PEARSON

Math Makes Sense 8

Practic

Reproducible Version

PEARSON

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Math Makes Sense 8

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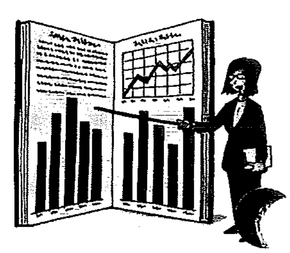

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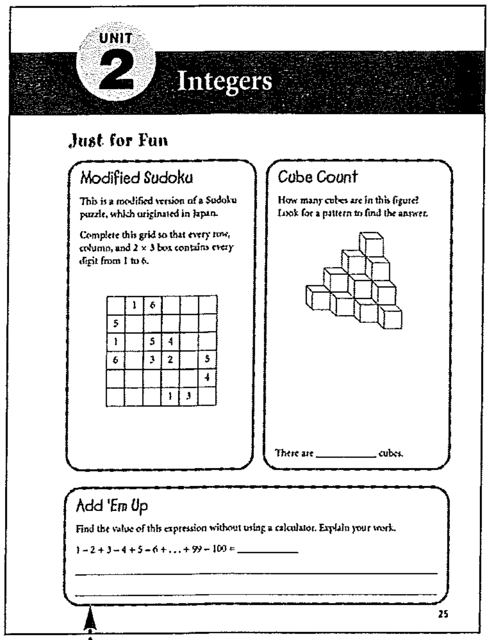
About Math Makes Sense 8 Practice and Homework Book

Welcome to *Pearson Math Makes Sense 8*. These pages describe how this Practice and Homework Book can support your progress through the year.

Each unit offers the following features.


UNIT 2 Integers

Just for Fun

Modified Sudoku

This is a modified version of a Sudoku puzzle, which originated in Japan.

Complete this grid so that every row, column, and 2 × 3 box contains every digit from 1 to 6.

Cube Count

How many cubes are in this figure? Look for a pattern to find the answer.

There are ________ cubes.

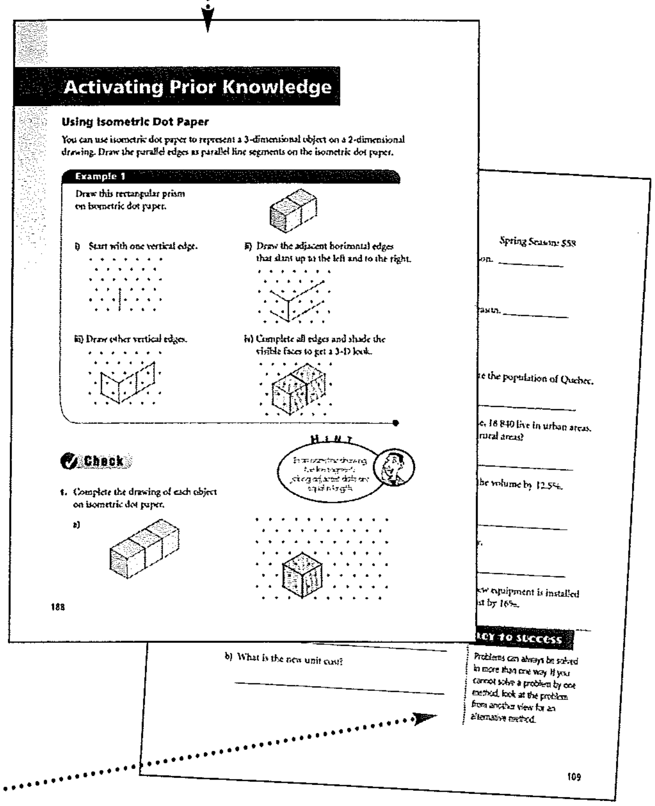
Add 'Em Up

Find the value of this expression without using a calculator. Explain your work.

1 − 2 + 3 − 4 + 5 − 6 + … + 99 − 100 = ________

25

Activating Prior Knowledge provides a brief introduction and Examples to refresh your skills, and Check questions to let you reinforce these prerequisite skills.


Activating Prior Knowledge

Using Isometric Dot Paper

You can use isometric dot paper to represent a 3-dimensional object on a 2-dimensional drawing. Draw the parallel edges as parallel line segments on the isometric dot paper.

Example 1

Draw this rectangular prism on isometric dot paper.

i) Start with one vertical edge.

ii) Draw the adjacent horizontal edges that slant up to the left and to the right.

iii) Draw other vertical edges.

iv) Complete all edges and shade the visible faces to get a 3-D look.

Check

1. Complete the drawing of each object on isometric dot paper.

a)

188

Key to Success

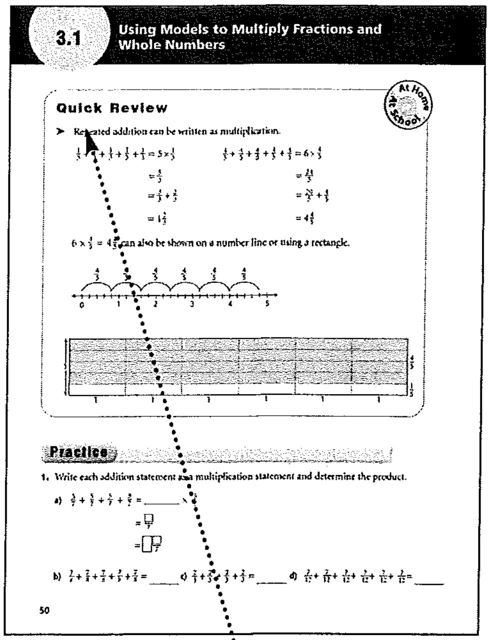
b) What is the new unit cost?

109

Just for Fun presents puzzles, games, and activities to help you warm up for the content to come. You may work with key words, numeracy skills, and creative and critical thinking skills.

Key to Success highlights ways you can develop your study skills, test-taking skills, and overall independence as a grade 8 student.

For each lesson of the Student Book, the workbook provides 2 to 4 pages of support.



3.1 Using Models to Multiply Fractions and Whole Numbers

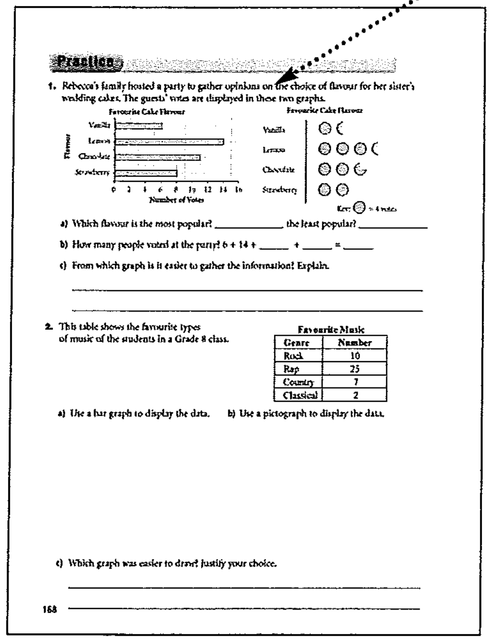
Quick Review

Repeated addition can be written as multiplication.

Practice

1. Write each addition statement as a multiplication statement and determine the product.

50



Practice

1. Rebecca's family hosted a party to gather opinions on the choice of flavour for her sister's wedding cake. The guests' votes are displayed in these two graphs.

a) Which flavour is the most popular? ________ the least popular? ________

b) How many people voted at the party? 6 + 14 + ____ + ____ = ____

c) From which graph is it easier to gather the information? Explain.

2. This table shows the favourite types of music of the students in a Grade 8 class.

Favourite Music

Genre	Number
Rock	10
Rap	25
Country	7
Classical	2

a) Use a bar graph to display the data. b) Use a pictograph to display the data.

c) Which graph was easier to draw? Justify your choice.

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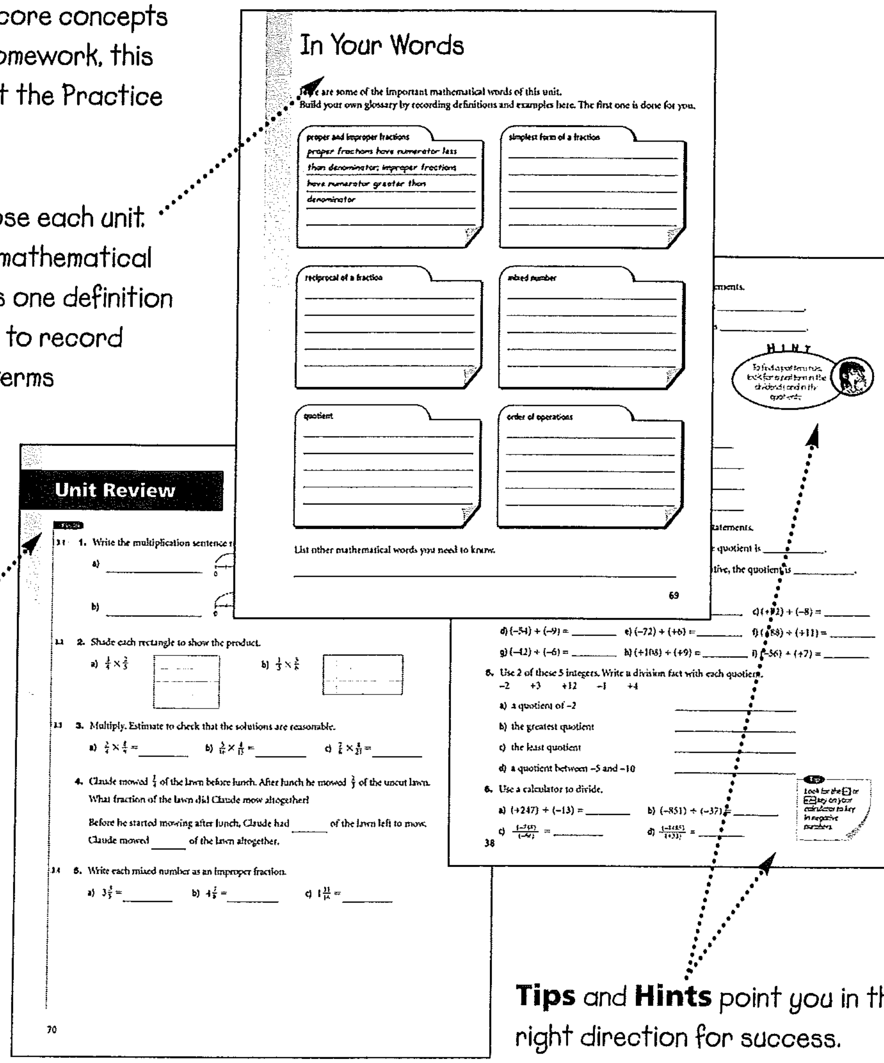
Practice questions provide a structure for your work, gradually leaving more steps for you to complete on your own.

Quick Review covers the core concepts from the lesson. If used for homework, this Quick Review lets you take just the Practice and Homework Book home.

In Your Words helps to close each unit. This page identifies essential mathematical vocabulary from the unit, gives one definition as an example, and allows you to record your understanding of other terms in your own words.



In Your Words

Here are some of the important mathematical words of this unit. Build your own glossary by recording definitions and examples here. The first one is done for you.

proper and improper fractions: proper fractions have numerator less than denominator; improper fractions have numerator greater than denominator

simplest form of a fraction

reciprocal of a fraction

mixed number

quotient

order of operations

List other mathematical words you need to know.

69

Unit Review

1. Write the multiplication sentence

2. Shade each rectangle to show the product.

3. Multiply. Estimate to check that the solutions are reasonable.

4. Claude mowed of the lawn before lunch. After lunch he mowed of the uncut lawn. What fraction of the lawn did Claude mow altogether?

5. Write each mixed number as an improper fraction.

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5. Use 2 of these 5 integers. Write a division fact with each quotient.

6. Use a calculator to divide.

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Unit Review pages provide the same level of support as lesson Practice. Each Unit Review question is referenced to the relevant lesson where related concepts are developed.

Tips and **Hints** point you in the right direction for success.

UNIT 1

Square Roots and the Pythagorean Theorem

Just for Fun

What Do You Notice?

Follow the steps. An example is given.

	Example
1. Pick a 4-digit number with different digits.	3078
2. Find the greatest number that can be made with these digits.	8730
3. Find the least number that can be made with these digits.	0378
4. Subtract the least from the greatest.	8730 – 0378 = 8352
5. Repeat steps 2, 3, and 4 with the result.	8532 – 2358 = 6174
6. Continue to repeat steps 2, 3, and 4 until you notice something interesting.	7641 – 1476 = 6174

What do you notice?

____

Try these steps with the number 2395. What do you notice? Pick any 4-digit number. What do you notice?

____

Letter Symmetry

A letter has mirror symmetry if a straight line can be drawn through the letter so that one half of the letter is a mirror image of the other half. The straight lines can be vertical, horizontal, or slanted.
For example, the letter A has mirror symmetry, but the letter F does not.

Which letters have mirror symmetry?

Which letters have more than one line of symmetry?

____

Activating Prior Knowledge

Areas of Rectangles and Triangles

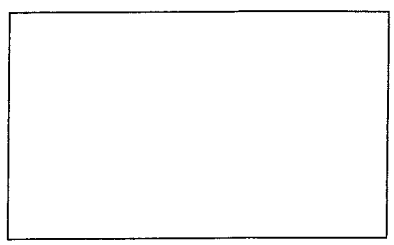
Area is the amount of surface a figure covers. It is measured in square units.

- To find the area of a rectangle, use the formula $A = bh$, where b is the base length and h the height of the rectangle.
- To find the area of a triangle use the formula $A = \frac{1}{2}bh$, where b is the base length and h is the height of the triangle.

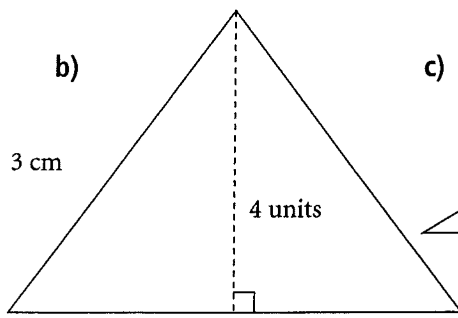
Example 1

Find the area of each figure.

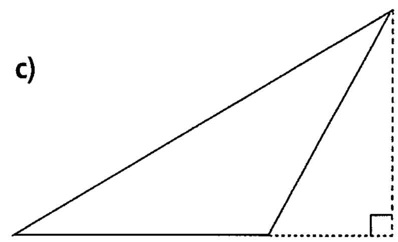
a)



b)



c)



Solution

a) The figure is a rectangle with base 5 cm and height 3 cm.
Substitute $b = 5$ cm and $h = 3$ cm into $A = bh$.

$A = 5 \text{ cm} \times 3 \text{ cm}$
$= 15 \text{ cm}^2$

The area is 15 cm^2. The abbreviation cm^2 stands for "square centimetres."

b) The figure is a triangle with base 6 units and height 4 units.
Substitute $b = 6$ units and $h = 4$ units into $A = \frac{1}{2}bh$.

$A = \frac{1}{2}(6 \text{ units} \times 4 \text{ units})$
$= 12$ square units

The area is 12 square units.

c) The figure is a triangle with base 5 cm and height 3 cm.
Substitute $b = 5$ cm and $h = 3$ cm into $A = \frac{1}{2}bh$.

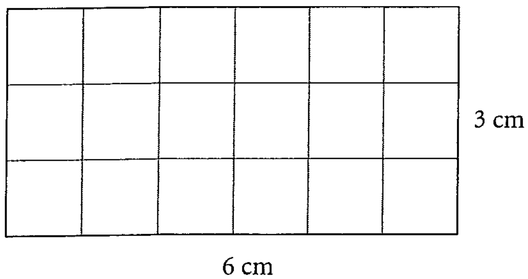
$A = \frac{1}{2}(5 \text{ cm} \times 3 \text{ cm})$
$= \frac{1}{2}(15 \text{ cm}^2)$
$= 7.5 \text{ cm}^2$

The area is 7.5 cm^2.

Check

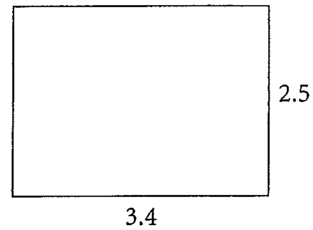
1. Find the area of each figure.

a)



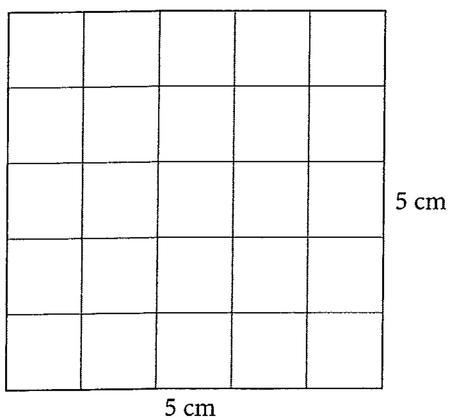
The area is ____ cm × ____ cm = ____

b)



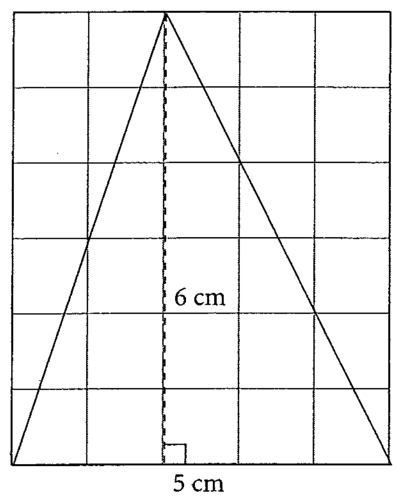
The area is ________________

c)



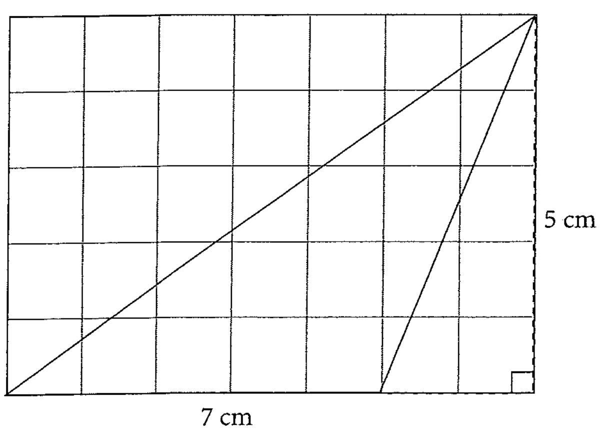
The area is ________________

d)



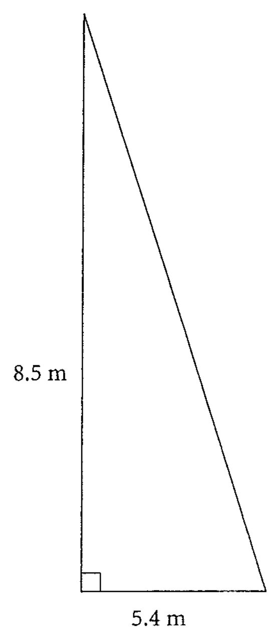
The area is $\frac{1}{2}$(____ cm × ____ cm) = ____

e)



The area is ________________

f)



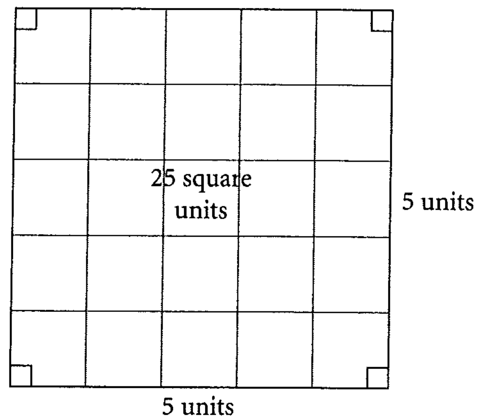
The area is

____

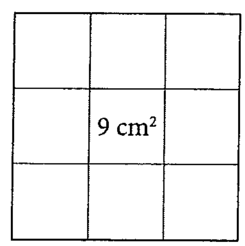
Quick Review

- When you multiply a number by itself, you *square* the number.
  The square of 5 is $5 \times 5 = 25$
  We write: $5^2 = 5 \times 5 = 25$
  We say: Five squared is twenty five.
  25 is a **square number**, or a **perfect square**.

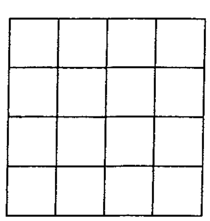
- You can model a square number by drawing a square whose area is equal to the square number.
  You can model 25 using a square with side length 5 units.



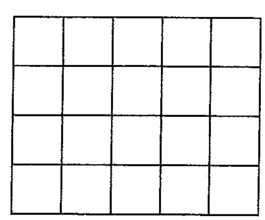
Find the perimeter of a square with area 9 cm².
First, find the side length of the square.
Since $3 \times 3 = 9$, the side length is 3 cm. So, the perimeter is 3 cm + 3 cm + 3 cm + 3 cm = 12 cm



16 is a perfect square because you can create a square with area 16 square units using square tiles.

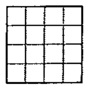


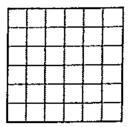
20 is not a perfect square because you cannot create a square with area 20 square units using square tiles. The 4×5 rectangle is the closest to a square that you can get.

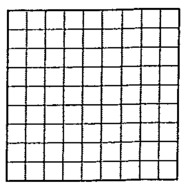


Practice

1. Match each diagram to the correct square number.

a) 

b) 

c) 

i) 36

ii) 81

iii) 16

2. Complete the statement for each square number.

a) 64 is a square number because 64 = ______ × ______ = ______

b) 49 is a square number because 49 = ______ × ______ = ______

3. Complete the table. The first row has been done for you.

a)	4^2	4 × 4	16
b)	3^2	______ × ______	
c)	$\_\_\_\_\_\_^2$	7 × 7	
d)	11^2	______ × ______	

4. Match the area of the square with the correct side length.

a) 25 cm^2 — i) 2 cm
b) 64 cm^2 — ii) 10 cm
c) 4 cm^2 — iii) 5 cm
d) 100 cm^2 — iv) 8 cm

5. Use square tiles to decide whether 32 is a square number.

6. Use graph paper to decide whether 64 is a square number.

____

7. Which of the numbers are perfect squares? How do you know?

a) 81 — 81 is a perfect square because 81 = ______ × ______ = ______

b) 18 — 18 ______ a perfect square because I ______ draw a square with area 18 square units on grid paper.

c) 20 — ____

____

d) 25 — ____

8. Find the side length of the square with each area. Give the unit.

a) 49 cm^2 — _____ × _____ = 49, so the length of the side is 7 cm.

b) 900 mm^2 — ______________

c) 121 cm^2 — ______________

d) 169 m^2 — ______________

9. Find the perimeter of each square.

a) side length 6 cm — Perimeter = _____ cm + _____ cm + _____ cm + _____ cm = _____ cm

b) area 25 m^2 — Side length is _____ m, because _____ × _____ = 25. So,

Perimeter = ____________________ = ____________________

c) area 144 m^2 — ______________

10. If you multiply a perfect square by a different perfect square, is the answer also a perfect square? Give examples to explain your answer.

____

1.2 Squares and Square Roots

Quick Review

➤ When a number is multiplied by itself, the result is a square number.
For example, 9 is a square number because $3 \times 3 = 9$.

➤ A number is a square number if it has an *odd* number of factors.
For example, to check if 36 is a square number, first create a list of the factors of 36 in pairs as shown:

1×36
2×18
3×12
4×9
6×6

Write these factors in ascending order, starting at 1:

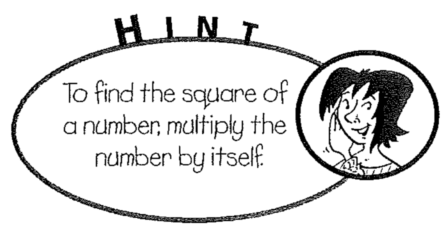
1, 2, 3, 4, (6), 9, 12, 18, 36

There are nine factors of 36. This is an odd number, so 36 is a square number.

Tip
A number with an even number of factors is not a square number.

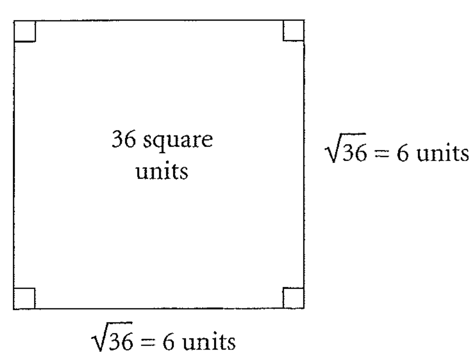
In the ordered list of factors, notice that 6 is the middle number, and that $6 \times 6 = 36$.
6 is called the **square root** of 36.
We write the square root of 36 as $\sqrt{36}$

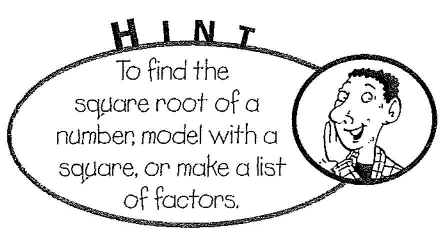
➤ Squaring and taking the square root are inverse operations.
So, $\sqrt{36} = 6$ because $6^2 = 6 \times 6 = 36$.
This means $\sqrt{6^2} = 6$



➤ You can find a square root using a diagram of square. The area is the square number.

➤ The side length of the square is the square root of the area.





Practice

1. List the factors of each number in ascending order. Which numbers are square numbers? For each of the square numbers, find the square root.

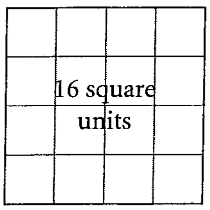
   a) 196: ______________________ ______________________

   b) 200: ______________________ ______________________

   c) 441: ______________________ ______________________

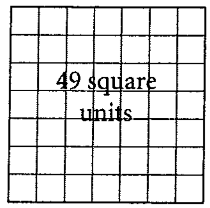
2. For each square, state the square number and the square root.

   a)



   square number ____________
   square root ____________

   b)



   square number ____________
   square root ____________

3. Complete the sentence for each square root. The first one has been done for you.

   a) $\sqrt{25} = 5$ because $5^2 = 25$

   b) $\sqrt{49} =$ ______ because ______ = ______

   c) $\sqrt{100} =$ ______ because ______ = ______

   d) $\sqrt{144} =$ ______ because ______ = ______

4. Complete each sentence. The first one has been done for you.

   a) $\sqrt{16} = 4$ because $4^2 = 16$

   b) ______ = 8 because $8^2 =$ ______

   c) ______ = 9 because ______ = ______

   d) ______ = 11 because ______ = ______

5. Match each number in column 1 to the number that is equal to it in column 2.

a) $\sqrt{9}$	i) 9
b) 81	ii) 9^2
c) 3^2	iii) $\sqrt{81}$
d) 9	iv) 3

6. Find each square root.

   a) $\sqrt{64} =$ ________ b) $\sqrt{400} =$ ________ c) $\sqrt{225} =$ ________ d) $\sqrt{324} =$ ________

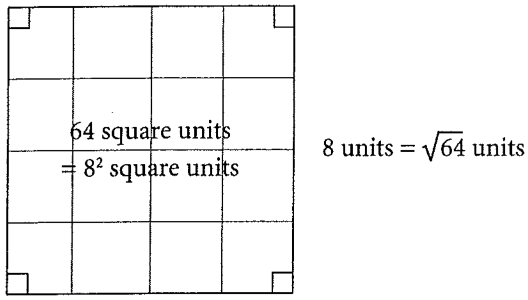
7. Find the square root of each number:

   a) $5^2 =$ ________ b) $8^2 =$ ________ c) $16^2 =$ ________ d) $54^2 =$ ________

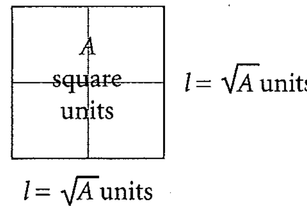
8. Find the number whose square root is

   a) 17 ________ = ________ b) 22 ________ c) 30 ________

1.3 Measuring Line Segments



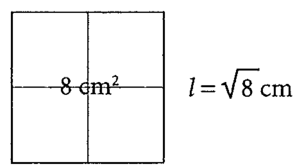
➤ This is true for all squares.



➤ In the square:
- the side length is 8 units and the area is 8^2 square units
- the area is 64 square units and the side length is $\sqrt{64}$ units

➤ In the square:
- the side length is l units and the area is l^2 square units
- the area is A square units and the side length is $\sqrt{A}$ units

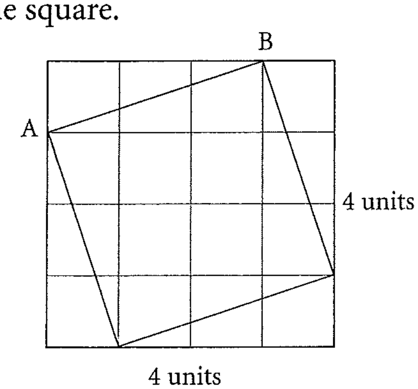
➤ Squares can have areas that are not square numbers. The side length of this square is $\sqrt{8}$ cm and the area is $(\sqrt{8})^2 = 8$ cm^2
The area is 8 cm^2 and the side length is $\sqrt{8}$ cm.



➤ You can find the length of a line segment AB on a grid by constructing a square on the segment. The length AB is the square root of the area of the square.

Tip
The square of the square root of a number is that number. For example, $(\sqrt{2})^2 = 2$. $\sqrt{8}$ is not a whole number. It is called an irrational number.

Draw an enclosing square around the square containing AB.
Then find the area of the enclosing square, and subtract the sum of the areas of the triangles.



The area of the enclosing square is 4^2 square units = 16 square units

Each triangle has area $\frac{1}{2} \times 1$ unit $\times$ 3 units = 1.5 square units

4 triangles have area 4×1.5 square units = 6 square units
The area of the square with AB as a side is
16 square units – 6 square units = 10 square units
So, the length of AB is $\sqrt{10}$ units.

HINT
Use the formulas $A = s^2$ for the area of a square and $A = \frac{1}{2}bh$ for the area of a triangle.

Practice

1. Circle the correct answer for each question.

a) $16^2 = ?$ 4 256

b) $\sqrt{100} = ?$ 50 10

c) $25^2 = ?$ 5 625

2. The area of a square is given. Find its side length. Which of the side lengths are whole numbers?

a) $A = 81\ \text{cm}^2$, $l =$ ____________________

b) $A = 30\ \text{cm}^2$, $l =$ ____________________

c) $A = 144\ \text{mm}^2$, $l =$ ____________________

d) $A = 58\ \text{m}^2$, $l =$ ____________________

3. The side length of a square is given. Find its area.

a) $l = 7$ cm, $A =$ ________

b) $l = 15$ m, $A =$ ________

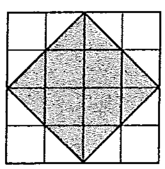
c) $l = \sqrt{36}$ cm, $A =$ ________

d) $l = \sqrt{50}$ mm, $A =$ ________

e) $l = \sqrt{24}$ cm, $A =$ ________

f) $l = \sqrt{121}$ mm, $A =$ ________

4. Find the area of each shaded square. Then write the side length of the square.

a) 

Area of large square = _____ square units

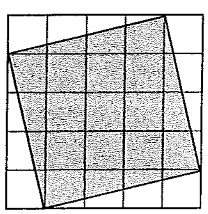
Area of each triangle = _____ square units

Area of shaded square = area of large square – _____ × area of each triangle

= ______________________________

= ______________________________

Side length = ______________________________

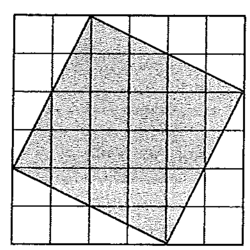
b) 

Area of square

____

Side length

____

c) 

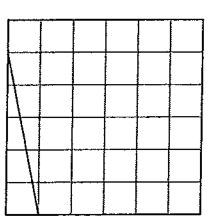
Area of square

____

Side length

____

5. Copy each line segment and square onto grid paper. Draw a square on each line segment. Find the area of the square and the length of the line segment.

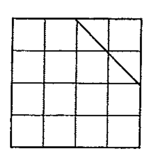
a) 

Area of square

____

Length of line segment

____

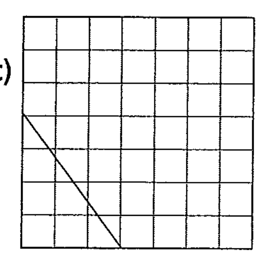
b) 

Area of square

____

Length of line segment

____

c) 

Area of square

____

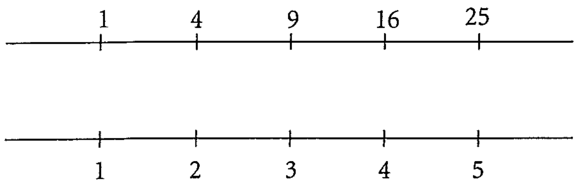
Length of line segment

____

1.4 Estimating Square Roots

Quick Review

➤ To estimate the square root of a number that is not a perfect square, you can use a number line.

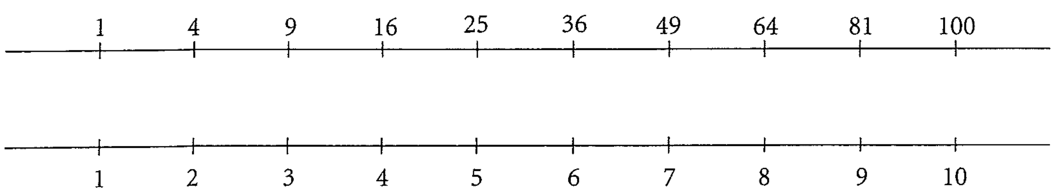


To estimate $\sqrt{10}$: Note that $\sqrt{10}$ lies between $\sqrt{9}$ and $\sqrt{16}$. So, $\sqrt{10}$ must have a value between 3 and 4, but closer to 3. Use trial and error and a calculator to get a closer approximation. Round to 2 decimal places.

Try 3.3: $3.3 \times 3.3 = 10.89$ too big
Try 3.2: $3.2 \times 3.2 = 10.24$ too big
Try 3.1: $3.1 \times 3.1 = 9.61$ too small
Try 3.16: $3.16 \times 3.16 = 9.99$ very close
$\sqrt{10}$ is approximately 3.16.

Practice

1. Use the number lines to complete each statement with whole numbers. The first one is done for you.

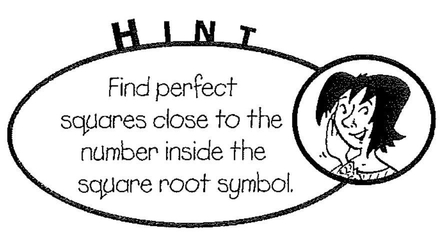


a) $\sqrt{5}$ lies between ______ and ______

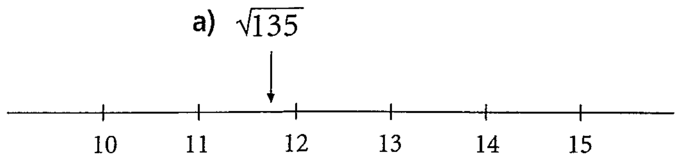
b) $\sqrt{20}$ lies between ______ and ______

c) $\sqrt{55}$ lies between ______ and ______

d) $\sqrt{2}$ lies between ______ and ______



2. Place the letter of the question on the number line below. The first one is done for you.



a) $\sqrt{135}$ b) $\sqrt{201}$ c) $\sqrt{108}$ d) $\sqrt{167}$ e) $\sqrt{188}$

3. Which statements are true, and which are false?

a) $\sqrt{20}$ is between 19 and 21. ________

b) $\sqrt{20}$ is between 4 and 5. ________

c) $\sqrt{20}$ is closer to 4 than 5. ________

d) $\sqrt{20}$ is between $\sqrt{19}$ and $\sqrt{21}$. ________

4. Which are good estimates of the square roots?

a) $\sqrt{19} = 4.75$ ________________

b) $\sqrt{220} = 14.83$ ________________

5. Use a calculator and the trial and error method to approximate each square root to 1 decimal place. Record each trial.

a) $\sqrt{20} =$ ______ b) $\sqrt{57} =$ ______ c) $\sqrt{115} =$ ______ d) $\sqrt{175} =$ ______

6. Find the approximate side length of the square with each area. Answer to 1 decimal place.

a) $A = 50$ cm^2
$s =$ ________

b) $A = 125$ cm^2
$s =$ ________

c) $A = 18$ cm^2
$s =$ ________

7. Which is the closest estimate of $\sqrt{99}$: 9.94 or 9.95 or 9.96? How did you find out?

8. What length of fencing is required to surround a square field with area 250 m^2? Answer to 2 decimal places.

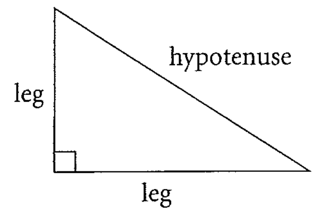
Side length = $\sqrt{\_\_\_\_\_\_}$ = ______

Perimeter = ______ + ______ + ______ + ______ = ______

1.5 The Pythagorean Theorem

Quick Review

- A right triangle has two **legs** that form the right angle. The side opposite the right angle is called the **hypotenuse**.

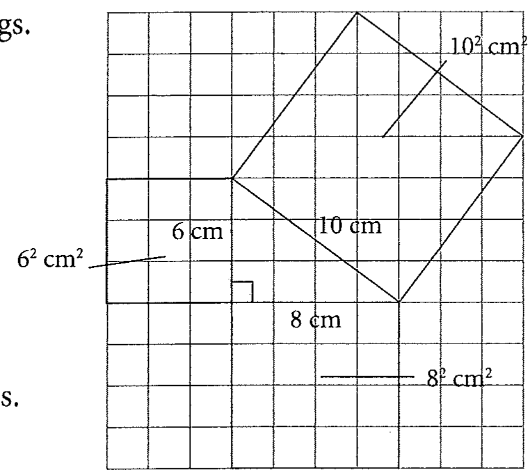


- The three sides of a right triangle form a relationship known as the **Pythagorean Theorem.**

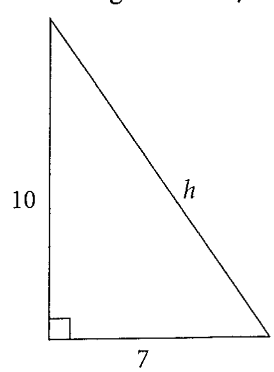
  Pythagorean Theorem: The area of the square on the hypotenuse is equal to the sum of the areas of the squares on the legs.

- In the diagram:
  Area of square on hypotenuse:
  10^2 cm^2 = 100 cm^2
  Areas of squares on legs:
  6^2 cm^2 + 8^2 cm^2 = 36 cm^2 + 64 cm^2
  = 100 cm^2
  Notice that $10^2 = 6^2 + 8^2$.

  This theorem is true for all right triangles.



- You can use the Pythagorean Theorem to find the length of any side of a right triangle when you know the lengths of the other two sides.



To calculate the hypotenuse h, solve for h in this equation:

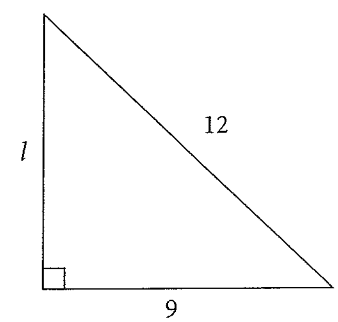
$h^2 = 7^2 + 10^2$

$h^2 = 49 + 100$

$h^2 = 149$

$h = \sqrt{149}$

Use a calculator: $h \doteq 12.2$



To calculate the leg with length l, solve for l in this equation:

$12^2 = l^2 + 9^2$

$144 = l^2 + 81$

$144 - 81 = l^2 + 81 - 81$

$63 = l^2$

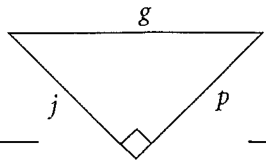
$\sqrt{63} = l$

Use a calculator: $l \doteq 7.9$ cm

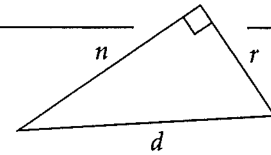
Practice

1. Identify the legs and the hypotenuse of each right triangle.

a)

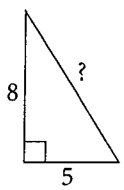


b)



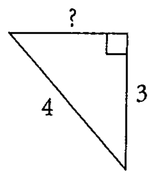
2. Circle the length of the unknown side in each right triangle.

a)



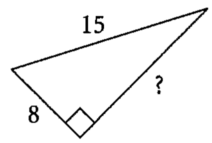
$\sqrt{13}$ $\sqrt{89}$

b)



$\sqrt{7}$ 5

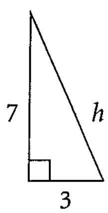
c)



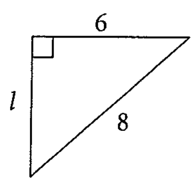
17 $\sqrt{161}$

3. Find the length of the unknown side in each right triangle. Use a calculator to approximate each length to 2 decimal places, if necessary.

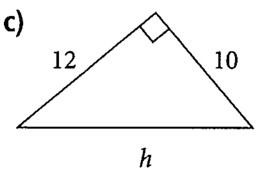
a)



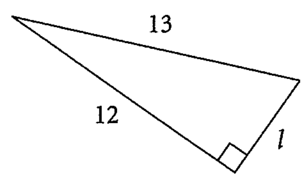
b)



c)



d)

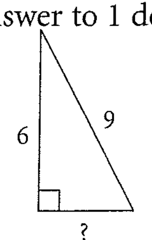


h^2 = _____ + _____   _____ = _____ + _____

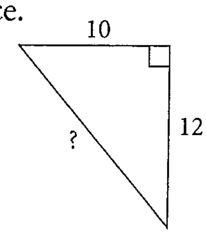
=   =

4. Find the length of the unknown side in each triangle. Use a calculator to approximate each answer to 1 decimal place.

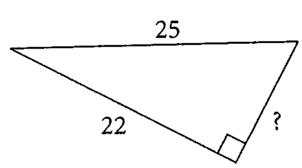
a)



b)



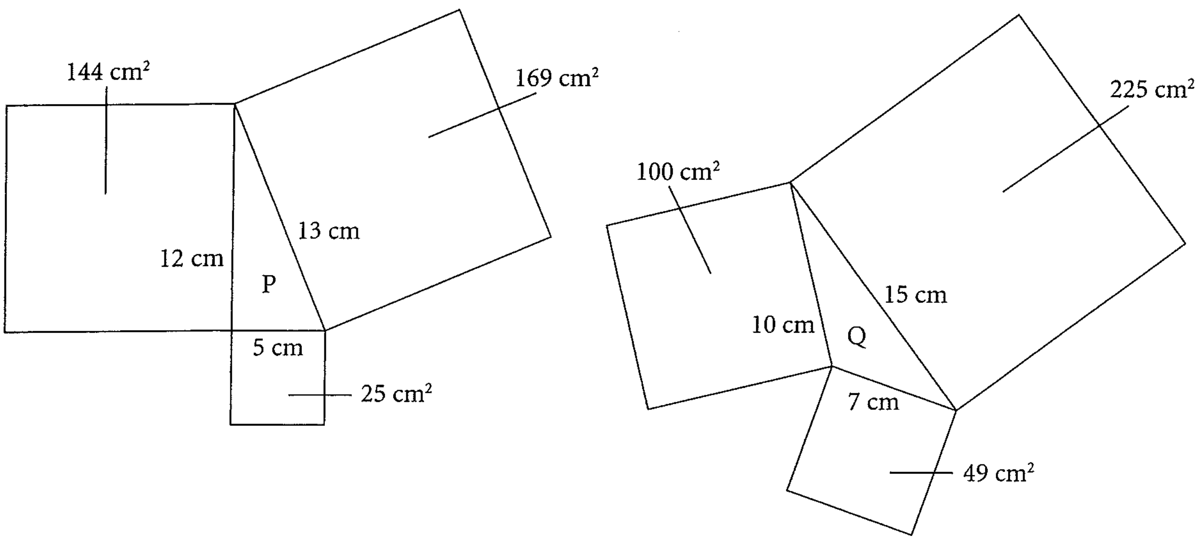
c)



Quick Review

At Home At School

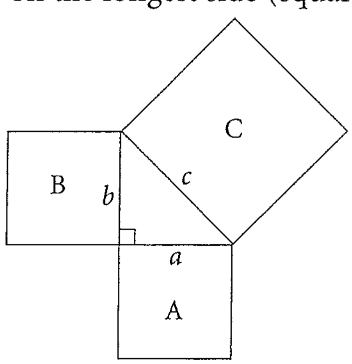
- The Pythagorean Theorem is true for right triangles only.
- To see which triangle is a right triangle, check to see if the area of the square on the longest side is equal to the sum of the areas of the squares on the other two sides.



$25\text{ cm}^2 + 144\text{ cm}^2 = 169\text{ cm}^2$ $\qquad$ $49\text{ cm}^2 + 100\text{ cm}^2 \neq 225\text{ cm}^2$

The Pythagorean Theorem applies to triangle P, but not to triangle Q.

- A set of three whole numbers that satisfy the Pythagorean Theorem is called a Pythagorean triple. For example, 5-12-13 is a Pythagorean triple because $5^2 + 12^2 = 13^2$
- For a right triangle:
  area of square on the longest side (square C) = area of square A + area of square B

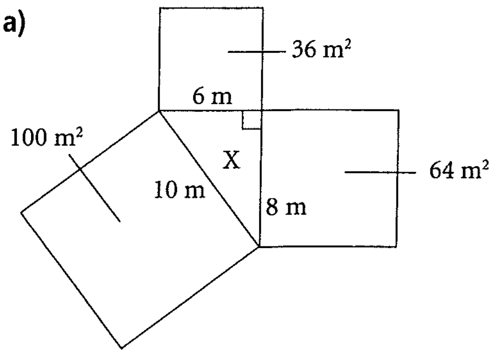


- For a Pythagorean triple a-b-c:
  $c^2 = a^2 + b^2$

Practice

1. Fill in the blanks from the list of choices to make the sentence true.

a)

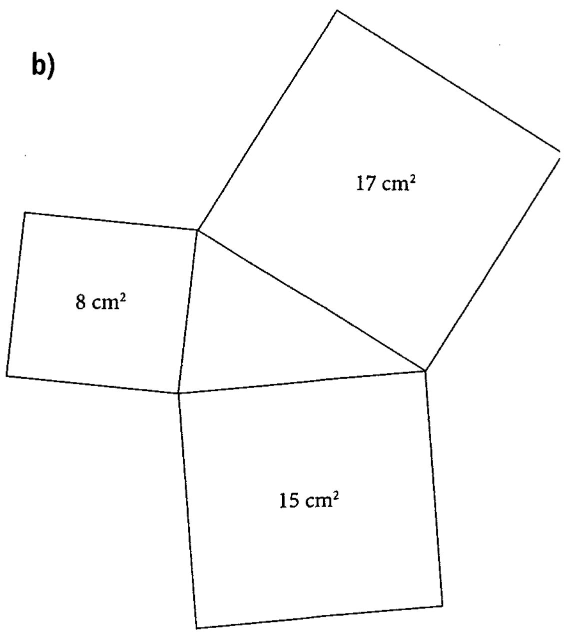


Triangle X ________ a right triangle because

____

is   is not $6 + 8 \neq 10$ $100 = 64 + 36$

b)



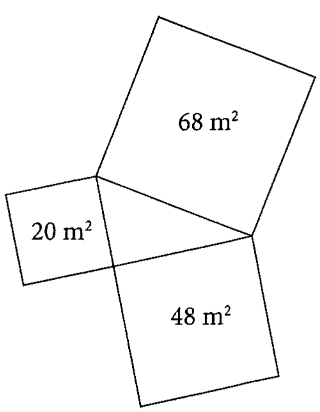
Triangle Y ________ a right triangle because

____

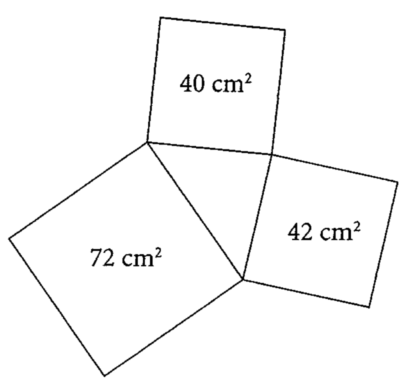
is   is not $8^2 + 15^2 = 17^2$ $8 + 15 \neq 17$

2. Which of the following triangles are right triangles? Explain.

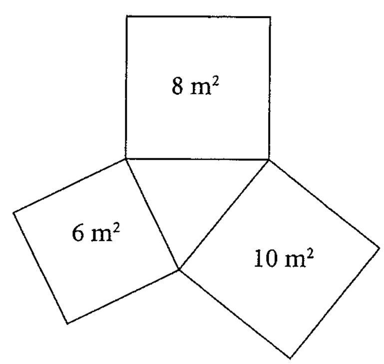
a)



b)



c)



______________ ______________ ______________

______________ ______________ ______________

3. Determine whether the triangle with each set of side lengths is a right triangle. Justify your answer.

a) 20 cm, 30 cm, 40 cm

_____ cm^2 + _____ cm^2 = ______ cm^2

_____ cm^2 = ______ cm^2

The triangle _______ a right triangle

because ____________________

b) 30 mm, 40 mm, 50 mm

____

____

c) 20 m, 21 m, 29 m

____

____

d) 60 cm, 11 cm, 62 cm

____

____

4. Fill in the blanks to make the sentence true.

The set of numbers 7, 24, 25 is a Pythagorean triple because ______ + ______ = ______

5. Which of these sets of numbers are Pythagorean triples? Explain.

a) 10, 50, 60

This __________ a Pythagorean triple

because $10^2 + 50^2$ ______ 60^2

b) 12, 35, 37

____

____

6. Two numbers of a Pythagorean triple are given. Find the missing number. The numbers are listed in ascending order.

a) 7, 24, _______

The missing number is the __________ of the sum of the ________ of the first two numbers.

____

= _____

= _____

b) 16, 30, ________

c) 10, ________, 26

7. Doug wants to check that a lawn he is planting is a rectangle. He measures the width of the lawn to be 10 m, the length to be 24 m, and the diagonal to be 26 m. Is the lawn a rectangle? Explain.

If the lawn is a rectangle, then the width, length, and diagonal form a __________ triangle.

____

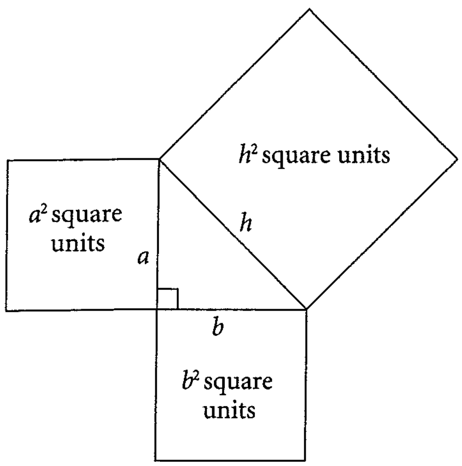
____

____

1.7 Applying the Pythagorean Theorem

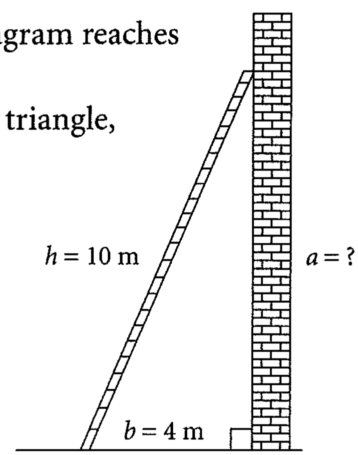
Quick Review

- The Pythagorean Theorem applies to right triangles.
- An algebraic equation for the Pythagorean Theorem is $h^2 = a^2 + b^2$, where h is the length of the hypotenuse and a and b are the lengths of the legs.



- You can apply the Pythagorean Theorem to problems involving right triangles.

You can calculate how high up the wall the ladder in the diagram reaches using the formula $h^2 = a^2 + b^2$
Since the length of the ladder is the hypotenuse of the right triangle, we label it h. The lengths of the two legs of this triangle are labelled a and b.



Substitute $b = 4$ and $h = 10$ into $h^2 = a^2 + b^2$

$10^2 = a^2 + 4^2$
$100 = a^2 + 16$
$100 - 16 = a^2 + 16 - 16$
$84 = a^2$
$\sqrt{84} = a$
$9.2 \doteq a$

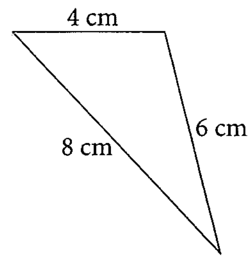
a is approximately 9.2 m. The ladder reaches approximately 9.2 m up the wall.

Tip

*It does not matter which leg is labelled a and which leg is labelled b, so long as a and b label the **legs** and h labels the **hypotenuse**.*

Practice

1. Use the Pythagorean Theorem to check if this is a right triangle.



Substitute $a =$ ____, $b =$ ____, and $h =$ ____ into the formula $h^2 = a^2 + b^2$

$h^2 =$ ______ $a^2 + b^2 =$ ______

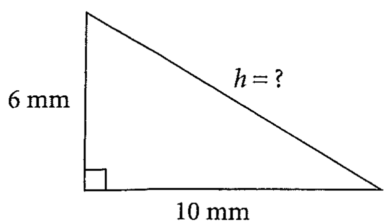
Circle the choices that make the sentence true.

Since h^2 *equals* / *does not equal* $a^2 + b^2$, the triangle *is* / *is not* a right triangle.

For questions 2 to 5, give each length to 1 decimal place.

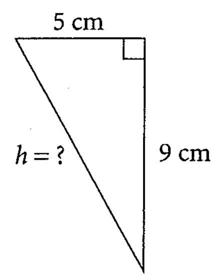
2. Use the equation $h^2 = a^2 + b^2$ to find the length of the unknown side.

a)



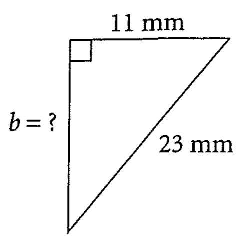
$h \doteq$ ________

b)



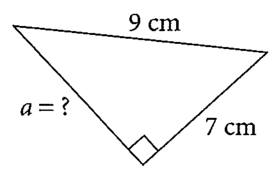
$h \doteq$ ________

c)



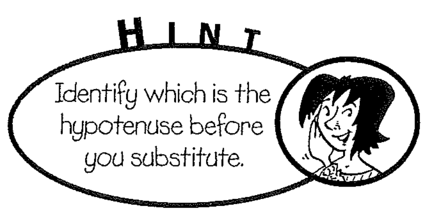
$b \doteq$ ________

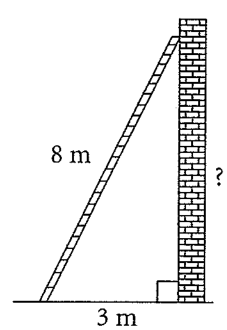
d)



$a \doteq$ ________

3. An 8-m ladder leans against a wall. How far up the wall does the ladder reach if the foot of the ladder is 3 m from the base of the wall? Show your work.

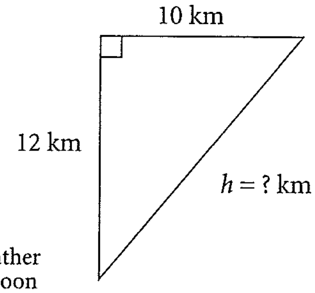




$b \doteq$ ________

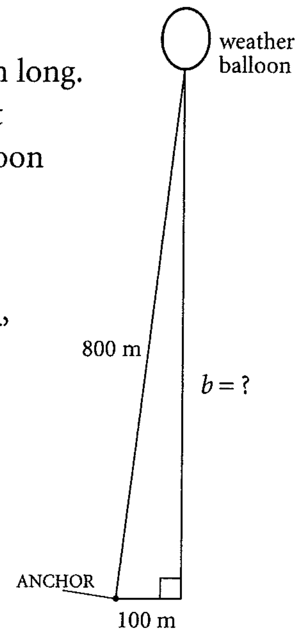
The ladder can reach a height of ______, to 1 decimal place.

4. A ship leaves port and travels 12 km due north. It then changes direction and travels due east for 10 km. How far must it travel to go directly back to port? Sketch a diagram to explain.



The ship must travel ________, to 1 decimal place, to go directly back to port.

5. A weather balloon is anchored by a cable 800 m long. The balloon is flying directly above a point that is 100 m from the anchor. How high is the balloon flying? Give your answer to the nearest metre.



The balloon is flying at a height of __________, to the nearest metre.

6. A rectangular field is 40 m long and 30 m wide. Carl walks from one corner of the field to the opposite corner, along the edge of the field. Jade walks across the field diagonally to arrive at the same corner. How much shorter is Jade's shortcut?

Tip
Sketch a diagram first.

The diagonal of the field measures ______.
Jade walks ______.
Carl walks ______ + ______ = ______
Jade's shortcut is ______ – ______ = ______ shorter.

7. What is the length of a diagonal of a square with area 100 cm^2? Give your answer to 1 decimal place.

The side length of the square is the square root of _______, or ______ cm.

The diagonal of the square is the __________ of the right triangle with sides ______ and ______.

The length of the diagonal of the square is _______, to 1 decimal place.

In Your Words

Here are some of the important mathematical words of this unit.
Build your own glossary by recording definitions and examples here. The first one is done for you.

perfect square (square number) ____

the product of a whole number multiplied by itself
For example, 25 is 5×5, so 25 is a perfect square.

square root ____

legs of a right triangle ____

hypotenuse ____

Pythagorean Theorem ____

Pythagorean triple ____

List other mathematical words you need to know.

____

Unit Review

LESSON

1.1 **1.** Circle the perfect squares. Use a diagram to support your answer.

a) 36 **b)** 63 **c)** 121 **d)** 99

1.2 **2.** Simplify without using a calculator.

a) $8^2 =$ ____________ **b)** $\sqrt{49} =$ ____________

c) $12^2 =$ ____________ **d)** $\sqrt{121} =$ ____________

3. List the factors of each number in ascending order. Circle the numbers that are perfect squares.

a) 50 **b)** 196

____________________ ____________________

c) 84 **d)** 225

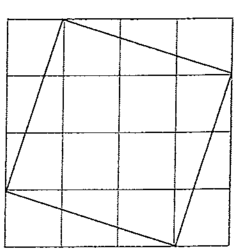
____________________ ____________________

1.3 **4.** The area of a square is given. Find its side length. Circle the side lengths that are whole numbers.

a) 18 cm^2 **b)** 169 cm^2 **c)** 200 cm^2

________ ________ ________

5. Find the area of the square. Then write the side length of the square.



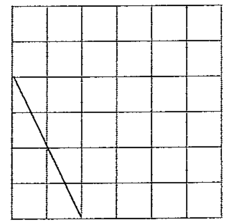
Area = ____________

Side length = ____________

LESSON

6. Construct a square on the line segment.
Find the length of the line segment.

Length = ________________



1.4 **7.** Evaluate.

a) $\sqrt{8 \times 8}$ = _____ b) $\sqrt{54 \times 54}$ = ______ c) $\sqrt{153 \times 153}$ = ______

8. Between which two whole numbers is each square root?

a) $\sqrt{45}$ ________ b) $\sqrt{18}$ ________ c) $\sqrt{55}$ ________ d) $\sqrt{135}$ ________

9. Estimate each root in question 8 to 1 decimal place.

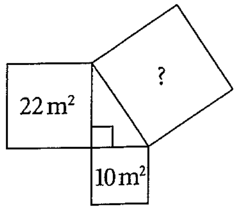
a) ________ b) ________ c) ________ d) ________

10. Circle the better estimate.

a) $\sqrt{75} \doteq 8.65$ or 8.66? b) $\sqrt{90} \doteq 9.49$ or 9.50?

1.5 **11.** Find the area of each indicated square.

a)



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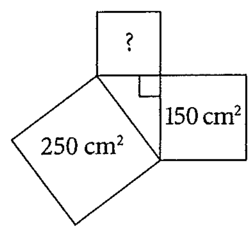
b)



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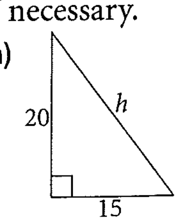
12. Find the length of each side labelled with a variable. Give answers to 1 decimal place, if necessary.

a)



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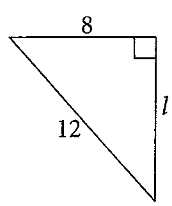
b)



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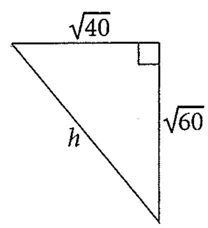
c)



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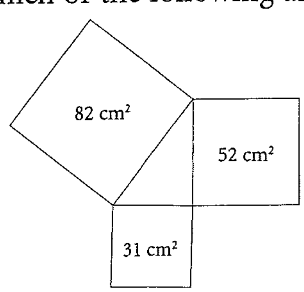
1.6 **13.** Which of the following are right triangles? Justify your answer.

a)



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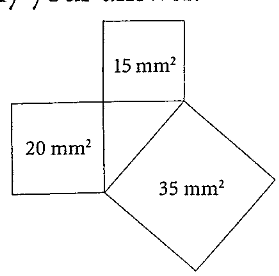
b)



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LESSON

14. Circle the sets of numbers that are Pythagorean triples.

a) 10, 24, 26

b) 12, 15, 20

c) 7, 24, 26

d) 11, 60, 61

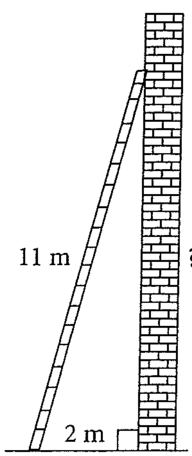
1.7 **15.** A ship travels for 14 km toward the south. It then changes direction and travels for 9 km toward the east. How far does the ship have to travel to return directly to its starting point? Answer correct to 2 decimal places.

Tip

Draw a diagram.

The ship must travel __________

16. How high up the wall does the ladder reach? Answer correct to 2 decimal places.



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Integers

Just for Fun

Modified Sudoku

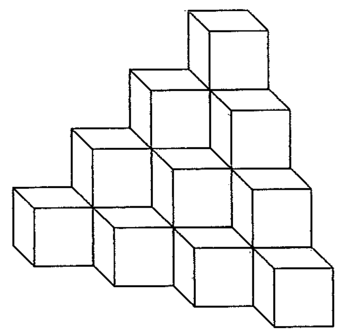
This is a modified version of a Sudoku puzzle, which originated in Japan.

Complete this grid so that every row, column, and 2 × 3 box contains every digit from 1 to 6.

	1	6			
5					
1		5	4		
6		3	2		5
					4
			1	3	

Cube Count

How many cubes are in this figure? Look for a pattern to find the answer.



There are ____________ cubes.

Add 'Em Up

Find the value of this expression without using a calculator. Explain your work.

$1 - 2 + 3 - 4 + 5 - 6 + \ldots + 99 - 100 =$ ____________

____

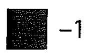
____

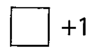
Activating Prior Knowledge

Using Models to Add Integers

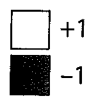
➤ You can use coloured tiles to model integers.

A black tile models –1. A white tile models +1.

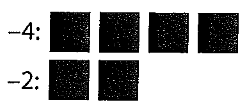




A black tile and a white tile combine to model 0.
They form a **zero pair**: (+1) + (–1) = 0

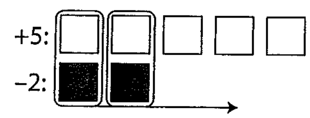


To add: (–4) + (–2)
Model –4 with 4 black tiles.
Model –2 with 2 black tiles.



There are 6 black tiles altogether.
They model –6.
So, (–4) + (–2) = –6

To add: (+5) + (–2)
Model +5 with 5 white tiles.
Model –2 with 2 black tiles.



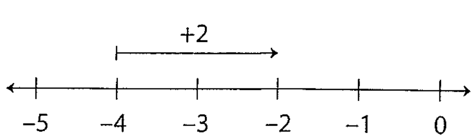
Circle zero pairs.
3 white tiles remain.
They model +3.
So, (+5) + (–2) = +3

➤ You can also use a number line to add integers.

Find the first integer on the number line.

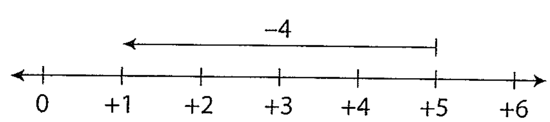
- To add a positive integer, move right on the number line.
- To add a negative integer, move left on the number line.

To add: (–4) + (+2)
Start at –4.
Move 2 units right to add +2.



The arrow ends at –2.
So, (–4) + (+2) = –2

To add: (+5) + (–4)
Start at +5.
Move 4 units left to add –4.



The arrow ends at +1.
So, (+5) + (–4) = +1

Check

1. Use tiles to add.

a) $(+1) + (+3) =$ ________ b) $(-2) + (-3) =$ ________

c) $(-4) + (+3) =$ ________ d) $(+4) + (-2) =$ ________

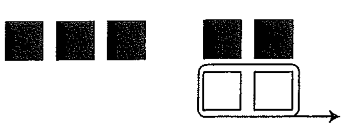
2. Use a number line to add.

a) $(+2) + (-2) =$ ________ b) $(+11) + (-5) =$ ________ c) $(+9) + (+7) =$ ________

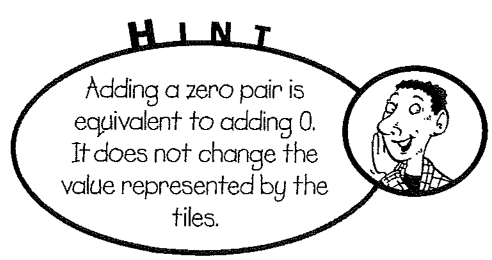
d) $(-2) + (-9) =$ ________ e) $(-12) + (+7) =$ ________ f) $(+7) + (-15) =$ ________

Using Models to Subtract Integers

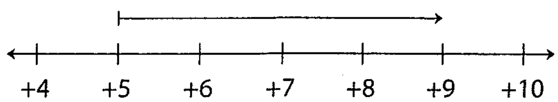
➤ To subtract, you take away tiles.
If there are not enough tiles to remove, add zero pairs.
To subtract: $(-3) - (+2)$
Model -3 with 3 black tiles.
To take away $+2$, 2 white tiles are needed.
Add 2 zero pairs of tiles to provide 2 white tiles.



5 black tiles remain. They model -5.
So, $(-3) - (+2) = -5$



➤ To subtract an integer on a number line, move in the opposite direction of adding the same integer.
To subtract a positive integer, move left.
To subtract a negative integer, move right.
To subtract: (+5) – (–4)
Start at +5.
Move 4 units right to subtract –4.



The arrow ends at +9.
So, (+5) – (–4) = +9

Check

3. Subtract using a model of your choice

a) (+2) – (–7) = ________ b) (–3) – (–4) = ________ c) (–5) – (–5) = ________

d) (+10) – (–4) = ________ e) (–5) – (+6) = ________ f) (–3) – (–5) = ________

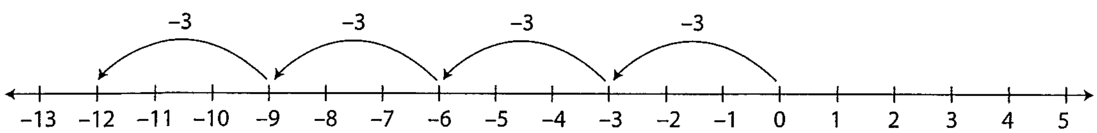
4. Match each description with the correct subtraction expression and answer.

Temperature Change	Expression	Answer
From 8°C to 3°C	(–3) – (–8)	–11
From 8°C to –3°C	(–3) – (+8)	–5
From –8°C to 3°C	(+3) – (+8)	+5
From –8°C to –3°C	(+3) – (–8)	+11

2.1 Using Models to Multiply Integers

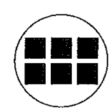
Quick Review

➤ You can think of multiplication as repeated addition.
$4 \times (-3)$ is the same as adding -3 four times.
As a sum: $(-3) + (-3) + (-3) + (-3) = -12$
As a product: $4 \times (-3) = -12$
On a number line:



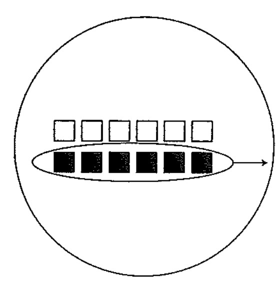
➤ You can use tiles to multiply integers.
Let a circle represent the bank. The bank has zero value at the start.

Multiply: $(+2) \times (-3)$
+2 is a positive integer.
-3 is modelled with 3 black tiles.
So, put 2 sets of 3 black tiles into the circle.



The 6 black tiles in the circle represent -6.
So, $(+2) \times (-3) = -6$

➤ Multiply: $(-2) \times (-3)$
-2 is a negative integer.
-3 is modelled with 3 black tiles.
So, we need to take 2 sets of 3 black tiles from the circle.



Add zero pairs until there are enough black tiles to remove.
Take out 2 sets of 3 black tiles.

There now are 6 white tiles left in the circle.
So, $(-2) \times (-3) = 6$

Practice

1. Write a multiplication expression for each repeated addition.

a) $(-2) + (-2) + (-2) + (-2) + (-2) = 5 \times$ ________

b) $(+11) + (+11) + (+11) =$ ____________

c) $(-5) + (-5) + (-5) =$ ____________

2. Write each multiplication expression as a repeated addition. Then use a number line to find each sum.

a) $(+2) \times (-4) = (-4) + (-4)$

$= \_\_\_\_\_\_\_\_$

b) $(+5) \times (+4) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

$= \_\_\_\_\_\_\_\_$

c) $(-3) \times (+2) = (+2) \times (-3)$

$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

$= \_\_\_\_\_\_\_\_$

3. Write a multiplication equation for each model. Find the product.

a) Deposit 3 sets of 2 black tiles.

$3 \times (-2) = \_\_\_\_\_\_\_\_$

b) Deposit 5 sets of 2 white tiles.

$\_\_\_\_\_\_\_\_ \times (+2) = \_\_\_\_\_\_\_\_$

c) Withdraw 2 sets of 3 black tiles.

$\_\_\_\_\_\_\_\_ \times \_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_$

d) Withdraw 9 sets of 2 black tiles.

____

e) Deposit 4 sets of 3 black tiles.

____

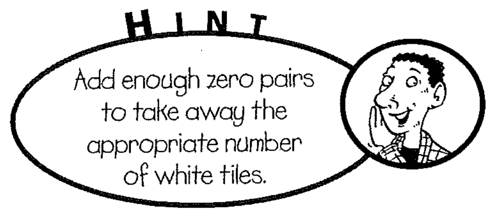
4. Use a tile model to find each product.

a) $(+7) \times (-2) = \_\_\_\_\_\_\_\_$

b) $(+3) \times (+5) = \_\_\_\_\_\_\_\_$

c) $(+2) \times (-3) =$ ________

d) $(-4) \times (+5) =$ ________



5. Use a model to represent each product. Draw the model you used each time.

a) $(-3) \times (-4) =$ ________

b) $(+2) \times (-5) =$ ________

c) $(+7) \times (+2) =$ ________

d) $(-3) \times (+6) =$ ________

6. The temperature dropped 2°C each hour for 4 h. Use integers to find the total change in temperature.

____

____

____

____

2.2 Developing Rules to Multiply Integers

Quick Review

➤ Integers have these properties of whole numbers.

- **Multiplying by 0:** $4 \times 0 = 0$ and $0 \times 4 = 0$
  So, $(-4) \times 0 = 0$ and $0 \times (-4) = 0$
- **Multiplying by 1:** $4 \times 1 = 4$ and $1 \times 4 = 4$
  So, $(-4) \times (+1) = -4$ and $(+1) \times (-4) = -4$
- **Commutative Property:** $4 \times 2 = 8$ and $2 \times 4 = 8$
  So, $(-4) \times (+2) = -8$ and $(+2) \times (-4) = -8$
- **Distributive Property:** $4 \times (2 + 3) = 4 \times 2 + 4 \times 3 = 20$
  So, $(-4) \times [(+2) + (+3)] = (-4) \times (+2) + (-4) \times (+3) = -20$

➤ You can write the product of integers without the use of the × sign.
$(-4) \times (+2)$ can simply be written as: $(-4)(+2)$

➤ When 2 integers with the same sign are multiplied, their product is positive.
$(+2)(+3) = +6$ $\quad$ $(-2)(-3) = +6$

When 2 integers with different signs are multiplied, their product is negative.
$(+2)(-3) = -6$ $\quad$ $(-2)(+3) = -6$

Practice

1. Find a pattern rule for each multiplication pattern.
Extend the pattern for 3 more rows.

HINT

To find a pattern rule, look for a pattern in the integer factors and in the products.

a)
$(+3)(+3) = +9$
$(+2)(+3) = +6$
$(+1)(+3) = +3$
$(0)(+3) =$ ________
(______)$(+3) =$ ______
____
____

b)
$(-3)(+3) = -9$
$(-3)(+2) = -6$
$(-3)(+1) = -3$
$(-3)(0) =$ ________
____
____
____

2. In this chart, write the sign of each product of multiplying 2 integers.

×	positive integer	negative integer
positive integer		
negative integer		

- When 2 integer factors have the same sign, their product is ________________.
- When 2 integer factors have different signs, their product is ________________.

3. Find each product.

a) (+7)(–2) = ________ **b)** (–4)(–3) = ________ **c)** (–8)(+9) = ________

d) (+10)(–5) = ________ **e)** (+5)(–7) = ________ **f)** (–9)(–4) = ________

i) (–7)(–1) = ________ **j)** (+5)(0) = ________ **k)** (+20)(–20) = ________

4. Fill in the blank to make each equation true.

a) (+7) × ________ = –35 **b)** ________ × (–9) = +99 **c)** (–10) × ________ = –320

d) ________ × (–5) = +20 **e)** (+7) × ________ = –49 **f)** ________ × (+13) = –65

g) ________ × (–15) = –180 **h)** (+14) × ________ = –140 **i)** ________ × (–7) = 56

5. Match each pattern rule with the corresponding pattern.
Complete each pattern and pattern rule.

Number Pattern	**Pattern Rule**
–3, +9, –27, + 81, . . .	Start at 2. Multiply by ________ each time.
+2, –10, +50, –250, . . .	Start at 1. Multiply by –10 each time.
+3, –3, ________, ________, . . .	Start at ________. Multiply by –3 each time.
+1, –10, ________, ________, . . .	Start at 3. Multiply by –1 each time.
–1, –2, –4, –8, –16, . . .	Start at –1. Multiply by ________ each time.

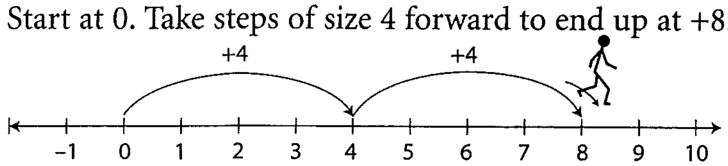
2.3 Using Models to Divide Integers

Quick Review

Division is the inverse of multiplication.
So, 10 ÷ 5 = ? is the same as ? × 5 = 10.
The product means, "how many sets of 5 produce 10?"

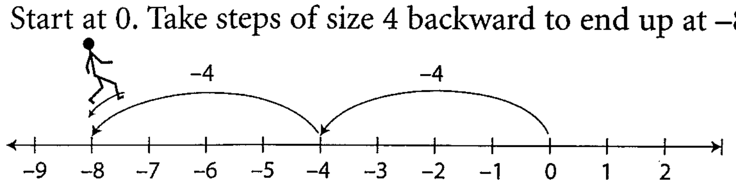
You can "walk" a number line to model the division of two integers.
If the step size is positive, walk forward. If the step size is negative, walk backward.
The number of steps is the quotient and the direction you are facing at the end determines its sign.

- **Positive ÷ Positive**
  Divide: (+8) ÷ (+4)
  Start at 0. Take steps of size 4 forward to end up at +8.



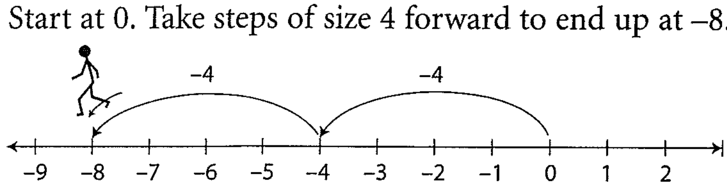
You took 2 steps and are facing the positive end of the line. So, (+8) ÷ (+4) = +2

- **Negative ÷ Negative:**
  Divide: (−8) ÷ (−4)
  Start at 0. Take steps of size 4 backward to end up at −8.



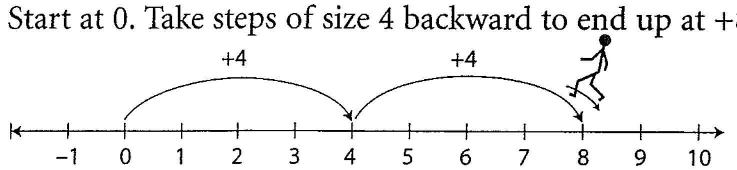
You took 2 steps and are facing the positive end of the line. So, (−8) ÷ (−4) = +2.

- **Negative ÷ Positive:**
  Divide: (−8) ÷ (+4)
  Start at 0. Take steps of size 4 forward to end up at −8.



You took 2 steps and are facing the negative end of the line. So, (−8) ÷ (+4) = −2.

- **Positive ÷ Negative:**
  Divide: (+8) ÷ (−4)
  Start at 0. Take steps of size 4 backward to end up at +8.



You took 2 steps. You are facing the negative end of the line. So, (−8) ÷ (+4) = −2.

Practice

1. Write 2 related multiplication equations for each division equation.

   a) $(+60) \div (+10) = +6$ ______________________, ______________________

   b) $(+36) \div (-9) = -4$ ______________________, ______________________

   c) $(-45) \div (-9) = +5$ ______________________, ______________________

   d) $(-16) \div (+2) = -8$ ______________________, ______________________

2. Suzanne wanted to model division using a number line. She started at zero and took steps backward of size 3. She ended up at –21.

   a) Illustrate this problem using a number line.

   b) Model this problem using a division equation. ______________________

   c) How many steps did Suzanne take? ______________________

3. Use a number line. Find each quotient.

   a) $(+24) \div (-8) =$ __________

   b) $(-20) \div (-5) =$ __________

   c) $(+25) \div (+5) =$ __________

   d) $(-18) \div (-9) =$ __________

4. Find each quotient.

a) $(-12) \div (+4) =$ _________ b) $(-12) \div (-6) =$ _________ c) $(-8) \div (+4) =$ _________

5. The water level in a well dropped 4 cm each hour. The total drop in the water level was 28 cm. Use an integer model to find out how long it took for the water level to change.

____

____

____

____

6. Use coloured tiles, a number line, or another model to clearly show your thinking. Find each quotient.

a) $(+10) \div (+2) =$ _________ b) $(-10) \div (-2) =$ _________

c) $(+10) \div (-2) =$ _________ d) $(-10) \div (+2) =$ _________

Compare the quotients. What do you notice? ____________________

7. The temperature dropped a total of 12°C over a 4-h period. The temperature dropped the same amount each hour. Using a model, show the hourly drop in temperature.

____

____

____

____

____

2.4 Developing Rules to Divide Integers

Quick Review

- For any multiplication of 2 different factors, there are 2 related division facts:
  For $4 \times 3 = 12$, the related division facts are: $12 \div 3 = 4$ and $12 \div 4 = 3$

  The same rules apply to the product of 2 integers.
  For $(-2)(+5) = -10$, the related division facts are:

  $(-10) \div (-2) = +5$ and $(-10) \div (+5) = -2$

  dividend (−10) **divisor** (−2) **quotient** (+5)

- The quotient of 2 integers with the same sign is positive.
  $(+10) \div (+2) = +5$ $(-10) \div (-2) = +5$
- The quotient of 2 integers with different signs is negative.
  $(+10) \div (-2) = -5$ $(-10) \div (+2) = -5$
- A division expression can be written using a division sign, $(-24) \div (-6)$, or it can be written as a fraction, $\frac{(-24)}{(-6)}$.

Practice

1. For each product, complete the 2 related division facts and name the sign of the quotient.

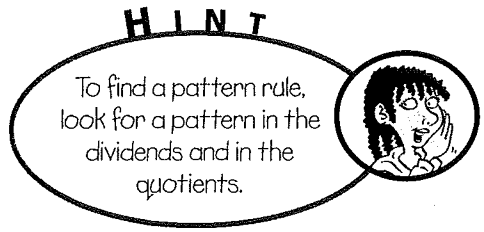
Multiplication Fact	Related Division Facts	Sign of Quotient
$(+2)(+3) = +6$	$(+6) \div (+2) =$ ________	________
	$(+6) \div (+3) =$ ________	________
$(-2)(-3) = +6$	$(+6) \div (-2) =$ ________	________
	$(+6) \div (-3) =$ ________	________
$(+2)(-3) = -6$	$(-6) \div (+2) =$ ________	________
	$(-6) \div (-3) =$ ________	________
$(-2)(+3) = -6$	$(-6) \div (-2) =$ ________	________
	$(-6) \div (+3) =$ ________	________

2. Use your results in question 1. Complete these 2 statements.

When 2 integers have the same sign, their quotient is ________________.

When 2 integers have different signs, their quotient is ________________.

3. Find a pattern rule for each division pattern.
Extend the pattern 3 more rows.



a) (+6) ÷ (–2) = –3
(+4) ÷ (–2) = –2
(+2) ÷ (–2) = –1
(0) ÷ (–2) = ________
____
____
____

b) (–12) ÷ (–4) = +3
(–8) ÷ (–4) = +2
(–4) ÷ (–4) = +1
(0) ÷ (–4) = ________
____
____
____

Use the last 3 rows of each pattern. Complete these statements.

When both the dividend and divisor are negative, the quotient is ____________.

When the dividend is positive and the divisor is negative, the quotient is ____________.

4. Find each quotient.

a) (+15) ÷ (–3) = __________ **b)** (–32) ÷ (+4) = __________ **c)** (+72) ÷ (–8) = __________

d) (–54) ÷ (–9) = __________ **e)** (–72) ÷ (+6) = __________ **f)** (+88) ÷ (+11) = __________

g) (–42) ÷ (–6) = __________ **h)** (+108) ÷ (+9) = __________ **i)** (–56) ÷ (+7) = __________

5. Use 2 of these 5 integers. Write a division fact with each quotient.
–2 +3 +12 –1 +4

a) a quotient of –2 ____________________

b) the greatest quotient ____________________

c) the least quotient ____________________

d) a quotient between –5 and –10 ____________________

6. Use a calculator to divide.

a) (+247) ÷ (–13) = __________ **b)** (–851) ÷ (–37) = __________

c) $\frac{(-748)}{(-68)}$ = __________ **d)** $\frac{(-1485)}{(+33)}$ = __________

Tip
Look for the [–] or [+/–] key on your calculator to key in negative numbers.

2.5 Order of Operations with Integers

Quick Review

➤ The order of operations with whole numbers also applies to integers.

① Perform operations in brackets first.
② Divide and multiply, in order, from left to right.
③ Add and subtract, in order, from left to right.

	① ③ ② $(1 + 2) - 3 \times 4$
① B	$= 3 - 3 \times 4$
② DM	$= 3 - 12$
③ AS	$= -9$

Tip
The letters BDMAS can help you remember the order of operations.
***B**–Brackets*
***DM**–Divide, Multiply*
***AS**–Add, Subtract*

➤ A fraction bar indicates division.
It also acts like brackets.
Evaluate the numerator and denominator separately before dividing.

For example, $\frac{12 + 8}{2 - 6} = \frac{20}{-4} = -5$

➤ If an integer does not have a sign, it is assumed to be positive: 2 = +2

Practice

1. Simplify.

a) $5 - 2 - 6$

$=$ ______ $- 6$

$=$ ________

b) $3(8 - 12)$

$= 3 \times$ ______

$=$ ________

Tip
Brackets symbolize multiplication as well as grouping.
3(8 – 12) means
3 × (8 – 12).

c) $-4 + 2 \times 3$

$= -4 +$ ______

$=$ ________

d) $21 \div (-7) \times 5$

$=$ ________

$=$ ________

e) $10 - [(5 - 3) + 9]$

____

f) $-8 + 15 \div (-3) + 7$

____

g) $(-3)(-8) + 24 \div (-2)$

____

2. Match each expression with its answer.

Expression	Answer
$30 \div (5 - 10) \times 2$	-14
$30 \div (5 - 10 \times 2)$	-12
$(30 \div 5 - 10) \times 2$	-8
$30 \div 5 - 10 \times 2$	-2

3. Simplify.

a) $\frac{3(5-9)}{2}$

$= \frac{3(\quad)}{2}$

= ____________

= ____________

b) $\frac{(-4)(-2)}{-8}$

____

c) $\frac{(-6)(4)+8}{(-2)\times 4}$

____

4. Evaluate each expression. Write the letter for the answer in the corresponding blank at the bottom to find out what one wall said to the other.

$2(-7+3)$ = 2________ = ________ A	$-8 + 12 \div 4$ = −8 + ________ = ________ C	$3(10 \div 2) - (-4)$ = 3______ + ______ = ________ = ________ E
$(-6)(-6) \div (-4)$ H	$4 \times (-3) + 24 \div 2$ M	$-5 + 12 \div 4 \times (-2)$ N
$19 - 3 \times 4 \div (-6)$ O	$\frac{6(-8)}{-12} - 1$ R	$\frac{10 - 2(-3)}{2 \times 4}$ T

___ ___ ___ ___   ___ ___   ___ ___   ___ ___ ___   ___ ___ ___ ___ ___ ___

0 19 19 2   0 19   −8 2   2 −9 19   −5 21 3 −11 19 3

In Your Words

Here are some of the important mathematical words of this unit.
Build your own glossary by recording definitions and examples here. The first one is done for you.

integer *the numbers ..., -3, -2, -1, 0, 1, 2, 3, ...*
For example, 1, 2, 3, ... are positive integers and -1, -2, -3, ... are negative integers. 0 is neither positive nor negative.

quotient ____________________

zero pair ____________________

commutative property ____________________

zero property ____________________

order of operations ____________________

List other mathematical words you need to know.

Unit Review

LESSON

2.1 **1.** Write each multiplication as a repeated addition. Then illustrate using coloured tiles to find each sum.

a) $(+5) \times (-2) =$ ______________________________

$=$ __________________

b) $(+3) \times (+5) =$ ______________________________

$=$ __________________

c) $(+3) \times (-3) =$ ______________________________

$=$ __________________

d) $(-4) \times (+2) = (+2) \times$ _________

$=$ ______________________________

$=$ __________________

2. Use a number line. Find each product.

a) $(+5) \times (-1) =$ _________

b) $(+3) \times (+4) =$ _________

c) $(-2) \times (+6) =$ _________

d) $(+4) \times (-5) =$ _________

LESSON

3. a) The temperature rose 2°C each hour for 6 h. Use integers to find the total change in temperature.

____

____

____

____

b) If the starting temperature was –4°C, what was the temperature after 6 h?

____

____

4. Show how to model (–2) × (–5). Explain why you chose that model.

2.2 **5.** Complete each statement using positive, negative, or zero.

a) The product of a positive integer and a negative integer is ______________.

b) The product of a negative integer and zero is ______________.

c) The product of an two negative integers is ______________.

6. Find each product.

a) (+2)(+3) = ________ **b)** (–6)(+4) = ________

c) (–22)(–10) = ________ **d)** (+24)(–30) = ________

e) (–36)(–5) = ________ **f)** (+42)(+3) = ________

g) (–81)(+2) = ________ **h)** (–237)(0) = ________

7. Fill in the blank to make each equation true.

a) (–6) × ______ = –24 **b)** (–9) × ______ = +27

c) ______ × (–3) = (–21) **d)** (–4) × ______ = +24

e) (+20) × ______ = +300 **f)** (–32) × ______ = –160

LESSON

2.3 **8.** Write a related multiplication equation for each division equation.

a) $(+100) \div (-25) = -4$

____

b) $(-28) \div (-7) = +4$

____

c) $\frac{(-15)}{(-5)} = +3$

____

d) $\frac{(+48)}{(+12)} = +4$

____

9. Show how to model $(-12) \div 4$.

2.4 **10.** Decide whether each quotient will be positive, negative, or zero. Then evaluate each quotient.

a) $(-25) \div (-5)$ ____________

b) $(-36) \div (+9)$ ____________

c) $\frac{(+42)}{(-7)}$ ____________

d) $0 \div (-5)$ ____________

11. Evaluate each quotient and order the results from least to greatest.

a) $(-20) \div (+4) =$ ________ b) $(-18) \div (-6) =$ ________ c) $(+48) \div (-8) =$ ________

The quotients from least to greatest are: ____________________

12. Find all of the divisors of –16. Write a division equation each time. The first one has been done for you.

Divisor	Division Equation
–1	$(-16) \div (-1) = +16$

LESSON

13. Write the next 3 terms in each pattern. Then write the pattern rule.

a) +1, –4, +16, –64, ______, ______, ______, …

Pattern rule: Start at ____. ________________ each time.

b) –128, +64, –32, 16, ______, ______, ______, …

Pattern rule: Start at ____. ________________ each time.

c) –3125, +625, –125, ______, ______, ______, …

Pattern rule: Start at ____. ________________ each time.

2.5 **14.** State which operation you would do first. Do not evaluate.

a) $(+8) + (-2) \times (-3)$

____

b) $(-20) \div (-4) - (-2)$

____

c) $(-2)(4 - 5)$

____

d) $5 - 3 + (-4) \times (-2)$

____

15. Evaluate each expression in question 14. Show all your steps.

a)

b)

c)

d)

LESSON

16. Evaluate using the order of operations.

a) $17 - 4 \times 4 =$

b) $-48 \div 4 - 2(3 - 4) =$

c) $-2 - 4 \times 9 =$

d) $\frac{(-6)(8-2)}{-4} =$

e) $(-3) \times (-3) + (-4) \times (-4) =$

f) $\frac{21 + 2(3)}{(-3) \times (-3)} =$

UNIT 3

Operations with Fractions

Just for Fun

Fraction Word Search

Can you find this list of words in the word search table at the right?

Words can be horizontal, vertical, or diagonal.

SIMPLIFY

IMPROPER

NUMERATOR

EQUIVALENT

FRACTION

MIXED

PART

WHOLE

K	V	W	W	X	J	J	O	S	E
F	Q	P	U	K	M	P	P	Q	W
R	O	T	A	R	E	M	U	N	V
A	E	T	W	W	F	I	U	Z	X
C	D	P	H	P	V	X	Y	A	M
T	K	H	O	A	Q	E	N	N	L
I	E	Z	L	R	E	D	W	F	G
O	M	E	E	T	P	U	Z	M	Q
N	N	E	W	F	O	M	H	W	I
T	C	S	I	M	P	L	I	F	Y

Compose It

A Game for 2 or more

Make as many words as you can from the letters of the word "fraction." Words must contain at least four letters. The person with the most words after 3 minutes wins!

____

Winfrac

A Game for 2 or more

Play with one or more classmates.

You will need two 8- or 10-sided dice, a pencil, and paper.

Take turns to roll the two dice. Use the 2 numbers to create a proper fraction.

The player with the larger fraction wins a point.

The first player to reach 10 points wins the game.

Activating Prior Knowledge

Equivalent Fractions

➤ $\frac{1}{3}$, $\frac{2}{6}$, $\frac{3}{9}$, and $\frac{4}{12}$ are equivalent fractions. To find equivalent fractions, multiply or divide the numerator and denominator by the same number.

Check

1. Write 3 equivalent fractions for each fraction.

a) $\frac{6}{24}$ ________________ b) $\frac{21}{14}$ ________________ c) $\frac{30}{72}$ ________________

Relating Mixed Numbers and Improper Fractions

➤ To convert $3\frac{5}{8}$ to an improper fraction:

$$3\frac{5}{8} = 3 + \frac{5}{8}$$
$$= \frac{24}{8} + \frac{5}{8}$$
$$= \frac{29}{8}$$

➤ To convert $\frac{17}{5}$ to a mixed number:

$$\frac{17}{5} = \frac{15}{5} + \frac{2}{5}$$
$$= 3\frac{2}{5}$$

Check

2. Convert each mixed number to an improper fraction.

a) $3\frac{4}{5}$ = __________

$= \frac{\square}{5} +$ __________

= __________

b) $5\frac{4}{9}$ = __________

c) $3\frac{7}{20}$ = __________

d) $2\frac{1}{24}$ = __________

3. Convert each improper fraction to a mixed number.

a) $\frac{27}{8} = \frac{\square}{8} + \frac{\square}{8}$

= __________

b) $\frac{41}{18}$ = __________

c) $\frac{41}{15}$ = __________

d) $\frac{29}{12}$ = __________

Adding and Subtracting Fractions

➤ To add or subtract fractions with the same denominator, add or subtract the numerators.

$\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$ $\frac{9}{13} - \frac{3}{13} = \frac{6}{13}$

➤ To add or subtract fractions with different denominators:

- Use the least common multiple of the denominators as the common denominator.
- Write equivalent fractions with this common denominator.

To add $\frac{1}{4} + \frac{5}{6}$, find the least common multiple of 4 and 6.

The least common multiple of 4 and 6 is 12.

$$\frac{1}{4} + \frac{5}{6} = \frac{3}{12} + \frac{10}{12}$$
$$= \frac{13}{12}$$

To subtract $\frac{5}{8} - \frac{1}{12}$, find the least common multiple of 8 and 12.

The least common multiple of 8 and 12 is 24.

$$\frac{5}{8} - \frac{1}{12} = \frac{15}{24} - \frac{2}{24}$$
$$= \frac{13}{24}$$

➤ To add or subtract mixed numbers, add or subtract the fractions and then add or subtract the whole numbers. Sometimes, you need to regroup a whole number to subtract the fractions. Simplify if necessary.

$$2\frac{1}{4} + 3\frac{5}{6} = 2\frac{3}{12} + 3\frac{10}{12}$$
$$= 5\frac{13}{12}$$
$$= 5 + 1\frac{1}{12}$$
$$= 6\frac{1}{12}$$

$$5\frac{1}{8} - 3\frac{1}{2} = 5\frac{1}{8} - 3\frac{4}{8}$$
$$= 4\frac{9}{8} - 3\frac{4}{8}$$
$$= 1\frac{5}{8}$$

Check

4. Add. Write the answer in simplest form. Write improper fractions as mixed numbers.

a) $\frac{7}{10} + \frac{1}{6} =$ ________ + ________

= ________

= ________

b) $\frac{1}{2} + \frac{3}{7} =$ ________

c) $3\frac{1}{3} + 4\frac{1}{2} =$ ________________

= ________

d) $2\frac{5}{6} + 1\frac{3}{8} =$ ________

5. Subtract. Write the answer in simplest form. Write improper fractions as mixed numbers.

a) $\frac{3}{4} - \frac{3}{10} =$ ________

= ________

b) $\frac{5}{8} - \frac{1}{6} =$ ________

c) $4\frac{1}{9} - 2\frac{2}{3} = 4\frac{1}{9} - 2\frac{\square}{9}$

$= 3\frac{\square}{9} -$ ________

= ________

d) $7\frac{1}{4} - 3\frac{5}{6} =$ ________

3.1 Using Models to Multiply Fractions and Whole Numbers

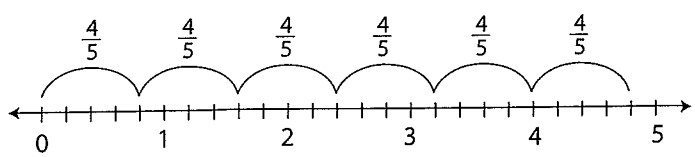
Quick Review

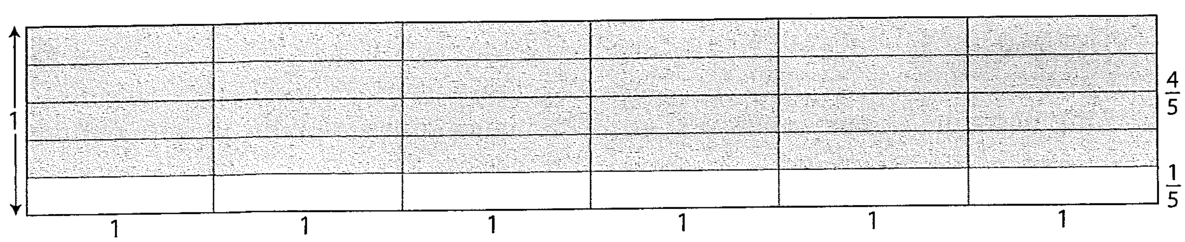
➤ Repeated addition can be written as multiplication.

$$\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=5\times\frac{1}{3}$$
$$=\frac{5}{3}$$
$$=\frac{3}{3}+\frac{2}{3}$$
$$=1\frac{2}{3}$$

$$\frac{4}{5}+\frac{4}{5}+\frac{4}{5}+\frac{4}{5}+\frac{4}{5}=6\times\frac{4}{5}$$
$$=\frac{24}{5}$$
$$=\frac{20}{5}+\frac{4}{5}$$
$$=4\frac{4}{5}$$

$6 \times \frac{4}{5} = 4\frac{4}{5}$ can also be shown on a number line or using a rectangle.





Practice

1. Write each addition statement as a multiplication statement and determine the product.

a) $\frac{5}{7}+\frac{5}{7}+\frac{5}{7}+\frac{5}{7}=$ ________ $\times \frac{5}{7}$

$=\frac{\square}{7}$

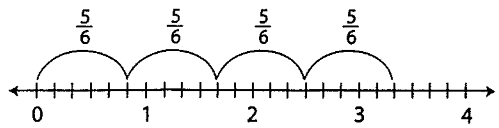
$=\square\frac{\square}{7}$

b) $\frac{7}{8}+\frac{7}{8}+\frac{7}{8}+\frac{7}{8}+\frac{7}{8}=$ ________

c) $\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}=$ ________

d) $\frac{7}{12}+\frac{7}{12}+\frac{7}{12}+\frac{7}{12}+\frac{7}{12}+\frac{7}{12}=$ ________

2. Write the multiplication sentence represented by each number line.

a) 

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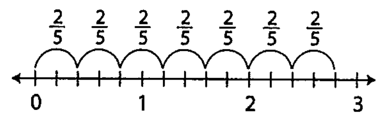
b) 

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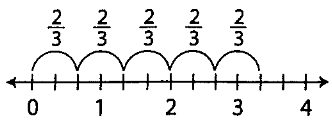
c) 

____

3. Multiply. Use a model to help.

a) $3 \times \frac{7}{12} =$ ________ b) $20 \times \frac{3}{4} =$ ________ c) $\frac{2}{3} \times 18 =$ ________

d) $\frac{4}{9} \times 10 =$ ________ e) $6 \times \frac{3}{4} =$ ________ f) $\frac{5}{8} \times 9 =$ ________

4. Match each multiplication to the correct product.

a) $2 \times \frac{4}{5}$	i) $7\frac{1}{2}$
b) $\frac{3}{8}$ of 13	ii) $4\frac{2}{3}$
c) $5 \times \frac{3}{4}$	iii) $1\frac{3}{5}$
d) $9 \times \frac{5}{6}$	iv) $3\frac{3}{4}$
e) $\frac{2}{3} \times 7$	v) $4\frac{7}{8}$

5. It takes $\frac{3}{4}$h to frame a picture. How long will it take to frame 13 pictures?

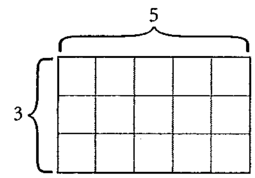
It will take ________ h to frame 13 pictures.

3.2 Using Models to Multiply Fractions

Quick Review

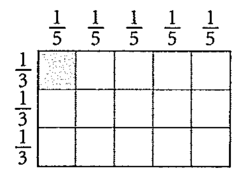
Area models are useful for visualizing multiplication.

➤ The area of a rectangle is length multiplied by width.
A 5 by 3 rectangle covers 15 unit squares.
So, $5 \times 3 = 15$.



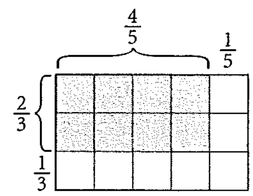
➤ To model $\frac{1}{5} \times \frac{1}{3}$, draw a 5 by 3 rectangle.
The rectangle has 15 equal parts.
A horizontal row of 5 squares represents $\frac{1}{3}$ of the rectangle.
$\frac{1}{5}$ of this row of $\frac{1}{3}$ covers 1 of the 15 parts.

So, $\frac{1}{5} \times \frac{1}{3} = \frac{1}{15}$.



➤ 2 horizontal rows of 5 squares represent $\frac{2}{3}$ of the rectangle.
$\frac{4}{5}$ of these 2 horizontal rows of 5 covers 8 of the 15 parts.

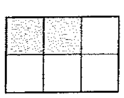
So, $\frac{4}{5} \times \frac{2}{3} = \frac{8}{15}$.



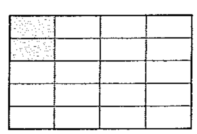
Practice

1. Write the multiplication sentence modelled by the shaded region in each rectangle.

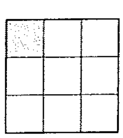
Tip
Write all fractions in simplest form.

a) 

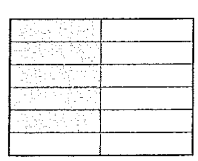
$\frac{2}{3} \times$ ________ = ________

b) 

$\frac{1}{4} \times$ ________ = ________

c) 

____

d) 

____

2. Draw an area model for each product. Then find the product. Write all fractions in simplest form.
Models may vary.

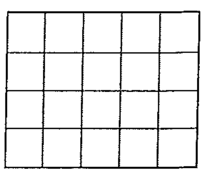
a) $\frac{1}{4} \times \frac{3}{4} =$ ____________ **b)** $\frac{1}{2} \times \frac{2}{3} =$ ____________

3. Tom took $\frac{3}{4}$ of a pie. He could only eat $\frac{1}{2}$ of what he took. What fraction of the pie did Tom eat?

HINT
$\frac{1}{2}$ of $\frac{3}{4}$ is $\frac{1}{2} \times \frac{3}{4}$.

____

4. Use the area model below to calculate each product.



$\frac{1}{5} \times \frac{1}{4} =$ ______ $\frac{2}{5} \times \frac{3}{4} =$ ______ $\frac{3}{5} \times \frac{1}{4} =$ ______ $\frac{1}{5} \times \frac{1}{2} =$ ______ $\frac{3}{5} \times \frac{1}{2} =$ ______

Look for a pattern in the numbers. Describe a relationship between the numerator and the denominator of each answer fraction and those of the fractions being multiplied.

____

____

____

____

5. Determine each product.

a) $\frac{3}{4} \times \frac{2}{5} =$ ________ **b)** $\frac{5}{8} \times \frac{2}{3} =$ ________ **c)** $\frac{4}{7} \times \frac{2}{3} =$ ________ **d)** $\frac{2}{3} \times \frac{7}{10} =$ ________

3.3 Multiplying Fractions

At Home At School

Quick Review

➤ To multiply fractions without using a model, multiply the numerators and multiply the denominators.

$\frac{3}{4} \times \frac{2}{5} = \frac{3 \times 2}{4 \times 5} = \frac{6}{20} = \frac{3}{10}$

➤ If the numerators and denominators have common factors, divide by the common factors before multiplying.

$$\frac{5}{12} \times \frac{8}{12} = \frac{5 \times 8}{12 \times 15}$$

$$= \frac{\cancel{5}^{1} \times \cancel{8}^{2}}{\cancel{12}^{3} \times \cancel{15}^{3}}$$

$$= \frac{1 \times 2}{3 \times 3}$$

$$= \frac{2}{9}$$

$5 \div 5 = 1$	$8 \div 4 = 2$
$12 \div 4 = 3$	$15 \div 5 = 3$

Practice

1. Multiply.

a) $\frac{3}{8} \times \frac{4}{15} = \frac{3 \times 4}{8 \times 15}$

_____ is the greatest common factor for 3 and 15.

_____ is the greatest common factor for 4 and 8.

$\frac{\overset{\square}{\cancel{3}} \times \overset{\square}{\cancel{4}}}{\overset{\square}{\cancel{8}} \times \overset{\square}{\cancel{15}}} =$ _________

$=$ _________

b) $\frac{3}{5} \times \frac{5}{6} =$ _________ c) $\frac{3}{4} \times \frac{7}{9} =$ _________ d) $\frac{8}{9} \times \frac{3}{10} =$ _________ d) $\frac{13}{9} \times \frac{3}{26} =$ _________

2. Simplify before multiplying. Express products as proper fractions.

a) $\frac{7}{3} \times \frac{9}{14} =$ _______ **b)** $\frac{15}{8} \times \frac{10}{9} =$ _______ **c)** $\frac{15}{4} \times \frac{2}{9} =$ _______ **d)** $\frac{12}{5} \times \frac{10}{9} =$ _______

3. Multiply.

a) $\frac{15}{8} \times \frac{3}{5} =$ _______ **b)** $\frac{6}{7} \times \frac{2}{3} =$ _______ **c)** $\frac{5}{6} \times \frac{3}{10} =$ _______ **d)** $\frac{7}{15} \times \frac{10}{21} =$ _______

4. Multiply. Estimate to check that each product is reasonable.

a) $\frac{44}{35} \times \frac{7}{33} =$ _______ **b)** $\frac{34}{33} \times \frac{22}{17} =$ _______ **c)** $\frac{57}{91} \times \frac{14}{19} =$ _______ **d)** $\frac{39}{64} \times \frac{24}{13} =$ _______

5. Match each multiplication to the correct product.

a) $\frac{5}{6} \times \frac{2}{7}$ **i)** $\frac{1}{4}$

b) $\frac{3}{2} \times \frac{1}{6}$ **ii)** $\frac{1}{6}$

c) $\frac{8}{9} \times \frac{9}{8}$ **iii)** $\frac{5}{21}$

d) $\frac{3}{4} \times \frac{2}{9}$ **iv)** 1

6. In the school band, $\frac{3}{5}$ of the students play the trumpet. Of these, $\frac{1}{6}$ also play the trombone. What fraction of the students in the band play both trumpet and trombone?

_____ of the students in the band play both trumpet and trombone.

7. Jeremy ate $\frac{1}{3}$ of an apple pie. Sara ate $\frac{1}{4}$ of the remainder. What fraction of the pie did Sara eat?

_____ of the pie was left after Jeremy had his share. So, Sara ate ____ × ____ = ____ of the pie.

8. Leona spent $\frac{5}{8}$ of $\frac{2}{3}$ of her allowance on magazines. What fraction of her total allowance did she spend on magazines? What fraction did she have left?

Leona spent ____ of her allowance on magazines. She had 1 − ____ = ____ of her allowance left.

3.4 Multiplying Mixed Numbers

Quick Review

➤ An area model is often useful for visualizing a multiplication.

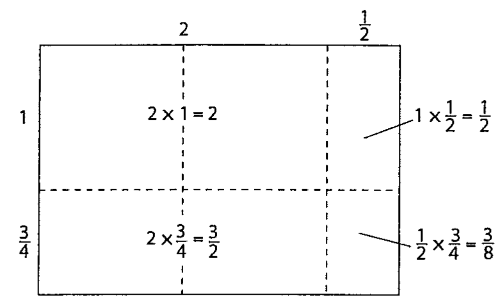
$$2\tfrac{1}{2} \times 1\tfrac{3}{4} = (2\times1)+\left(\tfrac{1}{2}\times 1\right)+\left(2\times\tfrac{3}{4}\right)+\left(\tfrac{1}{2}\times\tfrac{3}{4}\right)$$

$$= 2 + \tfrac{1}{2} + \tfrac{3}{2} + \tfrac{3}{8}$$

$$= \tfrac{16}{8} + \tfrac{4}{8} + \tfrac{12}{8} + \tfrac{3}{8}$$

$$= \tfrac{35}{8}$$

$$= 4\tfrac{3}{8}$$



➤ Another way to multiply mixed numbers is to first convert to improper fractions.

Multiply: $1\tfrac{1}{5} \times 3\tfrac{1}{8}$

$$1\tfrac{1}{5} \times 3\tfrac{1}{8} = \tfrac{6}{5} \times \tfrac{25}{8}$$

$$= \frac{\cancel{6}^{3} \times \cancel{25}^{5}}{\cancel{5}_{1} \times \cancel{8}_{4}}$$

$$= \tfrac{15}{4}$$

$$= 3\tfrac{3}{4}$$

$6 \div 2 = 3$	$25 \div 5 = 5$
$5 \div 5 = 1$	$8 \div 2 = 4$

Practice

1. Write each mixed number as an improper fraction.

a) $2\tfrac{3}{5} =$ ________ b) $4\tfrac{3}{4} =$ ________ c) $3\tfrac{1}{6} =$ ________ d) $1\tfrac{7}{12} =$ ________

2. Write each improper fraction as a mixed number.

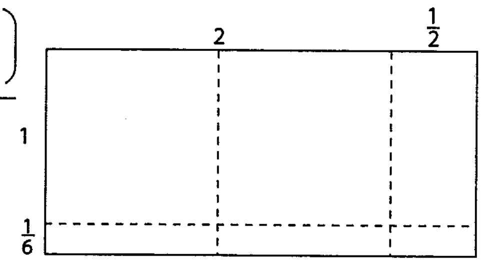
a) $\tfrac{43}{8} =$ ________ b) $\tfrac{19}{6} =$ ________ c) $\tfrac{17}{3} =$ ________ d) $\tfrac{27}{4} =$ ________

3. **a)** Show the product $1\frac{1}{6} \times 1\frac{1}{2}$ on the rectangle. State the product.

$1\frac{1}{6} \times 2\frac{1}{2} = (\_\_\_\_) + (\_\_\_\_) + (\_\_\_\_) + (\_\_\_\_)$

$= 2 + \_\_\_\_\_\_$

$= \_\_\_\_\_\_$



b) Draw an area model to show the product $2\frac{1}{4} \times 1\frac{1}{3}$. Determine the product.

$2\frac{1}{4} \times 1\frac{1}{3} = \_\_\_\_\_\_$

4. Multiply. Express answers as proper fractions.

a) $2\frac{5}{8} \times 1\frac{5}{7} = \_\_\_\_\_\_$

$= \_\_\_\_\_\_$

$= \_\_\_\_\_\_$

b) $2\frac{1}{10} \times 2\frac{2}{3} = \_\_\_\_\_\_$

c) $1\frac{1}{8} \times 3\frac{1}{3} = \_\_\_\_\_\_$

d) $2\frac{1}{4} \times 2\frac{2}{3} = \_\_\_\_\_\_$

e) $4\frac{2}{5} \times 2\frac{1}{7} = \_\_\_\_\_\_$

f) $4\frac{4}{5} \times 2\frac{1}{4} = \_\_\_\_\_\_$

5. George practises his guitar for $1\frac{1}{5}$ h per day on school days. On Saturdays, he increases his practice time to $2\frac{1}{2}$ times his normal time. How many hours does he practise on Saturdays?

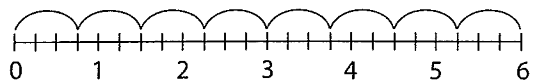
3.5 Dividing Whole Numbers and Fractions

At Home At School

Quick Review

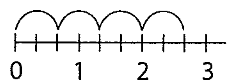
A number line can be used to help divide a whole number by a fraction.

➤ To determine how many three-quarters there are in 6, divide 6 into quarters.
Arrange 24 quarters into groups of three.
There are 8 groups of three-quarters.



$6 \div \frac{3}{4} = 8$

➤ To determine how many two-thirds there are in 3, divide 6 into thirds.
Arrange 18 thirds into groups of two-thirds.
There are 4 groups of two-thirds and one-third left over.



$\frac{1}{3}$ is $\frac{1}{2}$ of $\frac{2}{3}$.

So, $3 \div \frac{2}{3} = 4\frac{1}{2}$

To find $\frac{4}{5} \div 3$, think of sharing $\frac{4}{5}$ into 3 equal parts.

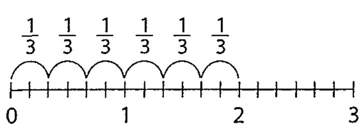
Each part is $\frac{1}{3}$ of $\frac{4}{5}$, or $\frac{1}{3} \times \frac{4}{5} = \frac{4}{15}$

So, $\frac{4}{5} \div 3 = \frac{4}{15}$

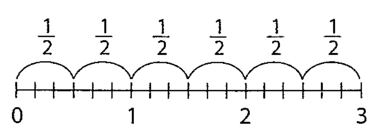
Practice

1. Use the number line to determine each quotient.

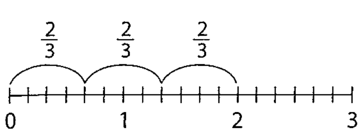
a) $2 \div \frac{1}{3} =$ _____



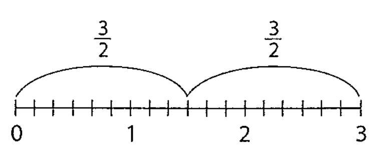
b) $3 \div \frac{1}{2} =$ _____



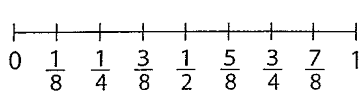
c) $2 \div \frac{2}{3} =$ _____



d) $3 \div \frac{3}{2} =$ _____



2. Use the number line to determine each quotient.



a) $\frac{1}{2} \div 4 =$ _____ **b)** $\frac{1}{4} \div 2 =$ _____ **c)** $\frac{3}{4} \div 2 =$ _____ **d)** $\frac{7}{8} \div 2 =$ _____

3. Use fraction circles, a number line, or a picture to determine each quotient.

a) $2 \div \frac{1}{7} =$ _____ b) $3 \div \frac{1}{3} =$ _____ c) $5 \div \frac{5}{6} =$ _____ d) $6 \div \frac{3}{5} =$ _____

4. Determine each quotient.

a) $2 \div \frac{3}{4} =$ _____ b) $3 \div \frac{2}{3} =$ _____ c) $2 \div \frac{3}{8} =$ _____ d) $2 \div \frac{3}{5} =$ _____

e) $\frac{3}{5} \div 2 =$ _____ f) $\frac{3}{4} \div 5 =$ _____ g) $\frac{5}{6} \div 2 =$ _____ h) $\frac{1}{2} \div 2 =$ _____

6. a) Two-thirds of a bag of candies is shared equally among 6 people. What fraction of the candies does each person receive?

Each person receives _____ of the bag of candies.

b) How many two-thirds cup servings are in 12 cups of fruit?

There are _____ two-thirds cup servings in 12 cups of fruit.

7. a) Write the digits 3, 4, and 12 in the boxes to obtain the greatest quotient. Is there more than one answer?

$\square \div \frac{\square}{\square} =$ _____

b) Write the digits 3, 4, and 12 in the boxes to obtain the least quotient. Is there more than one answer?

$\square \div \frac{\square}{\square} =$ _____

3.6 Dividing Fractions

At Home At School

Quick Review

There are at least two ways to divide fractions.

➤ Use common denominators

To divide: $\frac{3}{4} \div \frac{1}{6}$

Write the fractions with common denominator 12:

$\frac{3}{4} \div \frac{1}{6} = \frac{9}{12} \div \frac{2}{12}$

How many two-twelfths are in nine-twelfths?

$9 \div 2 = 4\frac{1}{2}$

So, $\frac{3}{4} \div \frac{1}{6} = 4\frac{1}{2}$

➤ You can also divide by a fraction by multiplying by the reciprocal.

To divide: $\frac{4}{5} \div \frac{2}{3}$

The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.

$$\frac{4}{5} \div \frac{2}{3} = \frac{4}{5} \times \frac{3}{2}$$
$$= \frac{6}{5}$$
$$= 1\frac{1}{5}$$

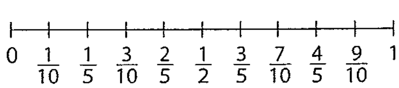
Practice

1. Write the reciprocal of each fraction.

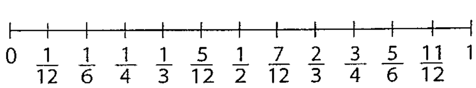
a) $\frac{4}{7}$ _____ b) $\frac{3}{8}$ _____ c) $\frac{11}{15}$ _____ d) $\frac{7}{8}$ _____

2. Use the number line to determine each quotient.

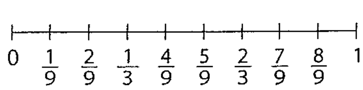
a) $\frac{9}{10} \div \frac{2}{5} =$ _____



b) $\frac{2}{3} \div \frac{1}{4} =$ _____



c) $\frac{2}{3} \div \frac{4}{9} =$ _____



3. Divide. Estimate to check that each quotient is reasonable.

a) $\frac{6}{7} \div \frac{3}{7}$ There are ______ three-sevenths in six-sevenths. So, $\frac{6}{7} \div \frac{3}{7} =$ ______

b) $\frac{8}{9} \div \frac{5}{9} =$ ______ **c)** $\frac{4}{5} \div \frac{3}{5} =$ ______ **d)** $\frac{7}{8} \div \frac{3}{8} =$ ______

4. Use common denominators to determine each quotient.

a) $\frac{7}{8} \div \frac{1}{4} = \frac{7}{8} \div \frac{\square}{8}$ There are ________-eighths in seven-eighths. So, $\frac{7}{8} \div \frac{1}{4} =$ ______

b) $\frac{4}{5} \div \frac{1}{10} =$ ______ **c)** $\frac{3}{4} \div \frac{2}{5} =$ ______ **d)** $\frac{6}{7} \div \frac{1}{3} =$ ______

5. Divide by multiplying by the reciprocal.

a) $\frac{9}{4} \div \frac{2}{3} = \frac{9}{4} \times$ ______ **b)** $\frac{7}{3} \div \frac{4}{5} =$ ______ **c)** $\frac{5}{2} \div \frac{3}{8} =$ ______ **d)** $\frac{3}{4} \div \frac{9}{2} =$ ______

= ______

= ______

6. Suppose you have $\frac{3}{4}$ of a cake. How many servings of each size can you make?

a) $\frac{1}{4}$ of the cake ______________ **b)** $\frac{1}{6}$ of the cake ______________

c) $\frac{1}{3}$ of the cake ______________ **d)** $\frac{1}{2}$ of the cake ______________

7. How many pieces of ribbon, each $\frac{1}{6}$ m long, can be cut from a ribbon $\frac{7}{8}$ m long?

______ pieces of ribbon can be cut. That is 5 whole pieces of ribbon with ______ of a piece,

or ______ m, left over.

3.7 Dividing Mixed Numbers

Quick Review

At Home At School

To divide mixed numbers without using a model, use either of the following methods.

Divide: $2\frac{5}{6} \div 1\frac{1}{4}$

- Write each mixed number as an improper fraction and then use common denominators.

$$2\frac{5}{6} \div 1\frac{1}{4} = \frac{17}{6} \div \frac{5}{4}$$
$$= \frac{34}{12} \div \frac{15}{12}$$
$$= 34 \div 15$$
$$= \frac{34}{15}\text{, or } 2\frac{4}{15}$$

- Use multiplication.

$$2\frac{5}{6} \div 1\frac{1}{4} = \frac{17}{6} \div \frac{5}{4}$$

Dividing by $\frac{5}{4}$ is the same as multiplying by $\frac{4}{5}$.

So, $\frac{17}{6} \div \frac{5}{4} = \frac{17}{6} \times \frac{4}{5}$

$$= \frac{17 \times \cancel{4}^{2}}{\cancel{6}_{3} \times 5}$$
$$= \frac{34}{15}\text{, or } 2\frac{4}{15}$$

Practice

1. Write each mixed number as an improper fraction.

a) $4\frac{5}{8} =$ ______ b) $3\frac{5}{7} =$ ______ c) $2\frac{5}{12} =$ ______ d) $3\frac{4}{9} =$ ______

2. Write each pair of mixed numbers as improper fractions with the same denominator.

a) $2\frac{1}{3}, 4\frac{1}{6}$

Write the fraction part of each mixed number with the same denominator, ____:

$2\frac{1}{3} = 2$____ $4\frac{1}{6} = 4$____

Write each mixed number as an improper fraction: ______________________

b) $3\frac{3}{5}, 2\frac{1}{10}$ ______________ c) $4\frac{1}{5}, 2\frac{1}{2}$ ______________ d) $2\frac{1}{4}, 1\frac{2}{3}$ ______________

3.9 Order of Operations with Fractions

Quick Review

At Home At School

➤ The order of operations for fractions is the same as for whole numbers.
Do the operations in brackets first.
Then divide and multiply, in order, from left to right.
Then add and subtract, in order, from left to right.

$\frac{3}{14} \div \left(\frac{5}{8} - \frac{1}{4}\right) + \frac{2}{7} = \frac{3}{14} \div \left(\frac{5}{8} - \frac{2}{8}\right) + \frac{2}{7}$ — Write the fractions in the brackets with common denominators.

$= \frac{3}{14} \div \frac{3}{8} + \frac{2}{7}$ — Do the operation in the brackets first.

$= \frac{3}{14} \times \frac{8}{3} + \frac{2}{7}$ — Divide by multiplying by the reciprocal.

$= \frac{\cancel{3}^{1} \times \cancel{8}^{4}}{\cancel{14}_{7} \times \cancel{3}_{1}} + \frac{2}{7}$

$= \frac{4}{7} + \frac{2}{7}$

$= \frac{6}{7}$ — Add.

Practice

1. Which operation would you do first?

a) $\frac{7}{8} \div \left(\frac{3}{4} + \frac{3}{8}\right)$ ________________

b) $\frac{7}{9} - \frac{5}{9} \times \frac{1}{4}$ ________________

c) $\left(\frac{9}{16} - \frac{3}{4}\right) \times \frac{5}{8}$ ________________

d) $\frac{3}{4} \times \left(\frac{3}{4} - \frac{1}{4} \div \frac{1}{2}\right)$ ________________

2. Elise was asked to evaluate $1\frac{1}{3} \div \frac{3}{4} \times \frac{2}{3}$. Her work is shown below. Is her answer correct? Explain.

$1\frac{1}{3} \div \frac{3}{4} \times \frac{2}{3} = \frac{4}{3} \div \frac{1}{2}$

$= \frac{4}{3} \times \frac{2}{1}$

$= \frac{8}{3}$

$= 2\frac{2}{3}$

Her answer *is/is not* correct.

____

3. Evaluate. Show all steps.

a) $\left(\frac{1}{2}+\frac{2}{3}\right)\times\frac{1}{7}=$

b) $\left(1-\frac{1}{4}\right)\div\left(1+\frac{3}{4}\right)=$

c) $\frac{1}{3}\div\left(\frac{5}{6}\times\frac{1}{4}\right)=$

d) $\frac{4}{7}\times\frac{3}{5}-\frac{1}{5}=$

4. Evaluate.

a) $\frac{7}{9}\times\frac{3}{5}-\frac{1}{6}\div\frac{5}{2}=$ __________

b) $\frac{1}{8}+\frac{3}{4}\div\frac{5}{8}-\frac{4}{5}=$ __________

c) $\frac{6}{7}\div\frac{3}{22}\times\frac{7}{11}\div\frac{8}{9}=$ __________

d) $\frac{11}{12}+\frac{5}{6}\times\frac{3}{4}-\frac{5}{6}=$ __________

5. Evaluate.

a) $3\frac{1}{3}\div4\frac{1}{6}\times2\frac{1}{4}=$ _____

b) $\frac{4}{5}\times\frac{5}{8}\div\frac{5}{8}\times\frac{3}{4}=$ _____

c) $\frac{5}{12}\div\frac{3}{8}\div\frac{3}{4}\times\frac{9}{10}=$ _____

d) $3\frac{1}{2}\div5\frac{1}{3}\times1\frac{1}{3}\div1\frac{1}{6}=$ _____

HINT

Convert mixed numbers to improper fractions first.

6. Evaluate.

a) $\left(\frac{5}{9}+\frac{2}{3}\right)\div\left(\frac{3}{4}+\frac{5}{8}\right)=$ _____

b) $\frac{9}{16}-\left(\frac{3}{4}-\frac{2}{3}\right)\times\frac{3}{4}=$ _____

c) $1\frac{3}{5}\times\left(\frac{5}{8}+\frac{3}{4}-\frac{5}{6}\right)=$ _____

d) $\left(\frac{9}{16}\div\frac{5}{12}\right)-\left(\frac{2}{5}\times\frac{7}{8}\right)=$ _____

e) $2\frac{2}{3}\times1\frac{1}{8}+\left(2\frac{3}{4}+1\frac{3}{8}\right)=$ _____

f) $\left(4\frac{5}{8}-2\frac{3}{4}\right)\div\left(2\frac{1}{3}+1\frac{1}{6}\right)=$ _____

In Your Words

Here are some of the important mathematical words of this unit.
Build your own glossary by recording definitions and examples here. The first one is done for you.

proper and improper fractions

proper fractions have numerator less than denominator; improper fractions have numerator greater than denominator

simplest form of a fraction

reciprocal of a fraction

mixed number

quotient

order of operations

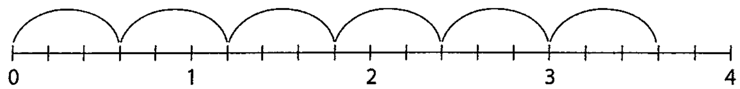
List other mathematical words you need to know.

Unit Review

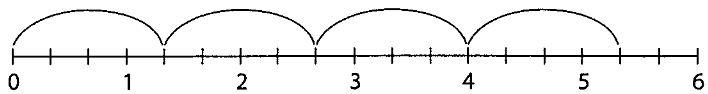
LESSON

3.1 **1.** Write the multiplication sentence represented by each number line.

a) ______________________

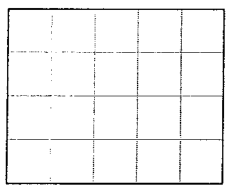


b) ______________________

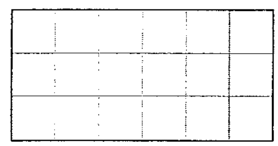


3.2 **2.** Shade each rectangle to show the product.

a) $\frac{3}{4} \times \frac{2}{5}$



b) $\frac{1}{3} \times \frac{5}{6}$



3.3 **3.** Multiply. Estimate to check that the solutions are reasonable.

a) $\frac{3}{4} \times \frac{8}{9} =$ ____________ b) $\frac{5}{16} \times \frac{4}{15} =$ ____________ c) $\frac{7}{6} \times \frac{8}{21} =$ ____________

4. Claude mowed $\frac{1}{4}$ of the lawn before lunch. After lunch he mowed $\frac{2}{3}$ of the uncut lawn. What fraction of the lawn did Claude mow altogether?

Before he started mowing after lunch, Claude had ________ of the lawn left to mow.

Claude mowed ________ of the lawn altogether.

3.4 **5.** Write each mixed number as an improper fraction.

a) $3\frac{3}{5} =$ ____________ b) $4\frac{7}{8} =$ ____________ c) $1\frac{11}{16} =$ ____________

LESSON

6. Multiply.

a) $3\frac{3}{8} \times 3\frac{1}{3} =$ ________ b) $2\frac{2}{5} \times 6\frac{2}{3} =$ ________ c) $1\frac{5}{12} \times 2\frac{5}{8} =$ ________

3.5 **7.** Use a model to determine each quotient.

a) $4 \div \frac{2}{3} =$ ________ b) $5 \div \frac{3}{4} =$ ________ c) $\frac{3}{5} \div \frac{3}{4} =$ ________

3.6 **8.** Divide.

a) $\frac{5}{12} \div \frac{10}{11} =$ ________ b) $\frac{3}{7} \div \frac{9}{14} =$ ________ c) $\frac{3}{5} \div \frac{5}{6} =$ ________

3.7 **9.** Divide. Estimate to check that the quotients are reasonable.

a) $2\frac{1}{4} \div 1\frac{7}{8} =$ ________ b) $1\frac{3}{4} \div 2\frac{4}{5} =$ ________ c) $3\frac{3}{4} \div 2\frac{1}{12} =$ ________

10. A recipe for chocolate cake calls for $1\frac{1}{4}$ cups of chocolate chips. Hasim has $7\frac{1}{2}$ cups of chocolate chips. How many cakes can he make?

Hasim can make ________ cakes.

LESSON

3.8 **11.** On Tuesday, $\frac{5}{12}$ of the grade 8 students attended the computer club meeting and $\frac{3}{8}$ of the grade 8 students attended the science club meeting. The meetings were at the same time. What fraction of the grade 8 students attended one of the meetings? What fraction did not attend either of the meetings?

________ of the grade 8 students attended one of the meetings.

________ of the grade 8 students did not attend either of the meetings.

12. Grace has $6\frac{3}{4}$ L of maple syrup that she wants to pour into $\frac{3}{4}$-L containers. How many containers can she fill?

Grace can fill ________ containers.

3.9 **13.** Evaluate.

a) $\frac{3}{5} + \frac{7}{15} \times \frac{9}{14} =$ ________ **b)** $\left(\frac{3}{5} + \frac{7}{15} \times \frac{9}{14}\right) =$ ________

14. Evaluate: $\frac{4}{7} \times \left(\frac{9}{5} - \frac{3}{4}\right) \div \frac{3}{8} =$ ________

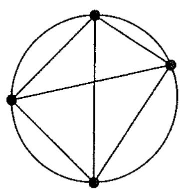
UNIT 4 Measuring Prisms and Cylinders

Just for Fun

Handshakes

People are standing in a circle.
Each person shakes hands with every other person in the circle.

Draw a circle.
Then draw dots to represent the people.
Join any 2 dots to represent a handshake.



Record your results in the table.

Write a pattern for the number of handshakes.

Number of People	Number of Handshakes
1	0
2	1
3	3
4	
5	
6	
7	

____

____

Word Search

1. Find the list of words in the word search table on the right. Words can be horizontal, vertical, or diagonal.

   ANGLE, AREA, BASE, BOX, CAPACITY, CUBE, DECAGON, FOUR, HEXAGON, METRE, NETS, ONE, PRISM, PYRAMID, RECTANGLE, SQUARE, TWO

2. Write all unused letters in order, row by row, from left to right. Separate the letters to form a phrase.

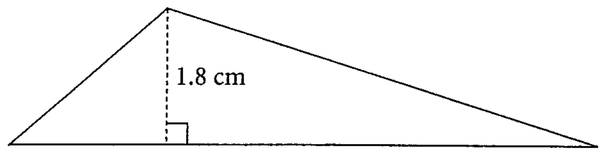
   __ __ __ __ __ __ __ __ __ __ __ __

C	U	B	E	E	M	S	A	T
R	U	O	F	L	H	T	D	I
N	S	X	G	G	G	E	I	H
O	N	E	R	N	S	N	M	E
G	S	Q	U	A	R	E	A	X
A	E	A	B	T	A	E	R	A
C	A	P	A	C	I	T	Y	G
E	E	R	T	E	M	W	P	O
D	M	S	I	R	P	O	T	N

Activating Prior Knowledge

Area of Two-Dimensional Shapes

To calculate the area of this triangle,
use the formula Area = $\frac{1}{2}$ × base × height
or $A = \frac{1}{2}bh$.



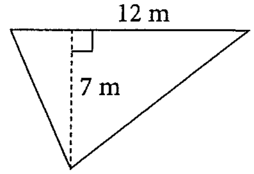
Substitute $b = 7.8$ and $h = 1.8$.

$A = \frac{1}{2}bh = \frac{1}{2}(7.8 \times 1.8) = 7.02$

The area is about 7 cm², to the nearest square centimetre.

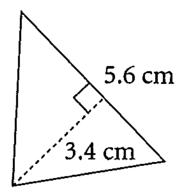
Check

1. Calculate the area of each triangle.



a) $A = \frac{bh}{2} = \frac{\square}{2} =$ ________

The area is ________ m².

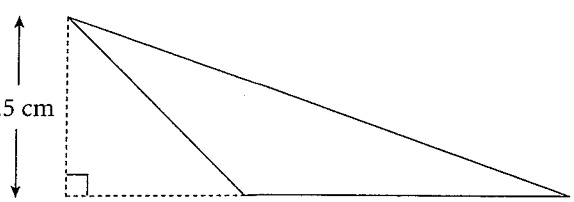


b) $A = \frac{bh}{2}$

The area is ________.

2. Calculate the area of each triangle.

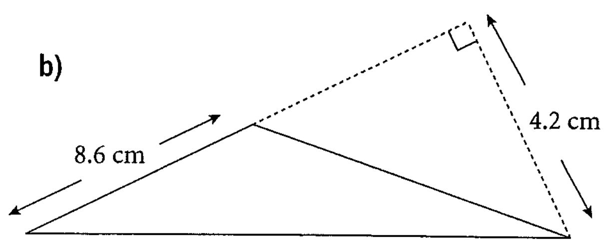
a)



$b =$ ________ $h =$ ________

$A =$ ________

b)



$b =$ ________ $h =$ ________

$A =$ ________

To calculate the area of a circle with diameter 14 cm, use the formula Area = π × radius² or $A = \pi r^2$. The diameter of the circle is 14 cm, so the radius is 7 cm.

Substitute $r = 7$ cm.

$A = \pi r^2 = \pi \times 7^2 \doteq 153.938$

The area is about 154 cm², to the nearest square centimetre.

Tip
For π, use the π key on a calculator.

Check

3. Calculate the area of each circle, to the nearest square unit.

a) diameter = 24 cm

$r = \frac{d}{2} = \frac{\square}{2} =$ ______

$A = \pi r^2 \doteq$ ________ The area of the circle is ______, to the nearest square __________.

b) radius = 9 m

$A = \pi r^2 \doteq$ ________ The area of the circle is ______, to the nearest square __________.

c) diameter = 11 mm The area of the circle is ______, to the nearest square __________.

d) radius = 8 km The area of the circle is ______, to the nearest square __________.

Circumference of a Circle

To calculate the circumference of a circle with diameter 4.8 cm, use the formula Circumference = pi × diameter, or $C = \pi d$.

Substitute $d = 4.8$.

$C = \pi \times d = \pi \times 4.8 \doteq 15.080$

The circumference of the circle is about 15.1 cm, to one decimal place.

To calculate the circumference of a circle with radius 5.2 cm, use the formula Circumference = 2 × pi × radius or $C = 2\pi r$.

Substitute $r = 5.2$.

$C = 2 \times \pi \times r = 2 \times \pi \times 5.2 \doteq 32.673$

The circumference of the circle is about 32.7 cm, to one decimal place.

Check

4. Calculate the circumference of each circle, to one decimal place.

a) $d = 12$ cm $C = \pi \times d = \pi \times$ __________ $\doteq$ __________

The circumference of the circle is _________, to one decimal place.

b) $r = 8$ m $C = 2 \times \pi \times r = 2 \times \pi \times$ __________ $\doteq$ __________

The circumference of the circle is _________, to one decimal place.

c) $d = 5.6$ mm The circumference of the circle is __________, to one decimal place.

d) $r = 3.8$ m The circumference of the circle is __________, to one decimal place.

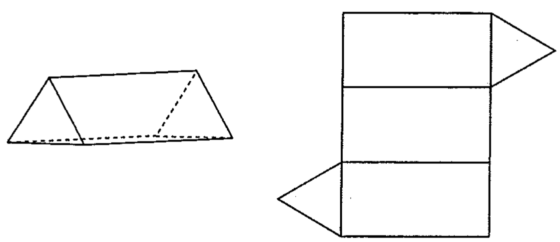
4.1 Exploring Nets

Quick Review

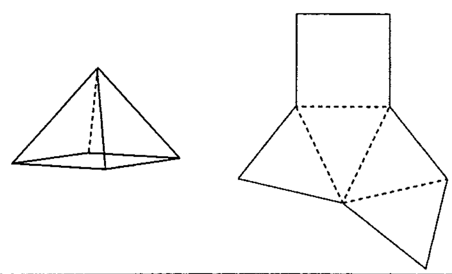
At Home At School

- ➤ A prism has two congruent bases and is named for its bases.
  A pyramid has one base and the other faces are congruent triangles.
- ➤ A net is a diagram that can be folded to make an object.

The diagram shows a triangular prism and its net.

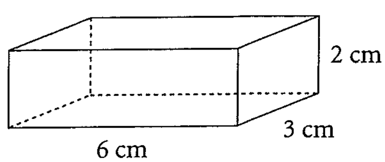


The diagram shows a square pyramid and its net.

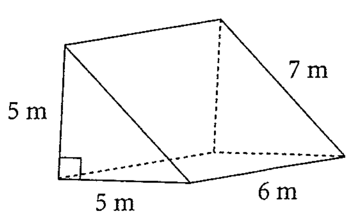


Practice

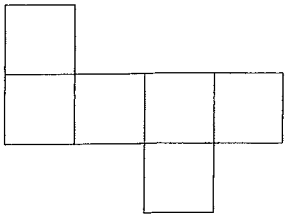
1. Sketch a net for the right rectangular prism. Identify and name each face.

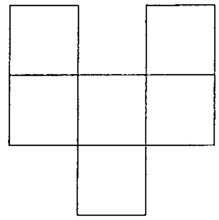


2. Sketch a net for the right triangular prism.
Identify and name each face.



3. Which of the following diagrams is **not** the net of a cube?

A


B


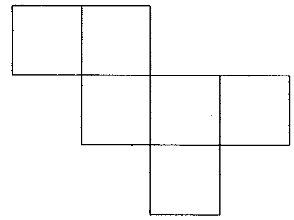
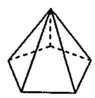
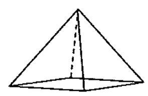
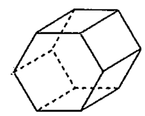
C


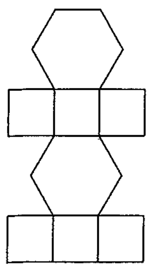
Diagram _____ is not the net of a cube.

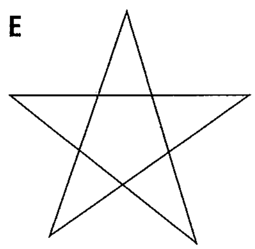
4. a) Match each object to its net.

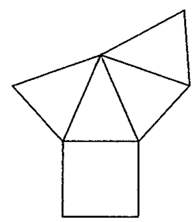
A


B


C


D


E


F


b) Identify and name each face of each object.

____

____

____

____

____

5. Use the descriptions to identify the object that has each set of faces.

a) six congruent triangles and one hexagon ________________

b) four congruent equilateral triangles ________________

c) two congruent squares and four congruent rectangles ________________

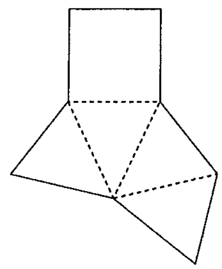
d) two congruent triangles and three rectangles ________________

4.2 Creating Objects from Nets

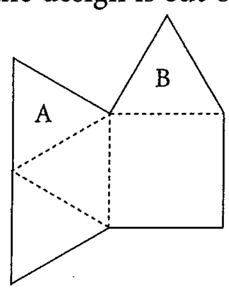
Quick Review

➤ To determine if a diagram is a net for an object, look at each shape and at how the shapes are arranged.

This is the net of a square pyramid.



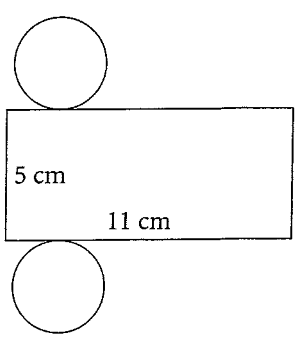
This is not the net of a square pyramid. If the design is cut out and folded, triangles A and B will coincide.



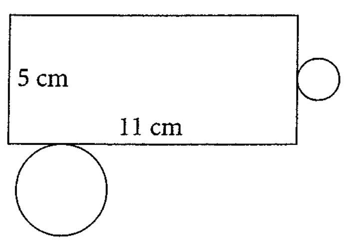
Practice

1. Which of the following diagrams is not the net of a right cylinder?

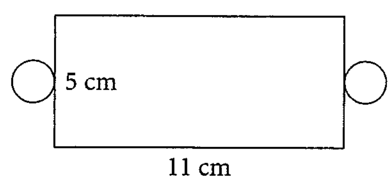
A



B



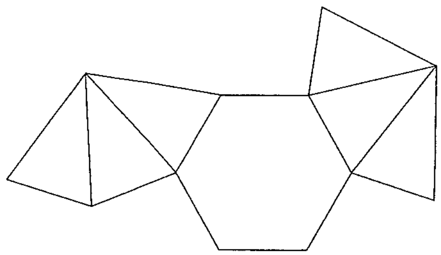
C



The figure in part ____ is not the net of a right cylinder.

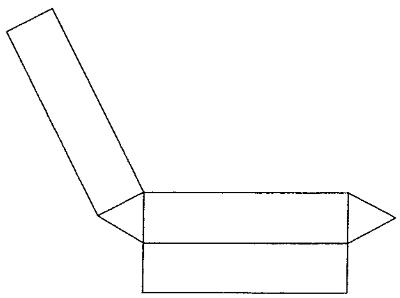
2. Is each diagram the net of an object?
If your answer is yes, name and describe the object.
If your answer is no, what changes could you make so it could be a net?

a)



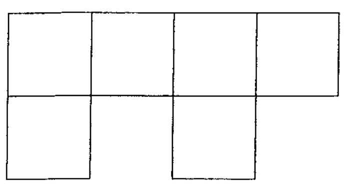
The diagram ________ the net of an object. ______________________________

b)



The diagram ________ the net of an object. ______________________________

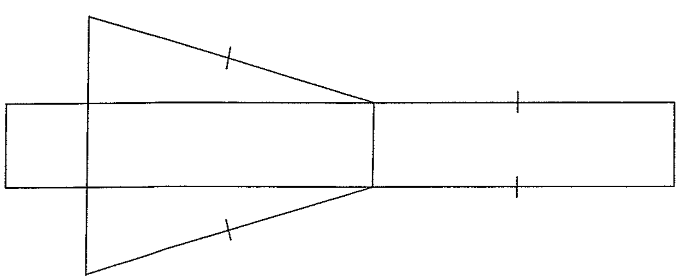
c)



The diagram ________ the net of an object. ______________________________

____

3. Name and describe the object that can be made from the net.

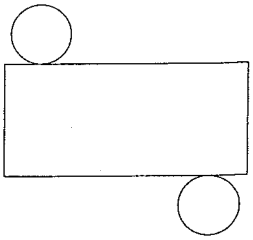


The object is a ____

____

4. Identify the object that each net folds to form.

a)



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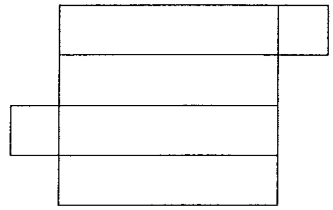
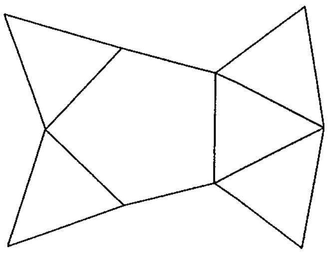
b)



____

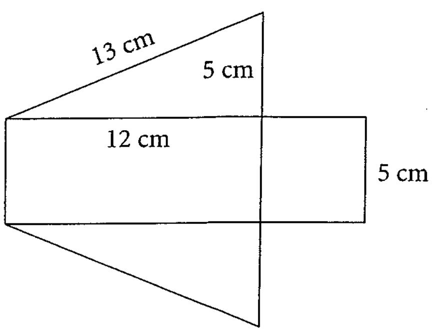
c)



____

5. Describe the changes that have to be made to each diagram to make it a net. Name the object that can be made from the new net.

a)



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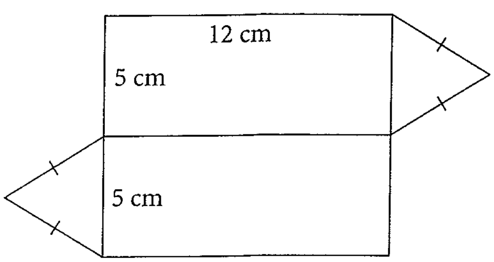
____

b)



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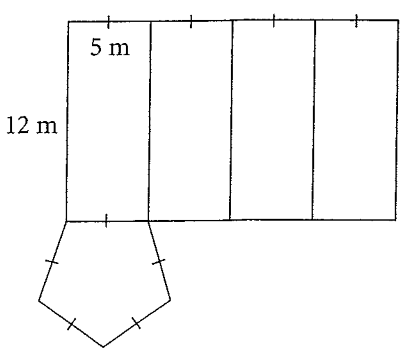
____

c)



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