac{41.49}{d} = \frac{83}{100} \quad \text{Multiply each side by 100.}$$
$$100 \times \frac{41.49}{d} = 100 \times \frac{83}{100}$$
$$\frac{4149}{d} = 83 \quad \text{Multiply each side by } d.$$
$$d \times \frac{4149}{d} = 83 \times d$$
$$4149 = 83d \quad \text{Divide each side by 83.}$$
$$\frac{4149}{83} = \frac{83d}{83}$$
$$49.99 \doteq d$$

- I could draw a diagram to help me reason the answer.

The original price is 100%.
Since the price has been reduced by 17%,
the sale price is 100% − 17%,
or 83% of the original price.
83% of the original price is \$41.49
So, 1% of the original price is $\frac{\$41.49}{83}$

And, 100%, which is the original price $= \frac{\$41.49}{83} \times 100$

$$= \frac{\$4149}{83}$$

$$\doteq \$49.99$$

The original price was \$49.99.

Check: The discount is:

$$17\% \text{ of } \$49.99 = 0.17 \times \$49.99 = \$8.50$$

$$\text{Sale price} = \text{original price} - \text{discount} = \$49.99 - \$8.50 = \$41.49$$

This is the same as the given sale price, so the answer is correct.

➤ Look back.
Which method do you find easiest? Why?
Would you have solved the problem a different way?
If your answer is yes, show your method.

Check

1. The price of gasoline increased by 6%. The new price is \$1.36/L.
   What was the price of gasoline before it increased?

2. Make up your own percent problem. Solve your problem.
   Trade problems with a classmate, then solve your classmate's problem.
   Compare your strategies for answering both problems.

Mid-Unit Review

6.1

1. For each equation, write the first operation you would use to isolate the variable. Justify your choice of operation.

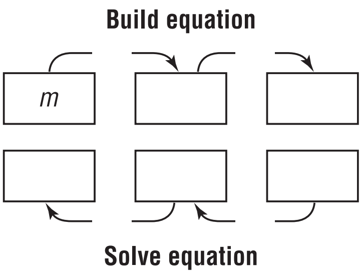
a) $-3j = 9.6$

b) $\frac{1}{4}r - 2 = 4$

c) $2(-3x + 1.5) = 6$

d) $3 = -2n + 9$

2. Marshall creates this arrow diagram to show the steps in the solution of $\frac{m}{10} + 20.3 = 45.5$.



a) Copy and complete the arrow diagram.

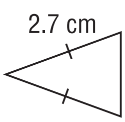
b) Record the solution algebraically.

3. Sheila is charged a fare of $27.70 for a cab ride to her friend's house. The fare is calculated using a flat fee of $2.50, plus $1.20 per kilometre. What distance did Sheila travel?

a) Let k kilometres represent the distance travelled. Write an equation to solve the problem. Solve the problem.

b) Verify the solution.

4. An isosceles triangle has two equal sides of length 2.7 cm and perimeter 7.3 cm.



a) Write an equation that can be used to determine the length of the third side.

b) Solve the equation.

c) Verify the solution.

5. Solve each equation. Verify the solution.

a) $\frac{k}{3} = -1.5$

b) $10.5 = 3b - 12.5$

c) $5(x - 7.2) = 14.5$

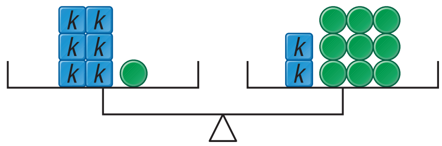
d) $8.4 = 1.2b$

e) $2 + \frac{n}{3} = 2.8$

f) $-8 = 0.4(3.2 + h)$

6.2

6. Write the equation modelled by these balance scales. Solve the equation. Verify the solution.



Each ● has a mass of 1 g.

7. Solve each equation. Verify the solution.

a) $\frac{56}{a} = -3.5, a \neq 0$

b) $8w - 12.8 = 6w$

c) $-8z + 11 = -10 - 5.5z$

d) $\frac{5x}{2} = 11 + \frac{2x}{3}$

e) $0.2(5 - 2r) = 0.3(1 - r)$

f) $12.9 + 2.3y = 4.5y + 19.5$

g) $\frac{2}{5}(m + 4) = \frac{1}{5}(3m + 9)$

8. Skateboards can be rented from two shops in a park.

Shop Y charges $15 plus $3 per hour
Shop Z charges $12 plus $4 per hour

Determine the time in hours for which the rental charges in both shops are equal.

a) Write an equation to determine the time.

b) Solve the equation.

c) Verify the solution.

GAME

Equation Persuasion

You will need

- a deck of playing cards

Number of Players

- 2

Goal of the Game

- To solve and verify your partner's end equation

How to Play

1. Each player picks a secret integer between -9 and 9.
Use the secret integer to write an equation of the form:
n = secret integer

2. Remove the face cards from the deck.
Each player draws 3 cards from the deck, one at a time.

Suit	Meaning
♣	Add the number on the card to each side of the equation.
♠	Add the indicated number of *n*s to each side of the equation.
♦	Multiply each side of the equation by the number on the card.
♥	Subtract the number on the card from each side of the equation.

For example:

- Suppose you choose a secret number of -2 and draw these three cards in the given order: 4 of ♣, 3 of ♠, and 5 of ♦.
- Secretly perform the operations indicated by the cards on the start equation: $n = -2$

  Add 4 to each side of the equation: $n + 4 = 2$

  Add $3n$ to each side of the equation: $4n + 4 = 2 + 3n$

  Multiply each side by 5: $5(4n + 4) = 5(2 + 3n)$

  This is the end equation.

3. Trade end equations with your partner.
Solve each other's end equation.

4. If you solved your partner's equation correctly, you receive 5 points. You get an additional 5 points if you verify the solution. But, if you gave your partner an incorrect end equation, you get 0 points.

5. The first player to get 50 points wins.

6.3 Introduction to Linear Inequalities

FOCUS
- Write and graph inequalities.

We use an **inequality** to model a situation that can be described by a range of numbers instead of a single number.
When one quantity is less than or equal to another quantity, we use this symbol: $\leq$
When one quantity is greater than or equal to another quantity, we use this symbol: $\geq$
Which of these inequalities describes the time, t minutes, for which a car could be legally parked?

$t > 30$
$t \geq 30$
$t < 30$
$t \leq 30$

Inequality signs
$<$ less than
$>$ greater than

Investigate

Define a variable and write an inequality for each situation.

a)


b)


c)


d)


Reflect & Share

Compare your inequalities with those of another pair of classmates.
If the inequalities are different, how can you find out which is correct?
Work together to describe three other situations that involve inequalities.
Write an inequality for each situation.

Connect

Here are some examples of inequality statements:

- One expression is less than another; a is less than 3: $a < 3$
- One expression is greater than another; b is greater than -4: $b > -4$
- One expression is less than or equal to another;
  c is less than or equal to $\frac{3}{4}$: $c \leq \frac{3}{4}$
- One expression is greater than or equal to another;
  d is greater than or equal to -5.4: $d \geq -5.4$

Many real-world situations can be modelled by inequalities.

Example 1 Writing an Inequality to Describe a Situation

Define a variable and write an inequality to describe each situation.

a) Contest entrants must be at least 18 years old.

b) The temperature has been below -5°C for the last week.

c) You must have 7 items or less to use the express checkout line at a grocery store.

d) Scientists have identified over 400 species of dinosaurs.

A Solution

a) Let a represent the age of a contest entrant.
"At least 18" means that entrants must be 18, or 19, or 20, and so on.
So, a can equal 18 or be greater than 18.
The inequality is $a \geq 18$.

b) Let t represent the temperature in degrees Celsius.
For the temperature to be "below -5°C", it must be less than -5°C.
The inequality is $t < -5$.

c) Let n represent the number of items.
The number of items must be 7 or less than 7.
The inequality is $n \leq 7$.

d) Let s represent the number of species of dinosaurs.
"Over 400" means greater than 400.
The inequality is $s > 400$.

A linear equation is true for only one value of the variable.
A linear inequality may be true for many values of the variable.
The solution of an inequality is any value of the variable that makes the inequality true.
There are usually too many numbers to list, so we may show them on a number line.

Example 2 Determining Whether a Number Is a Solution of an Inequality

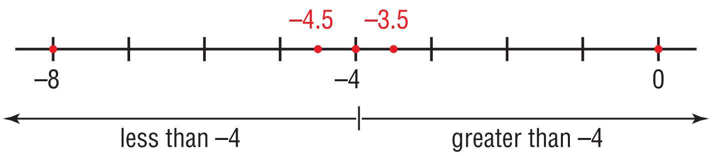
Is each number a solution of the inequality $b \geq -4$? Justify the answers.

a) -8 **b)** -3.5 **c)** -4 **d)** -4.5 **e)** 0

Solutions

Method 1

Use a number line. Show all the numbers on a line.
The solution of $b \geq -4$ is all numbers that are greater than or equal to -4.



For a number to be greater than -4, it must lie to the right of -4.

a) -8 is to the left of -4, so -8 is not a solution.

b) -3.5 is to the right of -4, so -3.5 is a solution.

c) -4 is equal to itself, so it is a solution.

d) -4.5 is to the left of -4, so -4.5 is not a solution.

e) 0 is to the right of -4, so 0 is a solution.

Method 2

Use substitution. Substitute each number for b in the inequality $b \geq -4$.
Determine whether the resulting inequality is true or false.

a) Since $-8 \geq -4$ is false, -8 is not a solution.

b) Since $-3.5 \geq -4$ is true, -3.5 is a solution.

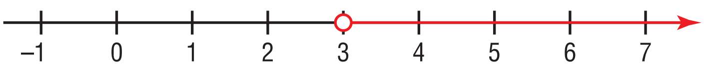
c) Since $-4 = -4$, -4 is a solution.

d) Since $-4.5 \geq -4$ is false, -4.5 is not a solution.

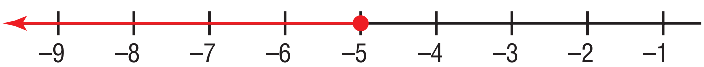
e) Since $0 \geq -4$ is true, 0 is a solution.

We can illustrate the solutions of an inequality by graphing them on a number line.

For $a > 3$, the solution is all numbers greater than 3. Since 3 is not part of the solution, we draw an open circle at 3 to indicate this.



For $b \leq -5$, the solution is all numbers less than or equal to -5. Since -5 is part of the solution, we draw a shaded circle at -5 to indicate this.



Example 3 Graphing Inequalities on a Number Line

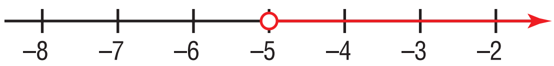
Graph each inequality on a number line.
Write 4 numbers that are solutions of the inequality.

a) $t > -5$ **b)** $-2 \geq x$ **c)** $0.5 \leq a$ **d)** $p < -\frac{25}{3}$

A Solution

a) $t > -5$

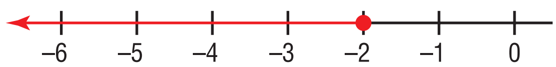
Any number greater than -5 satisfies the inequality.



Four possible solutions are:

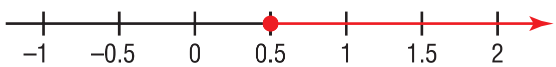
$-4, -2.1, 0, \frac{1}{2}$

b) $-2 \geq x$ means that -2 is greater than or equal to x, or x is less than or equal to -2; that is, $x \leq -2$



Four possible solutions are:
$-2, -4\frac{1}{4}, -6.8, -100$

c) $0.5 \leq a$ means that 0.5 is less than or equal to a, or a is greater than or equal to 0.5; that is, $a \geq 0.5$

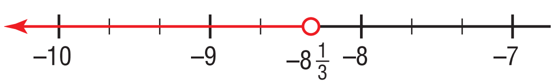


Four possible solutions are:
$0.5, 2, 3\frac{3}{4}, 1000$

d) $p < -\frac{25}{3}$

$-\frac{25}{3}$ is $-8\frac{1}{3}$.

The solution is all numbers that are less than $-8\frac{1}{3}$.



Four possible solutions are:
$-9, -15.8, -20\frac{2}{5}, -99$

Discuss the ideas

1. How is the solution of an inequality different from the solution of an equation?

2. How do you know whether to use an open circle or a shaded circle in the graph of an inequality?

Practice

Check

3. Is each inequality true or false? Explain your reasoning.

   a) $5 < 8$
   b) $-5 < -8$
   c) $5 < -8$
   d) $5 < 5$
   e) $5 \leq 5$
   f) $0 \geq -5$
   g) $5.01 < 5.1$
   h) $\frac{1}{5} < \frac{1}{8}$

4. Use a symbol to write an inequality that corresponds to each statement.

   a) x is less than -2.
   b) p is greater than or equal to 6.
   c) y is negative.
   d) m is positive.

5. Is each number a solution of $x < -2$? How do you know?

   a) 0
   b) -6.9
   c) -2.001
   d) -3
   e) -2
   f) $-\frac{1}{2}$

6. Write 4 numbers that are solutions of each inequality.

   a) $b > 5$
   b) $7 > x$
   c) $-2 \leq v$
   d) $w \leq -12$

Apply

7. Determine whether the given number is a solution of the inequality. If the number is not a solution, write an inequality for which the number is a solution.

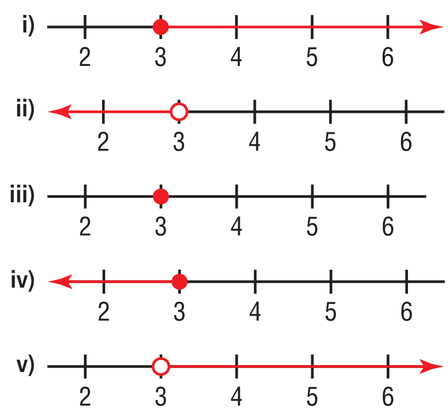
   a) $w < 3$; 3
   b) $-3.5 < y$; 0
   c) $m \geq 5\frac{1}{2}$; 5.05
   d) $a \leq -2$; -15

8. Define a variable and write an inequality to model each situation.

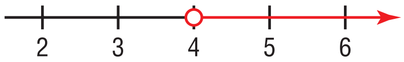
   a) A coffee maker can hold no more than 12 cups of water.
   b) You must be at least 15 years old to obtain a learner's permit to drive in Nunavut.
   c) The maximum seating capacity of a school bus is 48 students.
   d) Over 2500 people participate in the charity bike-a-thon each year.
   e) The shoe store sells sizes no larger than 13.

9. Match each equation or inequality with the graph of its solution below. Justify your choice.

   a) $m > 3$
   b) $p = 3$
   c) $k \leq 3$
   d) $t < 3$
   e) $v \geq 3$
   f) $3 < n$
   g) $3 \geq h$
   h) $3 \leq s$



10. Tom and Stevie write the inequality whose solution is shown on this graph:



Tom writes $a > 4$. Stevie writes $4 < b$. Can both of them be correct? Explain.

11. Assessment Focus

a) For each situation, define a variable and write an inequality to describe the situation.

i) In Canada, a child under 23 kg must ride in a car seat.

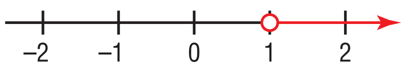
ii) A silicone oven mitt is heat resistant to temperatures up to 485°C.

iii) The minimum wage in Alberta is $8.40 an hour.

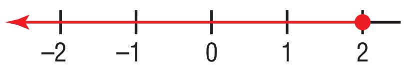
b) Graph the solution of each inequality on a number line.

12. Write an inequality whose solution is graphed on the number line. In each case, are 1 and -3 solutions of the inequality? Explain.

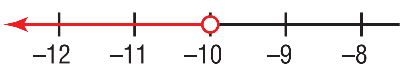
a)



b)



c)



13. Graph the solution of each inequality on a number line.

a) $w > 5.5$

b) $x \leq -2$

c) $z > -6$

d) $a < 6.8$

e) $b \leq 6.8$

f) $c > \frac{2}{3}$

g) $d \leq -\frac{2}{3}$

h) $x \leq \frac{18}{5}$

Take It Further

14. Joel is producing a one-hour TV show. An advertiser wants at least 12 min of commercials, and the station will not allow more than 20 min of commercials. Graph the possible show times on a number line. Write two inequalities to describe the situation.

15. The words "over," "under," "maximum," "minimum," "at least," and "no more than" can describe inequalities.

a) Which symbol describes each word?

b) Give a real-world situation that could be described by each word. Write the situation as an inequality.

16. Use a symbol to write an inequality for this statement: y is not negative.
Justify your inequality.

Reflect

An inequality can be described with words, symbols, or a graph. Which representation do you find easiest to understand? Explain. Include an example in your explanation.

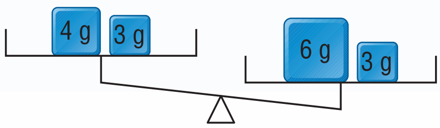
6.4 Solving Linear Inequalities by Using Addition and Subtraction

FOCUS

- Use addition and subtraction to solve inequalities.

Jamina places masses on balance scales.
Why is the right pan lower than the left pan?
Will the right pan remain lower than the left pan in each situation?

- Jamina places 2 g on each pan.
- Jamina removes 3 g from each pan.



Investigate

2

➤ Write two different numbers.
Write the symbol $<$ or $>$ between the numbers to make an inequality.

➤ Choose another number. Add that number to each side of the inequality.
Is the resulting inequality still true?

➤ Repeat the preceding step 3 more times with different numbers.

➤ Subtract the same number from each side of the original inequality.
Is the inequality still true?

➤ Repeat the preceding step 3 more times with different numbers.

Reflect & Share

Compare your results with those of another pair of classmates.
When the same number is added to or subtracted from each side of an inequality, is the resulting inequality still true? Explain.
How could you use this property to solve the inequality $x + 5 \geq 11$?
Work together to solve the inequality.
How do you know that your solution is correct?

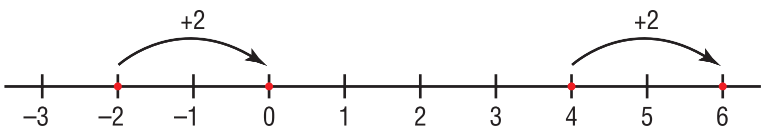
Connect

We can use a number line to investigate the effect of adding to and subtracting from each side of an inequality.

-2 is less than 4 because -2 is to the left of 4 on a number line.

➤ Adding the same number to each side of an inequality

$$-2 < 4 \qquad \text{Add 2 to each side.}$$
$$-2 + 2 < 4 + 2$$
$$0 < 6 \qquad \text{This resulting inequality is true.}$$



➤ Subtracting the same number from each side of an inequality

$$-2 < 4 \qquad \text{Subtract 1 from each side.}$$
$$-2 - 1 < 4 - 1$$
$$-3 < 3 \qquad \text{This resulting inequality is true.}$$

When we add the same number to, or subtract the same number from, each side of an inequality, the points move left or right, but their relative positions do not change.

The examples above illustrate this property of inequalities:

➤ When the same number is added to or subtracted from each side of an inequality, the resulting inequality is still true.

To solve an inequality, we use the same strategy as for solving an equation: isolate the variable by adding to or subtracting from each side of the inequality. Compare the following solutions of an equation and a related inequality.

Equation

$$h + 3 = 5$$
$$h + 3 - 3 = 5 - 3$$
$$h = 2$$

There is only one solution: $h = 2$

Inequality

$$h + 3 < 5$$
$$h + 3 - 3 < 5 - 3$$
$$h < 2$$

There are many solutions; too many to list. Any number that is less than 2 is a solution; for example, 0, -5.7, -3452, and so on

Example 1 Solving an Inequality

a) Solve the inequality: $6.2 \leq x - 4.5$ b) Verify the solution. c) Graph the solution.

A Solution

a) $6.2 \leq x - 4.5$ Add 4.5 to each side.

$6.2 + 4.5 \leq x - 4.5 + 4.5$

$10.7 \leq x$

b) The solution of the inequality $10.7 \leq x$ is all numbers greater than or equal to 10.7.

Choose several numbers greater than 10.7; for example, 11, 20, 30

Substitute $x = 11$ in the original inequality.

Left side $= 6.2$ Right side $= x - 4.5$

$= 11 - 4.5$

$= 6.5$

Since $6.2 < 6.5$, the left side is less than the right side, and $x = 11$ satisfies the inequality.

Substitute $x = 20$ in the original inequality.

Left side $= 6.2$ Right side $= x - 4.5$

$= 20 - 4.5$

$= 15.5$

Since $6.2 < 15.5$, the left side is less than the right side, and $x = 20$ satisfies the inequality.

Substitute $x = 30$ in the original inequality.

Left side $= 6.2$ Right side $= x - 4.5$

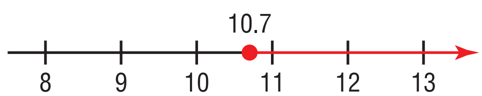
$= 30 - 4.5$

$= 25.5$

Since $6.2 < 25.5$, the left side is less than the right side, and $x = 30$ satisfies the inequality.

Since all 3 substitutions verify the inequality, it suggests that $x \geq 10.7$ is correct.

c) Graph the solution on a number line.



It is impossible to check all of the solutions of an inequality. We verified the inequality in *Example 1* by selecting several numbers from the solution and substituting them into the original inequality. Since the resulting statements were true, this suggests that the solution is correct.

A term containing a variable represents a number, so this term can be added to or subtracted from each side of an inequality.

Example 2 Using an Inequality to Model and Solve a Problem

Jake plans to board his dog while he is away on vacation.

- Boarding house A charges \$90 plus \$5 per day.
- Boarding house B charges \$100 plus \$4 per day.

For how many days must Jake board his dog for boarding house A to be less expensive than boarding house B?

a) Choose a variable and write an inequality that can be used to solve this problem.

b) Solve the problem.

c) Graph the solution.

A Solution

a) Let d represent the number of days Jake boards his dog. For house A: \$90 + \$5/day can be written, in dollars, as $90 + 5d$. For house B: \$100 + \$4/day can be written, in dollars, as $100 + 4d$. For house A to be less expensive than house B, $90 + 5d$ must be less than $100 + 4d$.

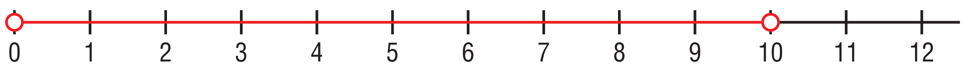
So, an inequality is: $90 + 5d < 100 + 4d$

b)

$$90 + 5d < 100 + 4d \qquad \text{Subtract } 4d \text{ from each side.}$$
$$90 + 5d - 4d < 100 + 4d - 4d$$
$$90 + d < 100 \qquad \text{Subtract 90 from each side.}$$
$$90 - 90 + d < 100 - 90$$
$$d < 10$$

Boarding house A is less expensive if Jake leaves his dog there for less than 10 days.

c) $d < 10$



In *Example 2*, the number line begins with a circle at 0 because a dog cannot be boarded for a negative number of days.

Discuss the ideas

1. Why is it impossible to check all the solutions of an inequality?

2. When a solution of an equation is verified, we say that the solution is correct. When a solution of an inequality is verified, we can only say that this suggests the solution is correct. Why?

3. Suppose the solution of an inequality is $r \geq 5.6$. How would you choose suitable values of r to substitute to check?

Practice

Check

4. Which operation will you perform on each side of the inequality to isolate the variable?

a) $a + 4 > 3$ **b)** $0 < -\frac{2}{3} + m$

c) $r - 4 \geq -3$ **d)** $k - 4.5 \leq 5.7$

e) $s + \frac{3}{10} \leq -3$ **f)** $6.1 > 4.9 + z$

5. What must you do to the first inequality to get the second inequality?

a) $x - 2 > 8$
$x > 10$

b) $12.9 \leq y + 4.2$
$y \geq 8.7$

c) $p - \frac{1}{2} \leq \frac{1}{2}$
$p \leq 1$

6. State three values of x that satisfy each inequality: one integer, one fraction, and one decimal.

a) $x + 3 \geq 7$ **b)** $x - 3 \leq 7$

c) $x + 7 < 3$ **d)** $x - 3 > 7$

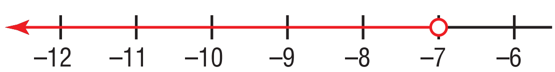
Apply

7. Match each inequality with the graph of its solution below. Is 3 a possible solution of each inequality? How can you find out?

a) $c - 2 > 2$ **b)** $8 \geq -5 + w$

c) $1 > r + 8$ **d)** $7 + m \leq -2$

i)



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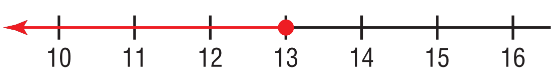

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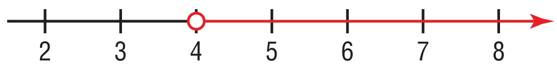

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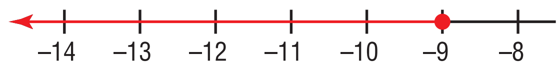


8. Solve each inequality. Graph the solution. Verify the solution.

a) $x + 5 > 2$ **b)** $-9 \geq y - 3$

c) $4 + a \leq 8$ **d)** $2 > x + 7$

e) $k + 8 < -13$ **f)** $q - 2.5 < 3.9$

9. Solve each inequality. Graph the solution. Show the steps in the solution.
Verify the solution by substituting 3 different numbers in each inequality.

a) $4t - 19 < 24 + 3t$

b) $3x < 2x - 11$

c) $5x - 7 < 4x + 4$

d) $2 + 3a \leq 2a - 5$

e) $1.7p + 2.8 \geq 0.7p - 7.6$

f) $2y + 13.3 \geq y - 24.1$

10. A student says $b \geq -9$ is the solution of $-7 \geq b + 2$ because substituting -9 into the original inequality gives the true statement $-7 \geq -7$. Do you agree? Justify your answer.

11. a) Solve the equation: $7.4 + 2p = p - 2.8$

b) Solve the inequality: $7.4 + 2p \geq p - 2.8$

c) Compare the processes in parts a and b. How is solving an inequality like solving the related equation? How is it different?

d) Compare the solutions in parts a and b. How is the solution of an inequality like the solution of the related equation? How is it different?

12. Joel currently has a balance of $212.35 in his bank account. He must maintain a minimum balance of $750 in the account to avoid paying a monthly fee. How much money can Joel deposit into his account to avoid paying this fee?

a) Choose a variable, then write an inequality that can be used to solve this problem.

b) Solve the problem.

c) Graph the solution.

13. Teagan is saving money to buy a snowmobile helmet. One weekend, she earned $20 to add to her savings, but she still did not have the $135.99 she needed for the helmet.

a) Choose a variable, then write an inequality to represent this situation.

b) Solve the inequality. What does the solution represent?

c) Verify the solution and graph it on a number line.

14. Assessment Focus Marie has $4.85. She wants to buy a muffin and a cake at a bake sale. The cake is on sale for $3.45. How much can Marie spend on a muffin?

a) Choose a variable, then write an inequality to solve the problem.

b) Use the inequality to solve the problem.

c) Graph the solution on a number line.

d) A deluxe muffin costs $1.45. Can Marie afford to buy this muffin? Justify your answer.

Show your work.

Take It Further

15. a) Solve each inequality. Graph the solution.

i) $2a - 5 \geq 2 + 3a$

ii) $0.7p - 7.6 \leq 1.7p + 2.8$

b) What strategies did you use to solve the inequalities in part a?

c) Compare your solution and graphs in part a with the solutions to questions 9d and 9e. Explain the differences.

16. a) Graph each inequality. Describe the solution in words.

i) $x < -2.57$

ii) $b \geq -10.25$

iii) $p \leq 1.005$

b) Explain how the graphs of these inequalities are different from those that you have graphed before.

c) Which is a more accurate way to describe a solution: using an inequality or using a graph? Explain.

Reflect

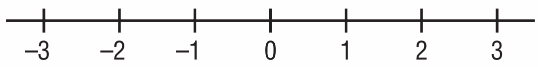
How is solving an inequality by using addition or subtraction similar to solving an equation by using addition or subtraction? How is it different? Use an example in your explanation.

6.5 Solving Linear Inequalities by Using Multiplication and Division

FOCUS

- Use multiplication and division to solve inequalities.

How does the position of a number on a number line determine whether it is greater than or less than another number?
How does this explain why $2 < 3$ but $-2 > -3$?



Investigate

In the patterns below, each side of the inequality $12 > 6$ is multiplied or divided by the same non-zero number.

Multiplication Pattern

$12 > 6$
$12(-3) \square 6(-3)$
$12(-2) \square 6(-2)$
$12(-1) \square 6(-1)$
$12(1) \square 6(1)$
$12(2) \square 6(2)$
$12(3) \square 6(3)$

Division Pattern

$12 > 6$
$12 \div (-3) \square 6 \div (-3)$
$12 \div (-2) \square 6 \div (-2)$
$12 \div (-1) \square 6 \div (-1)$
$12 \div 1 \square 6 \div 1$
$12 \div 2 \square 6 \div 2$
$12 \div 3 \square 6 \div 3$

➤ Copy and simplify each expression in the patterns.
➤ Replace each □ with < or > to create a true statement.
➤ Compare the inequality signs in the pattern with the inequality sign in $12 > 6$.
When did the inequality sign stay the same?
When did the inequality sign change?

Reflect & Share

Share your results with another pair of classmates.
What happens to an inequality when you multiply or divide each side by:
- a positive number?
- a negative number?

Work together to explain these results.

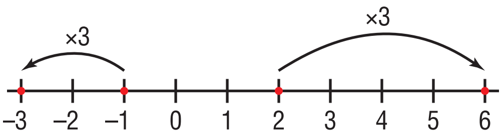
Connect

We can use a number line to investigate the effect of multiplying and dividing each side of an inequality by the same number.

Consider the inequality: $-1 < 2$

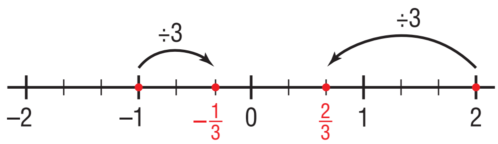
Multiply each side by 3.

$$-1 < 2$$
$$(-1)(3) < (2)(3)$$
$$-3 < 6$$



Divide each side by 3.

$$-1 < 2$$
$$(-1) \div 3 < 2 \div 3$$
$$-\frac{1}{3} < \frac{2}{3}$$



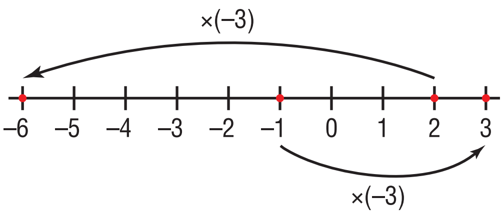
Multiply each side by -3.

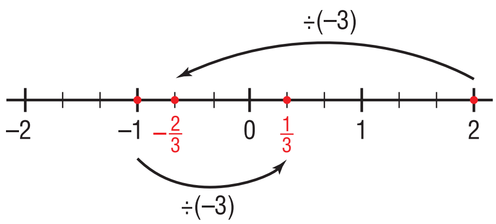
$$-1 < 2$$
$$(-1)(-3) > (2)(-3)$$
$$3 > -6$$

We must reverse the inequality sign for each inequality to remain true.

Divide each side by -3.

$$-1 < 2$$
$$(-1) \div (-3) > 2 \div (-3)$$
$$\frac{1}{3} > -\frac{2}{3}$$





The examples above illustrate these properties of inequalities:

- When each side of an inequality is multiplied or divided by the same positive number, the resulting inequality is still true.
- When each side of an inequality is multiplied or divided by the same negative number, the inequality sign must be reversed for the inequality to remain true.

To solve an inequality, we use the same strategy as for solving an equation. However, when we multiply or divide by a negative number, we reverse the inequality sign.

Example 1 Solving a One-step Inequality

Solve each inequality. Graph each solution.

a) $-5s \leq 25$

b) $7a < -21$

c) $\frac{y}{-4} > -3$

d) $\frac{k}{3} \geq -2$

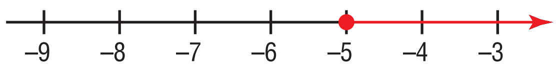
A Solution

a) $-5s \leq 25$

As you divide each side by -5, reverse the inequality sign.

$$\frac{-5s}{-5} \geq \frac{25}{-5}$$

$$s \geq -5$$

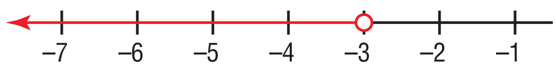


b) $7a < -21$

Divide each side by 7.

$$\frac{7a}{7} < \frac{-21}{7}$$

$$a < -3$$

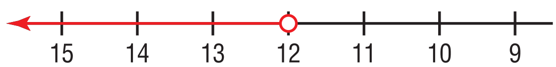


c) $\frac{y}{-4} > -3$

As you multiply each side by -4, reverse the inequality sign.

$$-4\left(\frac{y}{-4}\right) < -4\,(-3)$$

$$y < 12$$

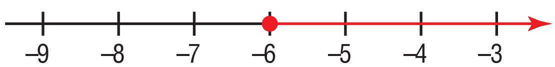


d) $\frac{k}{3} \geq -2$

Multiply each side by 3.

$$3\left(\frac{k}{3}\right) \geq 3(-2)$$

$$k \geq -6$$



Example 2 Solving a Multi-Step Inequality

a) Solve this inequality: $-2.6a + 14.6 > -5.2 + 1.8a$

b) Verify the solution.

A Solution

a)

$$-2.6a + 14.6 > -5.2 + 1.8a$$ Subtract 14.6 from each side.

$$-2.6a + 14.6 - 14.6 > -5.2 - 14.6 + 1.8a$$

$$-2.6a > -19.8 + 1.8a$$ Subtract 1.8a from each side.

$$-2.6a - 1.8a > -19.8 + 1.8a - 1.8a$$

$$-4.4a > -19.8$$ Divide each side by −4.4 and reverse the inequality sign.

$$\frac{-4.4a}{-4.4} < \frac{-19.8}{-4.4}$$

$$a < 4.5$$

b) The solution of the inequality $a < 4.5$ is all numbers less than 4.5.
Choose several numbers less than 4.5; for example, 4, 0, −2

Substitute $a = 4$ in the original inequality.

Left side	Right side
$= -2.6a + 14.6$	$= -5.2 + 1.8a$
$= -2.6(4) + 14.6$	$= -5.2 + (1.8)(4)$
$= -10.4 + 14.6$	$= -5.2 + 7.2$
$= 4.2$	$= 2$

Since $4.2 > 2$, the left side is greater than the right side, and $a = 4$ satisfies the inequality.

Substitute $a = 0$ in the original inequality.

Left side	Right side
$= -2.6a + 14.6$	$= -5.2 + 1.8a$
$= -2.6(0) + 14.6$	$= -5.2 + (1.8)(0)$
$= 14.6$	$= -5.2$

Since $14.6 > -5.2$, the left side is greater than the right side, and $a = 0$ satisfies the inequality.

Substitute $a = -2$ in the original inequality.

Left side	Right side
$= -2.6a + 14.6$	$= -5.2 + (1.8)(-2)$
$= -2.6(-2) + 14.6$	$= -5.2 - 3.6$
$= 5.2 + 14.6$	$= -8.8$
$= 19.8$	

Since $19.8 > -8.8$, the left side is greater than the right side, and $a = -2$ satisfies the inequality.

Since all 3 substitutions verify the inequality, it suggests that $a < 4.5$ is correct.

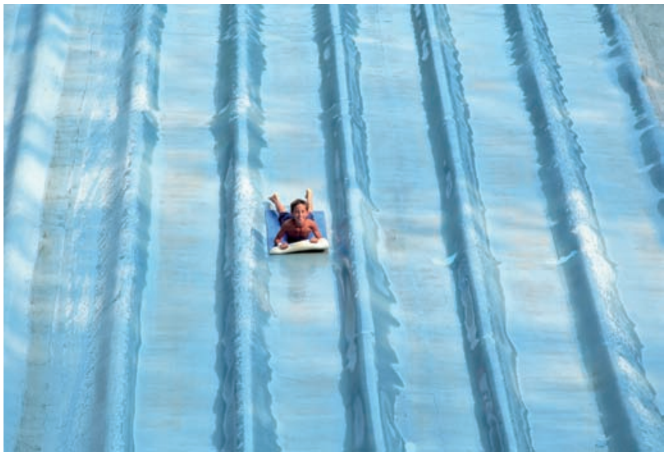
Example 3 Using an Inequality to Model and Solve a Problem

A super-slide charges \$1.25 to rent a mat and \$0.75 per ride. Haru has \$10.25. How many rides can Haru go on?

a) Choose a variable, then write an inequality to solve this problem.

b) Solve the problem.

c) Graph the solution.



A Solution

a) Let n represent the number of rides that Haru can go on.
The cost of n rides is $1.25 + 0.75n$.
This must be less than or equal to \$10.25.
So, the inequality is:
$1.25 + 0.75n \leq 10.25$

b)

$$1.25 + 0.75n \leq 10.25$$

Subtract 1.25 from each side.

$$1.25 - 1.25 + 0.75n \leq 10.25 - 1.25$$

$$0.75n \leq 9$$

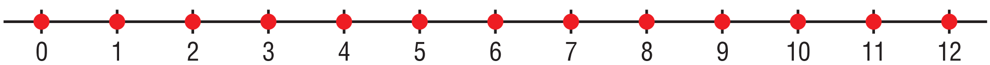
Divide each side by 0.75.

$$\frac{0.75n}{0.75} \leq \frac{9}{0.75}$$

$$n \leq 12$$

Haru can go on 12 or fewer rides.

c) Since the number of rides is a whole number, the number line is drawn with a shaded circle at each solution.



Discuss the ideas

1. How is multiplying or dividing each side of an inequality by the same positive number different from multiplying or dividing each side by the same negative number?

2. What is an advantage of substituting 0 for the variable to verify the solution of an inequality? Can you always substitute 0? Explain.

Practice

Check

3. Predict whether the direction of the inequality sign will change when you perform the indicated operation on each side of the inequality.
   a) $-9 < -2$; Multiply by 4.
   b) $14.5 > 11.5$; Multiply by -3.
   c) $6 > -12$; Divide by -4.
   d) $-4 < 10$; Divide by 4.
   Check your predictions. Were you correct? Explain.

4. Do not solve each inequality. Determine which of the given numbers are solutions of the inequality.
   a) $4w < 3$; $-2, 0, 2.5$
   b) $3d \geq 5d + 10$; $-5, 0, 5$

5. a) State whether you would reverse the inequality sign to solve each inequality. Then solve and graph the inequality.
   i) $10 - y \leq 4$ ii) $3c > -12$
   iii) $-6x < 30$ iv) $\frac{m}{-2} < 3$
   b) Refer to your solutions in part a. State three values of the variable that satisfy each inequality: one integer, one fraction, and one decimal.

6. A student says that if $c > 9$, then $-3c > -27$. Do you agree? Justify your answer.

Apply

7. Solve each inequality. Verify the solution by substituting 3 different numbers in each inequality.
   a) $4 - 2t < 7$
   b) $-5x + 2 > 24$
   c) $2m + 3 \leq -7$
   d) $-4x - 2 > 10$

8. Write, then solve an inequality to show how many cars you would have to wash at \$5 a car to raise at least \$300.

9. Solve each inequality. Graph the solution.
   a) $1 - k \leq 4 + k$
   b) $2 + 3g < g - 5$
   c) $4.5 - 2.5a > 6$
   d) $4.7b - 9 \geq 11 - 1.3b$
   e) $-6.4 + 3.6s \leq 1.8s + 1.7$
   f) $-2.5v + 4.7 \geq -3.8v + 1.58$

10. The Student Council decides to raise money by organizing a dance. The cost of hiring the video-DJ is \$1200 and the Student Council is charging \$7.50 per ticket. How many tickets can be sold to make a profit of more than \$1500?
    a) Choose a variable and write an inequality to solve this problem.
    b) Use the inequality to solve the problem.
    c) Verify the solution and graph it on a number line.

11. Solve each inequality. Graph the solution.
    a) $1 + \frac{3}{4}x > 17$
    b) $-2 \leq -6 + \frac{1}{4}c$
    c) $4 - \frac{2}{3}d \geq \frac{5}{6}d - 5$
    d) $\frac{3}{5}f - \frac{1}{2} < 2 + f$

12. Solve each inequality. Show the steps in the solution. Verify the solution by substituting 3 different numbers in each inequality.
    a) $4a - 5 \geq a + 2$
    b) $15t - 17 \geq 21 - 4t$
    c) $10.5z + 16 \leq 12.5z + 12$
    d) $7 + \frac{1}{3}b \leq 2b + 22$

13. Jake takes a taxi to tour a city. He is charged \$2.50, plus \$1.20 per kilometre.
Jake has \$12.00. How far can he travel?

a) Choose a variable and write an inequality for this problem.
b) Solve the inequality.
Explain the solution in words.
c) Verify the solution.
d) Graph the solution.

14. Assessment Focus

a) Solve the equation: $2 - \frac{3}{4}w = 3w + \frac{1}{2}$
b) Solve the inequality: $2 - \frac{3}{4}w \geq 3w + \frac{1}{2}$
c) Compare the processes in parts a and b. How is solving an inequality like solving the related equation? How is it different?
d) Compare the solutions in parts a and b. How is the solution of an inequality like the solution of the related equation? How is it different?

Show your work.

15. Janelle plans to replace the light bulbs in her house with energy saver bulbs.
A regular light bulb costs \$0.55 and has an electricity cost of \$0.004 20 per hour.
An energy saver bulb costs \$5.00 and has an electricity cost of \$0.001 05 per hour.
For how many hours of use is it cheaper to use an energy saver bulb than a regular bulb?

a) Write an inequality for this problem.
b) Solve the inequality.
Explain the solution in words.
c) Verify the solution.
d) Graph the solution.

16. Solve each inequality. Graph the solution.

a) $3(0.4h + 5) > 4(0.2h + 7)$
b) $-2(3 - 1.5n) \leq 3(2 - n)$
c) $-4(2.4v - 1.4) \geq -2(0.8 + 1.2v)$
d) $-5(3.2 + 2.3z) < 2(-1.5z - 4.75)$

Take It Further

17. Solve each inequality.
Verify and graph the solution.

a) $\frac{3}{2}a + \frac{1}{2} < \frac{7}{3}a - \frac{3}{4}$
b) $\frac{3}{5}(5.2 - 3m) > -\frac{7}{10}(2m + 7.5)$

18. A business must choose a company to print a promotional brochure.
Company A charges \$900 plus \$0.50 per copy.
Company B charges \$1500 plus \$0.38 per copy.

a) How many brochures must be printed for the cost to be the same at both companies?
b) How many brochures must be printed for Company A to be less expensive?
c) How many brochures must be printed for Company B to be less expensive?
d) Explain the strategies you used to solve these problems.

Reflect

A student says, "Solving inequalities is different from solving equations only when you multiply or divide each side of the inequality by a negative number."
Do you agree with this statement? Explain.

Study Guide

Solving Equations

- An equation is a statement that one quantity is equal to another. To solve an equation means to determine the value of the variable that makes the right side of the equation equal to the left side.

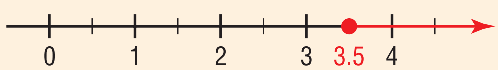
- To solve an equation, isolate the variable on one side of the equation. We use inverse operations or a balance strategy of performing the same operation on both sides of the equation. This can include:
  - adding the same quantity to each side of the equation
  - subtracting the same quantity from each side of the equation
  - multiplying or dividing each side of the equation by the same non-zero quantity

- Algebra tiles, arrow diagrams, and balance scales help model the steps in the solution.

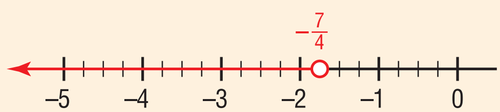
Solving Inequalities

- An inequality is a statement that:
  - one quantity is less than another; for example, $-4 < 3.2a$
  - one quantity is greater than another; for example, $\frac{3}{2}b + 8 > -7$
  - one quantity is greater than or equal to another; for example, $3.4 - 2.8c \geq 1.3c$
  - one quantity is less than or equal to another; for example, $-\frac{5}{8}d + \frac{1}{4} \leq \frac{3}{4} - \frac{1}{2}d$

- The solutions of an inequality are the values of the variable that make the inequality true. We can graph the solutions of an inequality on a number line; for example, $f \geq 3.5$:



and $g < -\frac{7}{4}$:



- The inequality sign reverses when you multiply or divide each side of the inequality by the same negative number.

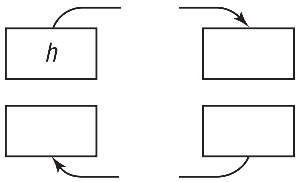
Review

6.1

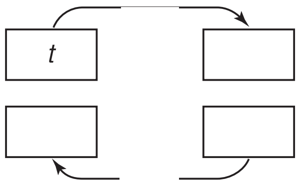
1. a) Copy and complete each arrow diagram to solve each equation.

b) Record the steps in the arrow diagram symbolically.

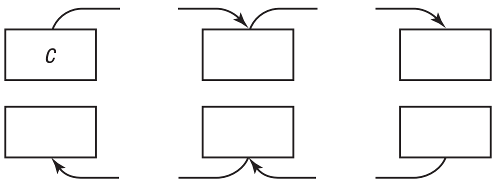
i) $8h = 7.2$



ii) $\frac{t}{5} = -7$



iii) $5c - 1 = 2.4$



2. Both Milan and Daria solve this equation:
$4(3.2s + 5.7) = -6$

a) Milan uses inverse operations to undo the steps used to build the equation. Show the steps in Milan's solution.

b) Daria uses the distributive property, then inverse operations. Show the steps in Daria's solution.

c) Describe an advantage and a disadvantage of each method.

3. Solve each equation. Verify the solution.

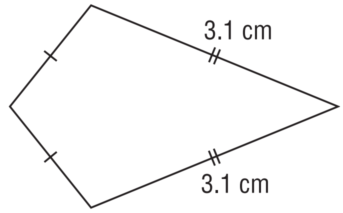
a) $-20.5 = 3b + 16.7$

b) $\frac{t}{3} + 1.2 = -2.2$

c) $-8.5 = 6.3 - \frac{w}{2}$

d) $-2.3(x + 25.5) = -52.9$

4. A kite has longer sides of length 3.1 cm and a perimeter of 8.4 cm.



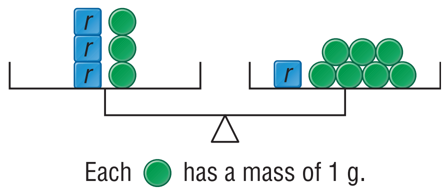
a) Write an equation that can be used to determine the length of a shorter side.

b) Solve the equation.

c) Verify the solution.

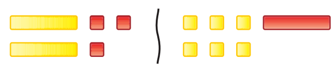
6.2

5. Write the equation represented by these balance scales:



Solve the equation. Record the steps algebraically.

6. Write the equation modelled by these algebra tiles:



Use algebra tiles to solve the equation. Record the steps algebraically.

7. Solve each equation. Verify the solution.

a) $\frac{-72}{a} = -4.5, a \neq 0$

b) $-\frac{1}{3} + 2m = -\frac{1}{5}$

c) $12.5x = 6.2x + 88$

d) $2.1g - 0.3 = -3.3g - 30$

e) $\frac{3}{2}x + \frac{4}{3} = \frac{5}{8}x + \frac{5}{2}$

f) $5.4(2 - p) = -1.4(p + 2)$

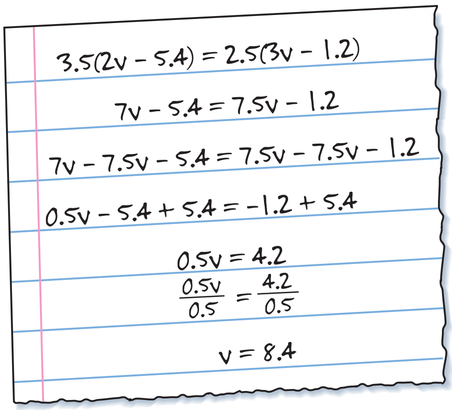
8. Kevin is planning to rent a car for one week. Company A charges \$200 per week, with no charge for the distance driven. For the same car, Company B charges a \$25 administration fee plus \$0.35 per kilometre. Determine the distance driven that will result in equal costs at the two companies.

a) Define a variable and write an equation that can be used to solve the problem.

b) Use the equation to solve the problem.

c) Verify the solution.

9. A student solves this equation:



What mistakes did the student make? Rewrite a correct and complete solution. How do you know your solution is correct?

6.3 **10.** Define a variable, then write an inequality that describes each situation.

a) Persons under 18 are not admitted.

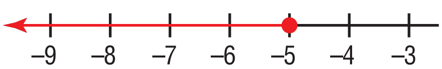
b) A person must be at least 90 cm tall to go on an amusement park ride.

c) Horton can spend a maximum of \$50.

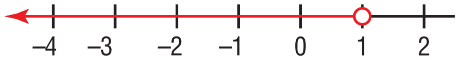
d) A game is recommended for players 5 years and older.

11. Write the inequality represented by each number line.

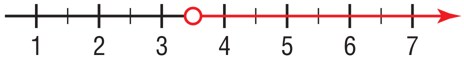
a)



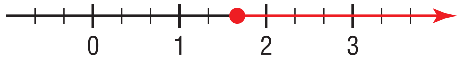
b)



c)



d)



12. a) Graph each inequality on a number line.

i) $a < -5.2$ **ii)** $b \leq 8.5$

iii) $c > -\frac{5}{3}$ **iv)** $d \geq \frac{25}{4}$

b) For each inequality in part a, are −3 and 5 possible solutions? Justify your answer.

6.4
6.5 **13.** Determine 3 values of the variable that satisfy each inequality: one integer, one fraction, and one decimal.

a) $h - 2 < -5$ **b)** $3k > -9$ **c)** $5 - y > 0$

14. State whether each operation on the inequality $-2x > 5$ will reverse the inequality sign.

a) Multiply each side by 4.

b) Add −5 to each side.

c) Subtract −2 from each side.

d) Divide each side by −6.

15. The cost of a prom is \$400 to rent a hall, and \$30 per person for the meal. The prom committee has \$10 000. How many students can attend?

a) Define a variable and write an inequality to model this problem.

b) Solve the inequality, then graph the solution.

16. Solve each inequality. Verify and graph the solution.

a) $7 + y < 25$ **b)** $-7y < 14$

c) $\frac{x}{4} > -2.5$ **d)** $5.2 - y < -5.5$

e) $13.5 + 2y \leq 18.5$ **f)** $24 + 3a \leq -6 + 7a$

Practice Test

1. Use a model of your choice to illustrate the steps to solve this equation:
$15 + 2d = 5d + 6$
Explain each step and record it algebraically.

2. Solve each equation.

a) $-3x - 0.7 = -7$

b) $\frac{26}{x} = 5 - 1.5$

c) $\frac{r}{3} + 5.4 = -3.2$

d) $2.4w - 5.6 = 3.7 + 1.9w$

e) $\frac{1}{4}c - \frac{7}{2} = \frac{1}{2}c + \frac{3}{4}$

f) $4.5(1.2 - m) = 2.4(-2m + 2.1)$

3. To cater a lunch, Tina's Catering charges \$100, plus \$15 per meal.
Norman's Catering charges \$25, plus \$20 per meal.
Determine the number of meals that will result in equal costs at the two companies.

a) Define a variable, then write an equation that can be used to solve this problem.

b) Solve the equation. Verify the solution.

4. Solve each inequality. Verify, then graph the solution.

a) $5 - t > 3$

b) $3(t + 2) \geq 11 - 5t$

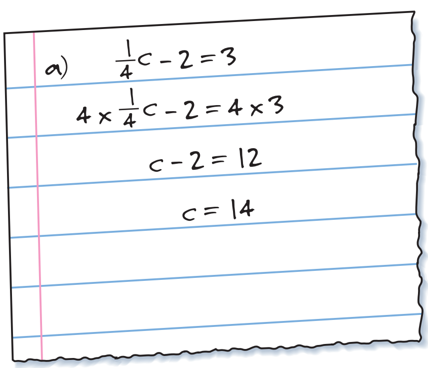
c) $\frac{m}{4} + 5 \leq \frac{1}{2} - m$

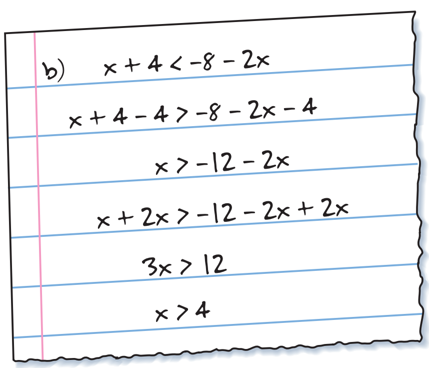
5. A car rental company charges \$24.95 per day plus \$0.35 per kilometre.
A business person is allowed \$50 each day for travel expenses.
How far can the business person travel without exceeding her daily budget?

a) Define a variable, then write an inequality to solve the problem.

b) Solve the problem. Graph the solution.
How do you know that your answer is correct?

6. Two students wrote these solutions on a test. Identify the errors.
Write a correct and complete algebraic solution.





Unit Problem — Raising Money for the Pep Club

There are 25 students in the school's Pep Club.

1. The Pep Club can buy new uniforms from 2 different suppliers:
Company A charges \$500, plus \$22 per uniform.
Company B charges \$360, plus \$28 per uniform.

a) Define a variable, then write an equation that can be used to determine the number of uniforms that will result in equal costs at both companies.

b) Solve the equation. Verify the solution.

c) Which company should the Pep Club choose? Justify your recommendation.

d) How much money must the Pep Club raise to purchase the uniforms?

2. The Pep Club decides to raise the money for the uniforms by selling snacks at lunch time. The snacks cost the Pep Club \$6.00 for a box of 30.

a) Determine the cost per snack.

b) The Pep Club makes a profit of \$0.25 on each snack sold. Suppose the club does raise the money it needs. Define a variable, then write an inequality that can be used to determine how many snacks might have been sold. How many boxes of snacks did the members of the Pep Club need?

c) Solve the inequality.

d) Verify the solution.

Your work should show:

- an equation and inequality and how you determined them
- how you determined the solutions of the equation and the inequality
- clear explanations of your reasoning

Reflect on Your Learning

How is solving a linear inequality like solving a linear equation? How is it different?
Include examples in your explanation.

Cumulative Review Units 1–6

1 **1.** Which numbers are perfect squares? Determine the square root of each perfect square. Estimate the square root of each non-perfect square.

a) 3.6 **b)** 0.81 **c)** $\frac{16}{25}$

d) 0.0004 **e)** $\frac{224}{9}$ **f)** 4.41

g) 2.56 **h)** 0.24

2 **2.** Simplify, then evaluate each expression.

a) $(-8)^4 \times (-8)^3 \div (-8)^6$

b) $(9^4 \times 9^3)^0$

c) $[(-2)^5]^3 - [(-3)^3]^2$

d) $[(-4)^1 + (-4)^2 - (-4)^3] \times (-4)^5 \div (-4)^4$

e) $\frac{3^5}{3^2} - (-3)^2$

3 **3.** Evaluate.

a) $1\frac{5}{8} + \left(-4\frac{1}{6}\right)$ **b)** $-3\frac{2}{5} - 7\frac{3}{4}$

c) $(-1.3)(3.4)$ **d)** $\left(-2\frac{1}{10}\right) \div \left(-5\frac{2}{5}\right)$

e) $-8.3 + 6.7 \times (-3.9)$

f) $1\frac{1}{2} \times \left[\left(-\frac{1}{3}\right) + \frac{1}{4}\right]$

g) $[-7.2 - (-9.1)] \div 0.5 + (-0.8)$

4 **4.** The pattern in this table continues.

Term Number, *n*	Term Value, *v*
1	5
2	7
3	9
4	11

a) Describe the patterns in the table.

b) Write an equation that relates v to n.

c) Verify the equation by substituting values from the table.

d) Determine the value of the 24th term.

e) Which term number has a value of 233?

5. a) Create a table of values for the linear relation $y = 3x - 2$.

b) Describe the patterns in the table.

c) Graph the data.

6. a) Does each equation describe a vertical line, a horizontal line, or an oblique line? How do you know?

i) $2x = 5$

ii) $y + 2 = -1$

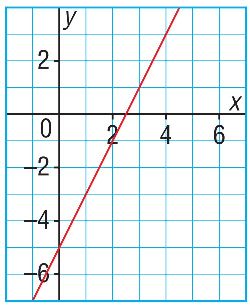
iii) $x + y = 3$

b) Graph each line in part a. Explain your work.

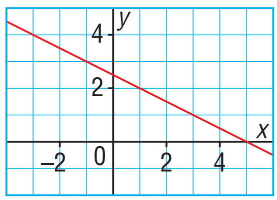
7. Match each equation with a graph below. Justify your answers.

a) $x + 2y = 5$ **b)** $2x + y = 5$ **c)** $2x - y = 5$

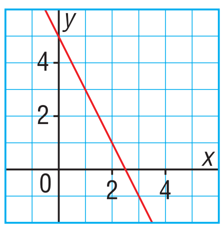
Graph A



Graph B

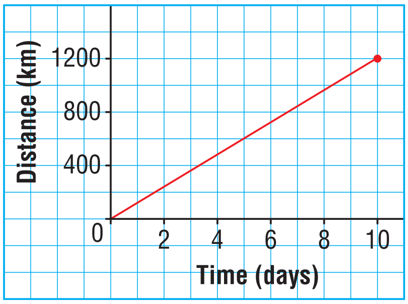


Graph C



8. Carl is cycling across Canada. This graph shows the distance he covers in 10 days.

Carl's Bike Trip



a) Estimate how many days it will take Carl to cycle 700 km.

b) Predict how far Carl will cycle in 13 days.

5 **9.** Name the coefficients, variable, and degree of each polynomial. Identify the constant term if there is one.

a) $3x - 6$ b) $4n^2 - 2n + 5$

c) 19 d) $-a^2 + 7 - 21a$

10. Simplify each polynomial.

a) $2a - 4 - 9a + 5$

b) $3y - 2y^2 + 4 - y + 3y^2 - 8$

c) $9c - 4cd + d - 6cd + 4 - 7c$

d) $4m^2 - 3n^2 + 2m - 3n + 2m^2 + n^2$

11. Add or subtract the polynomials.

a) $(3s^2 - 2s + 6) + (7s^2 - 4s - 3)$

b) $(8x^2 - 5x + 2) - (5x^2 + 3x - 4)$

c) $(9t - 4 + t^2) + (6 - 2t^2 + 5t)$

d) $(1 + 4n - n^2) - (3n - 2n^2 + 7)$

e) $(6y^2 + 3xy - 2x^2 + 1) + (3x^2 - 2y^2 - 8 + 6xy)$

f) $(8a - 6b - 3a^2 - 2ab) - (4b^2 - 7ab + 9b - 6)$

12. Determine each product or quotient.

a) $9(3s^2 - 7s + 4)$

b) $\dfrac{35 - 49w^2 - 56w}{-7}$

c) $7m(3m - 9)$

d) $(-12d^2 + 18d) \div (-6d)$

6 **13.** Solve each equation. Verify the solution.

a) $9x = 7.2$ b) $-2.7 = \frac{a}{4}$

c) $6.5s - 2.7 = -30$ d) $\frac{c}{4} - 0.2 = 5.8$

e) $6(n - 8.2) = -18.6$ f) $-8 = \frac{7}{c}, c \neq 0$

g) $22 - 7d = -8 - 2d$

h) $3.8v - 17.84 = 4.2v$

i) $2(t - 8) = 4(2t - 19)$

j) $\frac{3}{4}(2r - 4) = \frac{1}{5}(36 - r)$

14. a) Graph each inequality on a number line.

i) $a \leq 3$ ii) $-4.5 < b$

iii) $c < -\frac{7}{4}$ iv) $d \geq 2\frac{1}{3}$

b) State whether -4 and 2 are possible solutions for each inequality in part a. Justify your answer.

15. Solve each inequality. Graph the solution. Verify the solution.

a) $x + 7 < 3$ b) $-3x > 6$

c) $b - 4.8 \geq -1.5$ d) $\frac{n}{-8} + 2 \leq -7$

e) $7m + 23 \leq 6m - 15$

f) $6.5 - 0.2t > 8$

g) $-5(4 - 0.8s) \geq 3(19 - s)$

16. Daphne will sell her video game system for \$140 to Surinder. She also offers to sell him video games for \$15 each. Surinder has saved \$210 in total. How many video games can Surinder buy from Daphne?

a) Write an inequality to solve this problem.

b) Solve the inequality. Verify the solution.