Name: Notes Key
Date: _____

(GREEN TEXT) CHAPTER 4 NOTES – Polynomials

	Date:
4.1 – Polynomials	_____
4.2/4.3 – Adding and Subtracting Polynomials	_____
4.5 – Multiplying Monomials by Monomials	_____
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Review:	_____
Test:	_____

What You'll Learn:

4.1 – Using correct vocabulary to describe polynomials
4.2/4.3 – Performing addition and subtraction of polynomials using like terms
4.5/5.3 – Performing multiplication on monomials and polynomials using exponent laws
1.9 – Performing operations on polynomials involving exponents
4.7/5.4 – Performing division on monomials and polynomials using exponent laws

What is the difference between an expression and an equation?

An expression is a mathematical phrase that does not have an 'equals' sign.
An equation is a mathematical phrase that does have an 'equals' sign

What is a term?

A number, a variable, or a product of numbers and variables.
Terms are separated by add or subtract.

4.1 - Polynomials

Focus: Classifying and ordering polynomials.

Main Ideas:

Warmup:
How many terms does each expression have?
a) $y + 7$ b) $x^2 - 2x + 3$
c) -1 d) $2t - 9 + t^3 - t^2$
e) $2x^2y^5z^7$

(a) 2 (b) 3 (c) 1 (d) 4 (e) 1

What is a monomial?
What is a binomial?
What is a trinomial?
What is a polynomial?

Monomial: a one term expression ex. $2x$
Binomial: a two term expression ex. $2x+3$
Trinomial: a three term expression ex. x^2+2x+3
Polynomial: an expression with two or more terms.

ex1 – Classify each expression:
a) $x - 2y$
b) $-17x^2y^2z^2$
c) $4x$
d) $2x^2 - 5 + 16xy$
e) $x + y - z + 5$

(a) - binomial
- polynomial

(b) - monomial

(c) - monomial

(d) - trinomial
- polynomial

(e) - polynomial

What is the 'degree' of a monomial?

The degree of a monomial is the sum of the exponents of its variables.

ex2 – State the degree
a) $2x$
b) $-3x^2y^3z$
c) -7

(a) $2x$ or $2x^1$ degree: 1

(b) $-3x^2y^3z^1$ degree: $2+3+1=6$

(c) -7 no variables so degree = 0

What is the degree of a polynomial?

The degree of a polynomial is the highest exponent of the variable in any one term.

ex. $4x^2 - 3x + 5$
$\quad\;\; 2 \quad 1 \quad 0$
So degree is 2.

for 'x'

ex3 – State the degree of each polynomial:

a) $x^2 + 3x + 7$

b) $3y - 2y^3 + 2y^2$

c) $4x^6 + 2x^3y^4$

d) $25x^3y + 36x^2y^2$

(a) $x^2 + 3x + 7$ (2, 1, 0)

degree : 2

(c) $4x^6 + 2x^3y^4$ (6, 3)

degree : 6

(b) $3y - 2y^3 + 2y^2$ (1, 3, 2)

degree : 3

(d) $25x^3y + 36x^2y^2$ (3, 2)

degree : 3

How do you order a polynomial?

Arrange terms from highest degree to lowest degree.

Ex4 – Arrange in descending powers of x

a) $3x - x^2 + 2x^3 - 6$

b) $7 - 5x + 3x^4 - x^2$

c) $2x^2y^5 - 3x + 2x^3y^2$

(a) $3x - x^2 + 2x^3 - 6$ (1st, 2nd, 3rd, 4th)

$2x^3 - x^2 + 3x - 6$

(b) $7 - 5x + 3x^4 - x^2$ (4th, 3rd, 1st, 2nd)

$3x^4 - x^2 - 5x + 7$

(c) $2x^2y^5 - 3x + 2x^3y^2$ (2nd, 3rd, 1st)

$2x^3y^2 + 2x^2y^5 - 3x$

* make sure you bring the sign to the left of the term with the term when rearranging

Reflection: When rearranging terms, what do you have to make sure of? Explain using the example: $7 - 3x - 2x^2$

4.2/4.3 – Adding and Subtracting Polynomials

Focus: To add and subtract polynomials using like terms.

Main Ideas:

Warmup:
Simplify:
$x^2 - 2x + 5x - 3x^2$
What are like terms?
How do you add/subtract like terms?

$1x^2 - 2x + 5x - 3x^2$

$-2x^2 + 3x$

like terms are terms with the same variable(s) raised to the same exponent(s).

Add or subtract like terms by adding or subtracting the coefficient and keeping the variable(s) and exponent(s) the same.

ex1 - Add
$(2x^2 + 3x + 2) + (x^2 + 2x + 3)$

$2x^2 + 3x + 2 + 1x^2 + 2x + 3$

What steps are involved in adding polynomials?

1. Identify like terms
2. Add like terms (or subtract)

$3x^2 + 5x + 5$

ex2 – Simplify
a) $(3y^2 - 8y + 3) + (2y^2 + 8y - 9)$

(a) $3y^2 - 8y + 3 + 2y^2 + 8y - 9$

$5y^2 - 6$

b) $(5x^3 + 7x - 9) + (-8x + 11 + 4x^3)$

(b) $5x^3 + 7x - 9 - 8x^2 + 11 + 4x^3$

$9x^3 - 8x^2 + 7x - 9$

Subtracting Polynomials

ex3 - Simplify
$(4x^2 - 5x + 7) - (3x^2 + 2x - 5)$

$(4x^2 - 5x + 7) - 1(3x^2 + 2x - 5)$

$4x^2 - 5x + 7 - 3x^2 - 2x + 5$

$x^2 - 7x + 12$

What is the extra step needed when subtracting polynomials?

the subtraction sign must be distributed to every term in the second set of brackets.

ex4 - Simplify

a) $(3y^2 - y + 5) - (4y^2 - y + 1)$

(a) $(3y^2 - y + 5) - 1(4y^2 - y + 1)$

$3y^2 - 1y + 5 - 4y^2 + 1y - 1$

$-y^2 + 4$

b) $(-x^2y + 3xy) - (4xy^2 + xy - 2x^2y)$

(b) $(-1x^2y + 3xy) - 1(4xy^2 + 1xy - 2x^2y)$

$-1x^2y + 3xy - 4xy^2 - 1xy + 2x^2y$

$x^2y + 2xy - 4xy^2$

Reflection: What is different about adding polynomials compared to subtracting them?

4.5 – Multiplying Monomials by Monomials

Focus: How to multiply two monomials so only one monomial results.

Main Ideas:

Warmup:
Simplify in power form:
a) $3^2 \times 3^5$
b) $(-7)^4(-7)$
c) $y^6 \times y^3$

(a) $3^2 \times 3^5 = 3^{2+5} = 3^7$ (b) $(-7)^4(-7)^1 = (-7)^5$

(c) $y^6 \times y^3 = y^9$

What is the shortcut for multiplying powers with the same base?

Add exponents

ex1 - Multiply
$(6x^2)(-3x)$

$(6x^2)(-3x^1)$
$= -18x^3$

What are the steps to multiplying monomials?

① Multiply coefficients
② Add exponents of like variables.

ex2 - Multiply
a) $(3a)(4b^3)$
b) $-x^2(2xy)$
c) $-2y^3(4y^2)$

(a) $(3a)(4b^3)$ $= 12ab^3$

(b) $-1x^2(2x^1y^1)$ $= -2x^3y$

(c) $-2y^3(4y^2)$ $= -8y^5$

ex3 – Simplify
a) $(2x^2y)(-3xy)$
b) $(x^3)(-3xy^3)(4y)$
c) $(-2a^3bc^2)(-5ac^4y^2)$

(a) $(2x^2y^1)(-3x^1y^1)$ $= -6x^3y^2$

(b) $(1x^3)(-3x^1y^3)(4y^1)$ $-12x^4y^4$

(c) $(-2a^3b^1c^2)(-5a^1c^4y^2)$
$= 10a^4bc^6y^2$

Reflection: What are the steps to multiplying monomials?

5.3 – Multiplying a Polynomial by a Monomial

Focus: How to multiply a monomial to a polynomial.

Main Ideas:

Warmup:
Use distributive property
To expand the following:
x(x – 3)

$x^1(x^1-3)$

$= x^2-3x$

ex2 – Expand
a) $2x(x-6)$
b) $-y(y^2-7)$
c) $m(m+2)-3m(m-5)$

(a) $2x(x-6)$

$2x^2-12x$

(b) $-1y(y^2-7)$

$-1y^3+7y$

$= -y^3+7y$

(c) $m(m+2)-3m(m-5)$

$m^2+2m-3m^2+15$

$-2m^2+2m+15$

ex3 - Simplify
a) $4x(x^2-x+7)$
b) $y(y+3)-y(y-2)$
c) $3(x^2+2x-5)-x(x+1)$

(a) $4x(x^2-x+7)$

$4x^3-4x^2+28x$

(b) $y(y+3)-y(y-2)$

y^2+3y-y^2+2y

$= 5y$

(c) $3(x^2+2x-5)-1x(x+1)$

$3x^2+6x-15-x^2-x$

$2x^2+5x-15$

Reflection: When distributing into brackets, what exponent rule do you often use? Then, when adding and subtracting in the last step, explain how to collect like terms?

1.9 – Powers of Monomials

Focus: How to work with monomials that have an exponent.

Main Ideas:

Warmup:
Simplify in power form:
a) $(3^4)^5$
b) $(9^2)^3$
c) $(x^6)^2$

(a) $(3^4)^5 = 3^{4\times5} = 3^{20}$ (b) $(9^2)^3 = 9^6$

(c) $(x^6)^2 = x^{12}$

What is the shortcut when you have an exponent raised to an exponent?

multiply exponents

ex1 - Simplify
a) $(x^2)^3$
b) $(y)^4$

(a) $(x^2)^3 = x^{2\times3} = x^6$

(b) $(y^1)^4 = y^{1\times4} = y^4$

ex2 – Simplify
$(2x^2y^3)^2$

$(2x^2y^3)^2 = (2x^2y^3)(2x^2y^3) = 2\times2\times x^2\times x^2\times y^3\times y^3$
$= 4x^4y^6$

shortcut: $(2x^2y^3)^2 = 4x^4y^6$

ex3 - Simplify
a) $(-x^2y)^2$
b) $(-2a^2bc^3)^3$

(a) $(-1x^2y^1)^2 = 1x^4y^2 = x^4y^2$

(b) $(-2a^2b^1c^3)^3 = (-2)^3a^6b^3c^9 = -8a^6b^3c^9$

ex4 - Simplify
$-(m^2n^5)^0$

$-1(m^2n^5)^0 = -1(m^0n^0) = -1(1\times1) = -1$

ex5 - Simplify
a) $(2x^2y^3)^4(-xy^2)^3$
b) $(ab^3c)^2(2a^2bc^2)^3$

(a) $(2x^2y^3)^4(-1x^1y^2)^3$

$= (2^4x^8y^{12})(-1x^3y^6)$

$= 16x^8y^{12}(-1x^3y^6)$

$= -16x^{11}y^{18}$

(b) $(a^1b^3c^1)^2(2a^2b^1c^2)^3$

$= a^2b^6c^2(2^3a^6b^3c^6)$

$= a^2b^6c^2(8a^6b^3c^6)$

$= 8a^8b^9c^8$

ex6 - Simplify

$\left(\frac{2x}{3y^2}\right)^2$

$\left(\frac{2x}{3y^2}\right)^2 = \left(\frac{2x}{3y^2}\right)\left(\frac{2x}{3y^2}\right) = \frac{4x^2}{9y^4}$

OR ... shortcut...

$\left(\frac{2x^1}{3y^2}\right)^2 = \frac{2^2x^2}{3^2y^4} = \frac{4x^2}{9y^4}$

Reflection: What is the most difficult part for you and what can you do to improve upon it?

4.7 – Dividing Monomials by Monomials

Focus: How to divide monomials so only one monomial results.

Main Ideas:

Warmup:
Simplify in power form:
a) $4^5 \div 4^3$

b) $\frac{2^7}{2^6}$

c) $y^3 \div y$

What is the shortcut for dividing powers with the same base?

(a) $4^5 \div 4^3 = 4^{5-3} = 4^2$ (b) $\frac{2^7}{2^6} = 2^{7-6} = 2^1 = 2$

(c) $y^3 \div y^1 = y^{3-1} = y^2$

subtract exponents.

ex1 - Divide

$$\frac{8x^2}{2x}$$

$$\frac{8x^2}{2x^1} = 4x$$ ($8 \div 2$, $x^2 \div x^1$)

What are the steps for dividing monomials?

1. divide coefficients
2. divide like variables by subtracting exponents

ex2 - Simplify
a) $16y \div 4y$
b) $-12abc \div -2bc$
c) $18x^5y^2 \div -6x^2y$

(a) $16y \div 4y = 4$ (b) $-12abc \div -2bc = 6a$

(c) $18x^5y^2 \div -6x^2y^1 = -3x^3y$

ex3 – Divide
a) $\frac{21x^5yz^2}{-7xyz}$

b) $\frac{10m^8n^5}{5m^3n^2}$

(a) $\frac{21x^5y^1z^2}{-7x^1y^1z^1} = -3x^4z$ (b) $\frac{10m^8n^5}{5m^3n^2} = 2m^5n^3$

ex4 – Simplify

a) $\dfrac{2x^2y}{6x^{-2}y^4}$

b) $\dfrac{8a^2b^4c^3}{a^4bc^4}$

(a) $\dfrac{2x^2y^1}{6x^{-2}y^4}$ $\quad \dfrac{2}{6} = \dfrac{1}{3}$ $\quad \dfrac{x^2}{x^{-2}} = x^4$ $\quad \dfrac{y^1}{y^4} = y^{-3}$

$2-(-2) = 2+2 = 4$ $\quad 1-4 = -3$

$= \dfrac{1}{3}x^4y^{-3}$ OR $\dfrac{x^4y^{-3}}{3}$ $\quad$ Remember $y^{-3} = \dfrac{1}{y^3}$

SO ... $\dfrac{x^4}{3y^3}$

(b) $\dfrac{8a^2b^4c^3}{1a^4b^1c^4} = 8a^{-2}b^3c^{-1} = \dfrac{8b^3}{a^2c}$

Reflection: Describe the steps for multiplying monomials by monomials.

5.4 – Dividing Polynomials by Monomials

Focus: How to divide a polynomial by a monomial

Main Ideas:

Warmup:

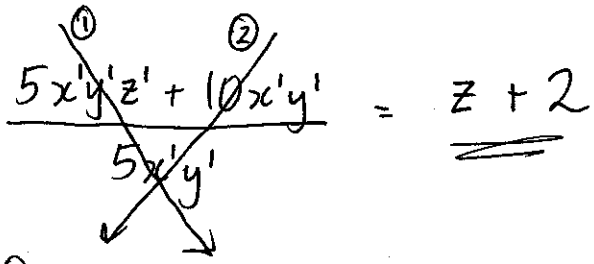
Divide $\frac{12x^3yz^2}{-3xz^2}$

$\frac{12x^3y^1z^2}{-3x^1z^2} = -4x^2y$

What steps did you use to solve the above division?

① Multiply coefficients
② Divide like variables by subtracting exponents.

ex1 - Divide $\frac{5xyz+10xy}{5xy}$



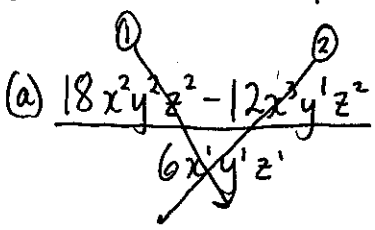
$= z + 2$

What are the steps involved?

① Divide every term on top by the term on the bottom
② Use the same 2 steps as above.

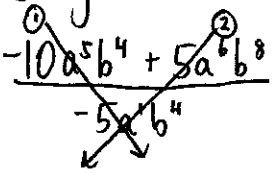
ex2 - Simplify

a) $\frac{18x^2y^2z^2 - 12x^3yz^2}{6xyz}$



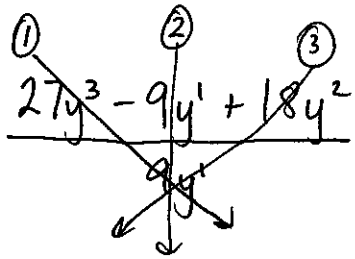
$= 3xyz - 2x^2z$

b) $\frac{-10a^5b^4 + 5a^6b^8}{-5ab^4}$



$= 2a^4 - a^5b^4$

c) $\frac{27y^3 - 9y + 18y^2}{9y}$



$= 3y^2 - 1 + 2y$

Reflection: What are the steps to dividing a polynomial by a monomial? What results when a top term and bottom term perfectly cancel?