



kzn education

Department:
Education
KWAZULU-NATAL

Mathematics
Saturday Workshops
Grade 9
Term 2

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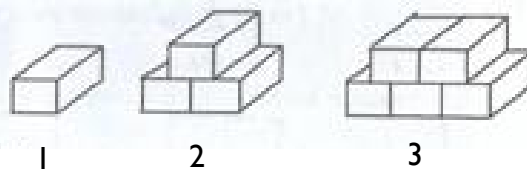
Factors and Products

Algebraic Reasoning

Worksheet A

Carefully study the patterns made below using bricks. Use them to answer questions

1.



1.1 Describe in words how the pattern continues

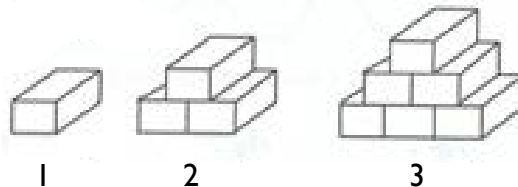
1.2 How many bricks are needed for:

- (a) Pattern 4
- (b) Pattern 5
- (c) Pattern 10
- (d) Pattern 50

1.3 Which pattern number can be built from:

- (a) 55 bricks
- (b) 101 bricks
- (c) 200 bricks

2.



2.1 Describe in words how the pattern continues

2.2 How many bricks are needed for:

- (a) Pattern 4
- (b) Pattern 5
- (c) Pattern 10
- (d) Pattern 50

2.3 Which pattern number can be built from:

- (a) 55 bricks
- (b) 101 bricks
- (c) 200 bricks

Worksheet B

I. Study the following pattern

Row 1				1			
Row 2			2	3	4		
Row 3		5	6	7	8	9	
Row 4	10	11	12	13	14	15	16

I.1 How many numbers are in row 50?

I.2 What is the first number in row 21?

2. Each of the following shapes is made from matchsticks. In each case:

2.1 Draw the next two pictures

2.2 Describe in words how many matchsticks are needed to move from one shape to the next

2.3 Complete the table in the next page.

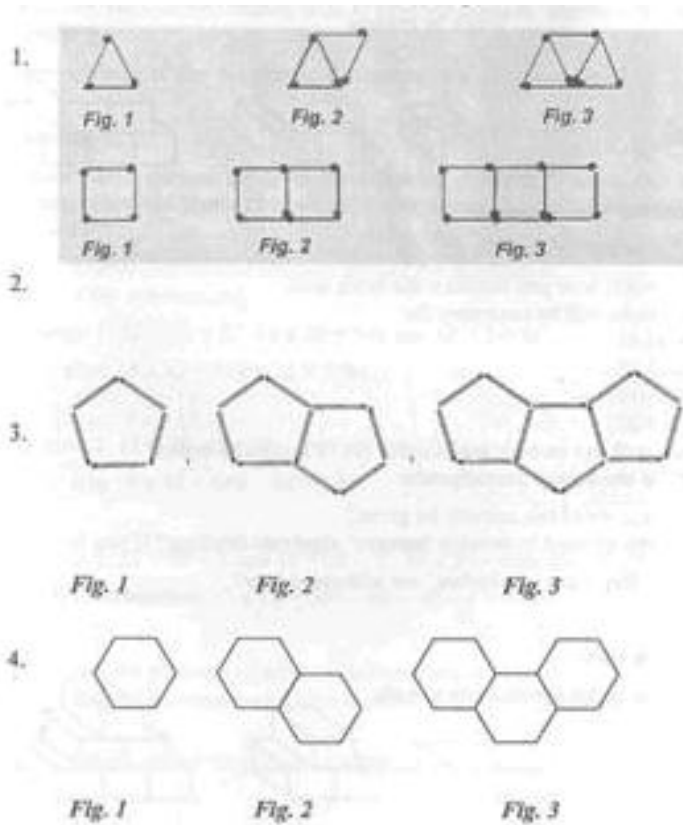


		Fig. 1	Fig. 2	Fig. 3	Fig. 4	Fig. 8	Fig. 100	Rule
Case 1	Number of matchsticks							
Case 2	Number of matchsticks							
Case 3	Number of matchsticks							
Case 4	Number of matchsticks							

Worksheet C

Complete

a.

Position in sequence	2	4	6	8	10	n
Term	-10	-8	-6	-4		

b.

Position in sequence	1	3	5	7	10	n
Term	-14	-12	-10	-8		

c.

Position in sequence	3	6	9	10	12	n
Term	-15	-12	-9		-6	

d.

Position in sequence	18	12	10	6	n	
Term	-4	-8		-10		

e.

Position in sequence	1	2	3	4	10	n
Term	-10	-12	-14	-16		

f.

Position in sequence	1	2	3	4	10	n
Term	-13	-12	-11	-10		

g.

Position in sequence	1	2	3	4	10	n
Term	-11	-10	-9	-8		

h.

Position in sequence	1	2	3	4	10	n
Term	-1	-2	-3	-4		

i.

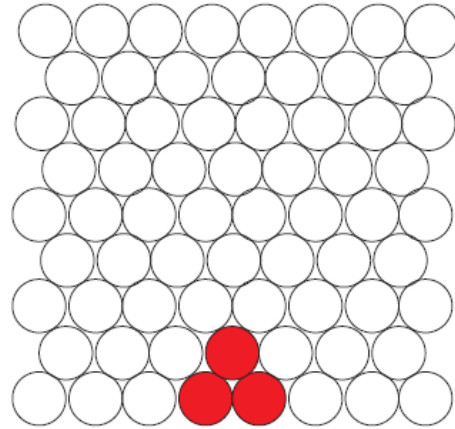
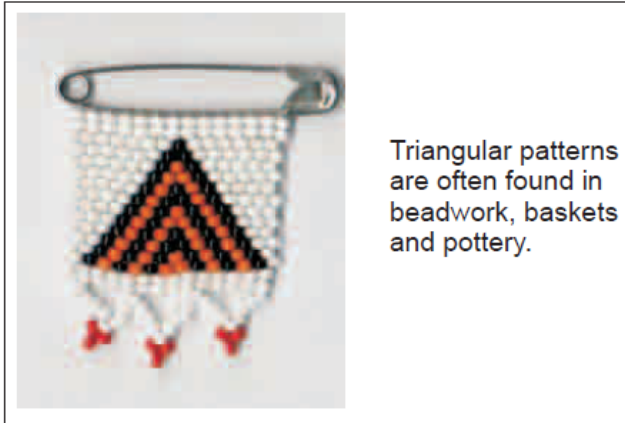
Position in sequence	1	2	3	4	10	n
Term	-8	-16	-24			

j.

Position in sequence	1	2	3	4	10	n
Term	-17	-16	-15	-14		

For consolidation, do workbook activities (Grade 9: Term 1, pp. 20 – 23 and Term 2, pp. 2 – 11)

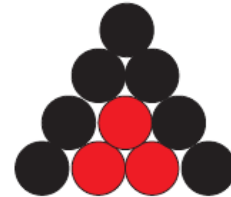
WORKSHEET1



- 1) Study the beaded love letter above. Use two different colours to copy the pattern onto the circles on the right.
- 2) How many red beads are there in Pattern 1? 3) How many black beads are there in Pattern 2?



Pattern 1



Pattern 2

- 4) Use your drawing to work out the number of red beads there would be in Pattern 3. Write the total on the table below.
- 5) How many black beads would there be in Pattern 4? Fill in the total on the table.
- 6) Use your table to work out how many beads there would be in Pattern 5 and Pattern 6. Check if you are right by drawing each pattern.

Pattern Number	Number of new beads
1	3
2	7
3	
4	
5	
6	

- 7) The learners in a Grade 9 class were asked to write down what they noticed about the patterns, and this is what they said:

A: MAKHOSI:

This is what I noticed about the pattern:

Pattern 1: Top bead + 1 bead on the left + 1 bead on the right

Pattern 2: Top bead + 3 beads on the left + 3 beads on the right

Pattern 3: Top bead + 5 beads on the left + 5 beads on the right

Pattern 4: Top bead + 7 beads on the left + 7 beads on the right

B: MARY:

This is what I noticed about the pattern:

Pattern 1: 2 beads on the left + 1 bead on the right

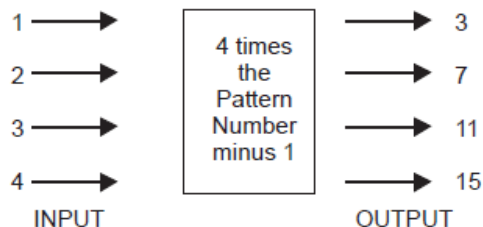
Pattern 2: 4 beads on the left + 3 beads on the right

Pattern 3: 6 beads on the left + 5 beads on the right

Pattern 4: 8 beads on the left + 7 beads on the right

C: TSHIDI

I worked out that the number of beads in each shape was equal to 4 times the pattern number minus 1. I showed this using a **flow diagram**:



D: KHWEZI

I used algebra and found the **general formula**. I used N for the *Number of Beads* and P for the *Pattern Number*. I could then say: $N=4P-1$

Work with a group of 4. Each member of your group should use one of the above methods to work out the number of beads in:

- | | |
|--------------|----------------|
| a) Pattern 7 | b) Pattern 8 |
| c) Pattern 9 | d) Pattern n |

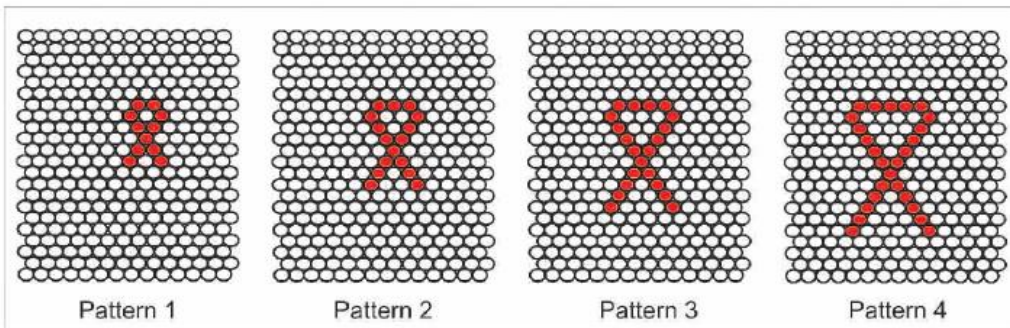
- 8) Compare your methods. Which method is quickest to use? Give a reason for your answer.

WORKSHEET 2

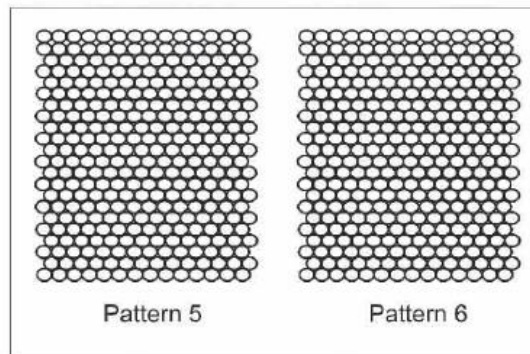
The AIDS ribbon can be made from beads in many different ways.



Different sized AIDS ribbons have been drawn below



1) Colour in the circles below to make Pattern 5 and Pattern 6.



2) Count the number of beads needed to make Patterns 1, 2, 3, 4, 5 and 6 and write the totals on the table:

- 3) Use your table to work out the number of beads needed to make Pattern 7 and Pattern 8. Check if you are right by drawing the patterns.

Pattern Number	Number of beads
1	
2	
3	
4	
5	
6	
7	
8	

- 4) Andile looked at the drawings of the different sized AIDS ribbons and discovered the following:

Pattern 1: Number of beads = $5 \times 2 + 1 = 11$
Pattern 2: Number of beads = $5 \times 3 + 1 = 16$
Pattern 3: Number of beads = $5 \times 4 + 1 = 21$

- a) Investigate whether Andile's formula holds for Patterns 4, 5, 6, 7, and 8

- b) Use Andile's formula to find the number of beads in

i) Pattern 9

ii) Pattern 10

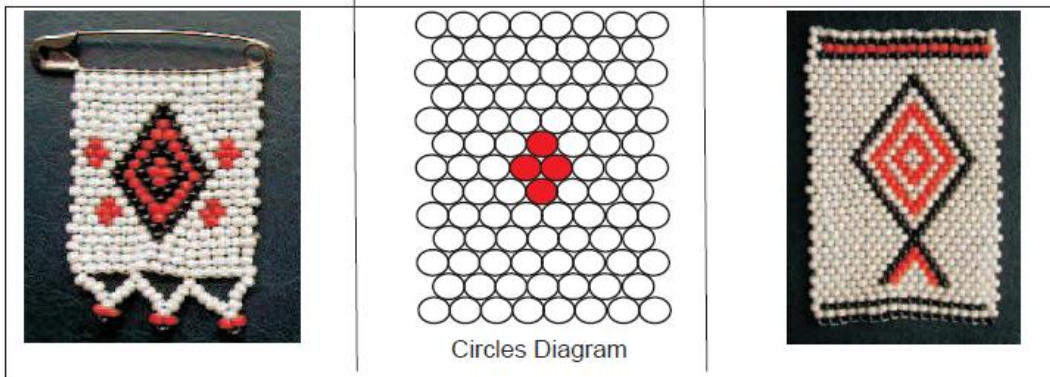
iii) Pattern 20

iv) Pattern x

- c) Write a general formula for finding the number of beads as an equation using N for *Number of beads* and P for the *Pattern Number*.

WORKSHEET 3

Some bead patterns use the rhombus (or diamond shape) as a basis for the design. Notice that all four sides of a rhombus are equal in length, and that opposite sides are parallel to one another



- 1) Use two colours to complete the pattern on the Circles Diagram.
- 2) Count the number of beads added on each time and fill in the answers on the table.

Pattern Number	Number of New Beads
1	4
2	12
3	
4	
5	
6	

- 3) The learners in a Grade 9 class were asked to look at the Circles Diagram and to write down any patterns they noticed. Mpume wrote

Pattern 1:	Number of beads = top bead + bottom bead + 1 bead to the left + 1 bead to the right = 1 + 1 + 1 + 1 = 4
Pattern 2:	Number of beads = top bead + bottom bead + 5 beads to the left + 5 beads to the right = 1 + 1 + 5 + 5 = 12
Pattern 3:	Number of beads = top bead + bottom bead + 9 beads to the left + 9 beads to the right = 1 + 1 + 9 + 9 = 20
Pattern 4:	Number of beads = top bead + bottom bead + 13 beads to the left + 13 beads to the right = 1 + 1 + 13 + 13 = 28

Use Mpume's pattern to work out the number of beads in

- | | |
|--------------|--------------|
| a) Pattern 5 | b) Pattern 6 |
| c) Pattern 7 | d) Pattern 8 |

WORKSHEET 4

Nomsa is making bead designs and builds patterns like this:



- 1) Use beads (or counters, stones, mealies, coins or small pieces of paper if you don't have beads) to make the same five patterns as Nomsa. Count the number of beads that you use for Pattern 1, Pattern 2, Pattern 3, Pattern 4 and Pattern 5, and fill in the number of beads you used on the table.
- 2) Study the table. Use your answers to work out the number of beads needed to make Patterns 6, 7, 8, 9 and 10. Fill in the numbers on the table.

Pattern Number	1	2	3	4	5	6	7	8	9	10
Number of beads	1	4								

- 3) Write down a formula that can be used to work out the Number of beads (N) in any Pattern Number (P)

- 4) Draw a flow diagram to show your general formula from question 3. Use 1, 2, 3 and 4 as input values. Calculate the output values and fill them in.

- 5) Use your formula to work out the number of beads needed for:
 - a) Pattern 50.....
 - b) Pattern 100.....
 - c) Pattern 2 500.....
 - d) Pattern n

ANSWERS TO WORKSHEET 1

- 1) Learners colour in the circles using two different colours
- 2) Pattern 1: 3 beads
- 3) Pattern 2: 7 beads
- 4) Pattern 3: 11 beads
- 5) Pattern 4: 15 beads

6)

Pattern Number	Number of New Beads
1	3
2	7
3	11
4	15
5	19
6	23

NOTE TO THE EDUCATOR:

$$7 = 3 + 4$$

$$11 = 7 + 4$$

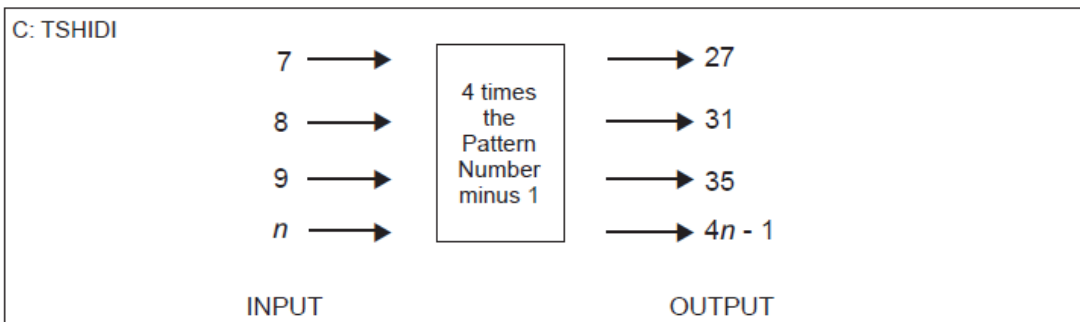
$$15 = 11 + 4$$

In other words, each time 4 is added on to get the new number.

7) The solutions for each method is as follows

A: MAKHOSI	
Pattern 1:	Top bead + 1 bead on the left + 1 bead on the right = $1 + (2 \times 1 - 1) + (2 \times 1 - 1) = 1 + 1 + 1 = 3$
Pattern 2:	Top bead + 3 beads on the left + 3 beads on the right = $1 + (2 \times 2 - 1) + (2 \times 2 - 1) = 1 + 3 + 3 = 7$
Pattern 3:	Top bead + 5 beads on the left + 5 beads on the right = $1 + (2 \times 3 - 1) + (2 \times 3 - 1) = 1 + 5 + 5 = 11$
Pattern 4:	Top bead + 7 beads on the left + 7 beads on the right = $1 + (2 \times 4 - 1) + (2 \times 4 - 1) = 1 + 7 + 7 = 15$
Pattern 5:	Top bead + 9 beads on the left + 9 beads on the right = $1 + (2 \times 5 - 1) + (2 \times 5 - 1) = 1 + 9 + 9 = 19$
Pattern 6:	Top bead + 11 beads on the left + 11 beads on the right = $1 + (2 \times 6 - 1) + (2 \times 6 - 1) = 1 + 11 + 11 = 23$
a) Pattern 7:	Top bead + 13 beads on the left + 13 beads on the right = $1 + (2 \times 7 - 1) + (2 \times 7 - 1) = 1 + 13 + 13 = 27$
b) Pattern 8:	Top bead + 15 beads on the left + 15 beads on the right = $1 + (2 \times 8 - 1) + (2 \times 8 - 1) = 1 + 15 + 15 = 31$
c) Pattern 9:	Top bead + 17 beads on the left + 17 beads on the right = $1 + (2 \times 9 - 1) + (2 \times 9 - 1) = 1 + 17 + 17 = 35$
d) Pattern n:	$1 + (2 \times n - 1) + (2 \times n - 1) = 1 + 2n - 1 + 2n - 1 = 4n - 1$

B: MARY	
Pattern 1:	2 beads on the left + 1 bead on the right = $(2 \times 1) + (2 \times 1 - 1) = 2 + 1 = 3$
Pattern 2:	4 beads on the left + 3 beads on the right = $(2 \times 2) + (2 \times 2 - 1) = 4 + 3 = 7$
Pattern 3:	6 beads on the left + 5 beads on the right = $(2 \times 3) + (2 \times 3 - 1) = 6 + 5 = 11$
Pattern 4:	8 beads on the left + 7 beads on the right = $(2 \times 4) + (2 \times 4 - 1) = 8 + 7 = 15$
Pattern 5:	10 beads on the left + 9 beads on the right = $(2 \times 5) + (2 \times 5 - 1) = 10 + 9 = 19$
Pattern 6:	12 beads on the left + 11 beads on the right = $(2 \times 6) + (2 \times 6 - 1) = 12 + 11 = 23$
a) Pattern 7:	14 beads on the left + 13 beads on the right = $(2 \times 7) + (2 \times 7 - 1) = 14 + 13 = 27$
b) Pattern 8:	16 beads on the left + 15 beads on the right = $(2 \times 8) + (2 \times 8 - 1) = 16 + 15 = 31$
c) Pattern 9:	18 beads on the left + 17 beads on the right = $(2 \times 9) + (2 \times 9 - 1) = 18 + 17 = 35$
d) Pattern n:	$2n + 2n - 1 = 4n - 1$



D: KHWEZI

Pattern 7: $N = 4P - 1 = 4 \times 7 - 1 = 28 - 1 = 27$

Pattern 8: $N = 4P - 1 = 4 \times 8 - 1 = 32 - 1 = 31$

Pattern 9: $N = 4P - 1 = 4 \times 9 - 1 = 36 - 1 = 35$

Pattern n : $N = 4P - 1 = 4n - 1$

- 8) Accept any answer as long as the reason is given for the answer. Generally, it is quicker to find the answer using a formula in other words using Khwezi's method.

ANSWERS TO WORKSHEET 2

- 1) Learners colour in the circles.
- 2) See the table below.
- 3)

Pattern number	Number of beads
1	11
2	16
3	21
4	26
5	31
6	36
7	41
8	46

NOTE TO THE EDUCATOR:

$$16 = 11 + 5$$

$$21 = 16 + 5$$

$$26 = 21 + 5$$

In other words, each time 5 is added on to get the new number.

4)

a)

Pattern 1: Number of beads = $5 \times 2 + 1 = 11$

Pattern 2: Number of beads = $5 \times 3 + 1 = 16$

Pattern 3: Number of beads = $5 \times 4 + 1 = 21$

Pattern 4: Number of beads = $5 \times 5 + 1 = 26$

Pattern 5: Number of beads = $5 \times 6 + 1 = 31$

Pattern 6: Number of beads = $5 \times 7 + 1 = 36$

Pattern 7: Number of beads = $5 \times 8 + 1 = 41$

Pattern 8: Number of beads = $5 \times 9 + 1 = 46$

b)

i) Pattern 9: Number of beads = $5 \times 10 + 1 = 51$

ii) Pattern 10: Number of beads = $5 \times 11 + 1 = 56$

iii) Pattern 20: Number of beads = $5 \times 21 + 1 = 105 + 1 = 106$

iv) Pattern x : Number of beads = $5 \times (x + 1) + 1 = 5x + 5 + 1 = 5x + 6$

c)

$$N = 5x(P+1) + 1$$

$$N = 5P + 5 + 1$$

$$N = 5P + 6$$

ANSWERS TO WORKSHEET 3

- 1) The learners use two colours to complete the pattern on the Circles Diagram.
- 2)

Pattern number	Number of new beads
1	4
2	12
3	20
4	28
5	36
6	44

3) Mpume's pattern:

Pattern 1:	No of beads = top bead + bottom bead + 1 bead to the left + 1 bead to the right = $1 + 1 + 1 + 1 = 4$
Pattern 2:	No of beads = top bead + bottom bead + 5 beads to the left + 5 beads to the right = $1 + 1 + 5 + 5 = 12$
Pattern 3:	No of beads = top bead + bottom bead + 9 beads to the left + 9 beads to the right = $1 + 1 + 9 + 9 = 20$
Pattern 4:	No of beads = top bead + bottom bead + 13 beads to the left + 13 beads to the right = $1 + 1 + 13 + 13 = 28$
Pattern 5:	No of beads = top bead + bottom bead + 17 beads to the left + 17 beads to the right = $1 + 1 + 17 + 17 = 36$
Pattern 6:	No of beads = top bead + bottom bead + 21 beads to the left + 21 beads to the right = $1 + 1 + 21 + 21 = 44$
Pattern 7:	No of beads = top bead + bottom bead + 25 beads to the left + 25 beads to the right = $1 + 1 + 25 + 25 = 52$
Pattern 8:	No of beads = top bead + bottom bead + 29 beads to the left + 29 beads to the right = $1 + 1 + 29 + 29 = 60$

4) Reuben's pattern:

Pattern 1:	Number of beads = 1 bead at each corner + 0 beads on each of the four sides = $4 + (4 \times 0) = 4$
Pattern 2:	Number of beads = 1 bead at each corner + 2 beads on each of the four sides = $4 + (4 \times 2) = 12$
Pattern 3:	Number of beads = 1 bead at each corner + 4 beads on each of the four sides = $4 + (4 \times 4) = 20$
Pattern 4:	Number of beads = 1 bead at each corner + 6 beads on each of the four sides = $4 + (4 \times 6) = 28$
Pattern 5:	Number of beads = 1 bead at each corner + 8 beads on each of the four sides = $4 + (4 \times 8) = 36$
Pattern 6:	Number of beads = 1 bead at each corner + 10 beads on each of the four sides = $4 + (4 \times 10) = 44$
Pattern 7:	Number of beads = 1 bead at each corner + 12 beads on each of the four sides = $4 + (4 \times 12) = 52$
Pattern 8:	Number of beads = 1 bead at each corner + 14 beads on each of the four sides = $4 + (4 \times 14) = 60$

5) Learners compare their answers for each Pattern in 2), 3) and 4). If they don't get the same number of beads for each Pattern, they should investigate where they went wrong.

6) Using Mpume's Pattern:

$$N = 1 + 1 + (4P - 3) + (4P - 3)$$

$$N = 2 + 4P - 3 + 4P - 3$$

$$N = 8P - 4$$

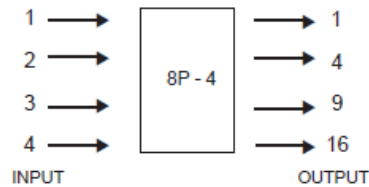
Using Reuben's Pattern

$$N = 4 + [4 \times (2P - 2)]$$

$$N = 4 + 8P - 8$$

$$N = 8P - 4$$

7)



ANSWERS TO WORKSHEET 4

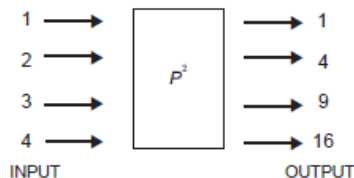
1) The learners make the five patterns.

2)

Pattern number	1	2	3	4	5	6	7	8	9	10
Number of beads	1	4	9	16	25	36	49	64	81	100

3) $N = P \times P = P^2$

4)



5)

a) Pattern 50: Number of beads = $50^2 = 2\,500$

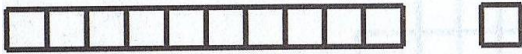
b) Pattern 100: Number of beads = $100^2 = 10\,000$

c) Pattern 2 500: Number of beads = $2\,500^2 = 6\,250\,000$

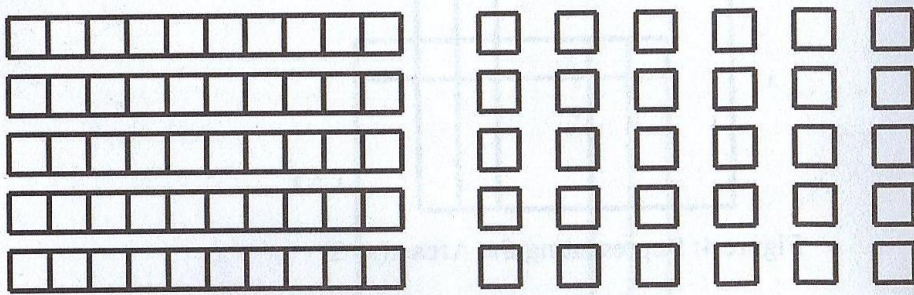
d) Pattern n : Number of beads = n^2

Products and Factors

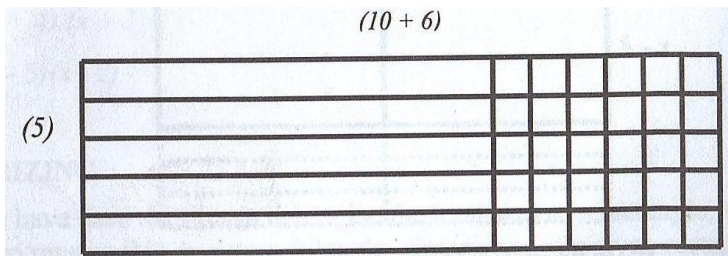
The following base 10 blocks may be used to determine products.



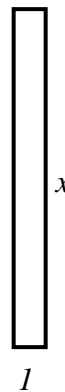
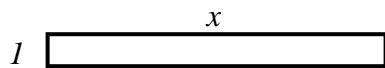
5×16 may be presented as follows:



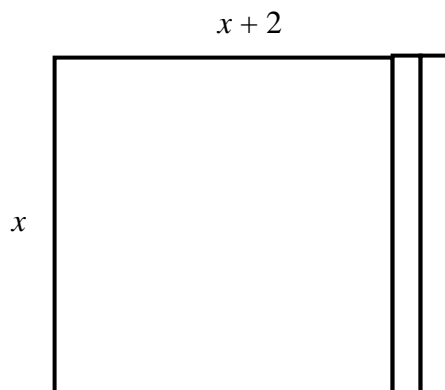
This shows 5×16 as $5 \times 10 + 5 \times 6$. Presented as area, this would look as follows:



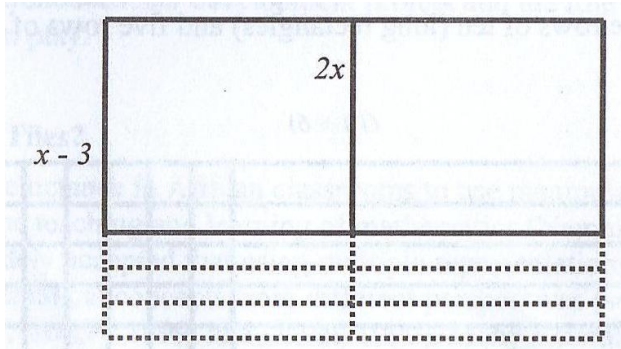
If we agree that the area of the long block is x , then the block could be presented as follows:



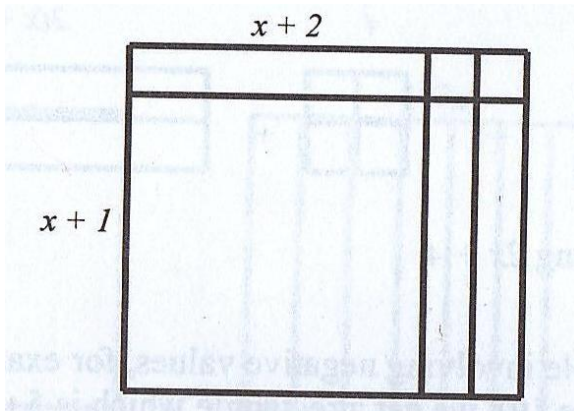
The product $x(x + 2)$ may be presented as follows



The product $2x(x-3)$ may be presented as follows:



The product $(x+2)(x+1)$ may be presented as follows:



Calculate:

1. $4(x+3)$

2. $5(x-4)$

3. $2x(3x+5)$

4. $5x(2x-3)$

5. $3x(2x+5)$

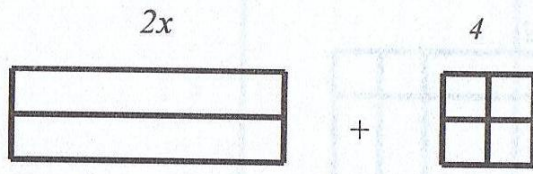
6. $(x+2)(x-2)$

7. $(x+3)(x-4)$

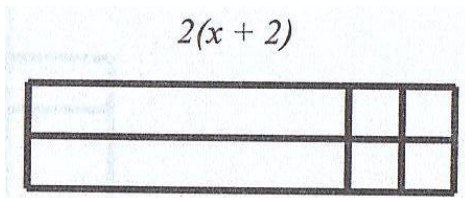
8. $(2x+4)(x+3)$

9. $(3x-5)(x-2)$

Factorising becomes more easier. $2x+4$ could be presented as follows:



Combining the diagram gives factors.



Factorise:

1. $3x + 12$

2. $2x - 6$

3. $x^2 + 4x$

4. $x^2 - 3x$

5. $3x^2 + 12$

6. $x^2 - 4$

For consolidation do activities in Grade 9 workbook: Term 3/4, pp. 16 - 27