Name: Notes Key
Date:

(GREEN TEXT) CHAPTER 4 NOTES – Polynomials

	Date:
4.1 – Polynomials	
4.2/4.3 – Adding and Subtracting Polynomials	
4.5 – Multiplying Monomials by Monomials	
5.3 – Multiplying a Polynomial by a Monomial	
1.9 – Powers of Monomials	
4.7 – Dividing Monomials by Monomials	
5.4 – Dividing Polynomials by Monomials	
Review:	
Test:	

What You'll Learn:

4.1 – Using correct vocabulary to describe polynomials
4.2/4.3 – Performing addition and subtraction of polynomials using like terms
4.5/5.3 – Performing multiplication on monomials and polynomials using exponent laws
1.9 – Performing operations on polynomials involving exponents
4.7/5.4 – Performing division on monomials and polynomials using exponent laws

What is the difference between an expression and an equation?

An expression is a mathematical phrase that does not have an 'equals' sign.
An equation is a mathematical phrase that does have an 'equals' sign

What is a term?

A number, a variable, or a product of numbers and variables.
Terms are separated by add or subtract.

4.1 - Polynomials

Focus: Classifying and ordering polynomials.

Main Ideas:

Warmup:
How many terms does each expression have?
a) $y + 7$ b) $x^2 - 2x + 3$
c) -1 d) $2t - 9 + t^3 - t^2$
e) $2x^2y^5z^7$

(a) 2 (b) 3 (c) 1 (d) 4 (e) 1

What is a monomial?
What is a binomial?
What is a trinomial?
What is a polynomial?

Monomial: a one term expression ex. $2x$
Binomial: a two term expression ex. $2x + 3$
Trinomial: a three term expression ex. $x^2 + 2x + 3$
Polynomial: an expression with two or more terms.

ex1 – Classify each expression:
a) $x - 2y$
b) $-17x^2y^2z^2$
c) $4x$
d) $2x^2 - 5 + 16xy$
e) $x + y - z + 5$

(a) -binomial
-polynomial
(b) -monomial
(c) - monomial
(d) -trinomial
-polynomial
(e) -polynomial

What is the 'degree' of a monomial?

The degree of a monomial is the sum of the exponents of its variables.

ex2 – State the degree
a) $2x$
b) $-3x^2y^3z$
c) -7

(a) $2x$ or $2x^1$
degree : 1
(b) $-3x^2y^3z^1$
degree: $2+3+1 = 6$
(c) -7
no variables so degree $= 0$

What is the degree of a polynomial?

The degree of a polynomial is the highest exponent of the variable in any one term.

ex. $\underbrace{4x^2}_{2} \underbrace{- 3x}_{1} \underbrace{+ 5}_{0}$ so degree is 2.

for 'x' ex3 – State the degree of each polynomial:

a) $x^2 + 3x + 7$
b) $3y - 2y^3 + 2y^2$
c) $4x^6 + 2x^3y^4$
d) $25x^3y + 36x^2y^2$

(a) $\underbrace{x^2}_{2} + \underbrace{3x}_{1} + \underbrace{7}_{0}$

degree : 2

(c) $\underbrace{4x^6}_{6} + \underbrace{2x^3y^4}_{3}$

degree : 6

(b) $\underbrace{3y}_{1} \underbrace{-2y^3}_{3} \underbrace{+2y^2}_{2}$

degree: 3

(d) $\underbrace{25x^3y}_{3} + \underbrace{36x^2y^2}_{2}$

degree : 3

How do you order a polynomial?

Arrange terms from highest degree to lowest degree.

Ex4 – Arrange in descending powers of x

a) $3x - x^2 + 2x^3 - 6$
b) $7 - 5x + 3x^4 - x^2$
c) $2x^2y^5 - 3x + 2x^3y^2$

(a) $\underbrace{3x}_{3^{rd}} \underbrace{-x^2}_{2^{nd}} \underbrace{+2x^3}_{1^{st}} \underbrace{-6}_{4^{th}}$

$2x^3 - x^2 + 3x - 6$

(c) $\underbrace{2x^2y^5}_{2^{nd}} \underbrace{-3x}_{3^{rd}} \underbrace{+2x^3y^2}_{1^{st}}$

$2x^3y^2 + 2x^2y^5 - 3x$

(b) $\underbrace{7}_{4^{th}} \underbrace{-5x}_{3^{rd}} \underbrace{+3x^4}_{1^{st}} \underbrace{-x^2}_{2^{nd}}$

$3x^4 - x^2 - 5x + 7$

* make sure you bring the sign to the left of the term <u>with</u> the term when rearranging

Reflection: When rearranging terms, what do you have to make sure of? Explain using the example: $7 - 3x - 2x^2$

4.2/4.3 – Adding and Subtracting Polynomials

Focus: To add and subtract polynomials using like terms.

Main Ideas:

Warmup:
Simplify:
$x^2 - 2x + 5x - 3x^2$
What are like terms?
How do you add/subtract like terms?

$1x^2 \boxed{-2x} \boxed{+5x} -3x^2$

like terms are terms with the same variable(s) raised to the same exponent(s).

$-2x^2 + 3x$

Add or subtract like terms by adding or subtracting the coefficient and keeping the variable(s) and exponent(s) the same.

ex1 - Add
$(2x^2 + 3x + 2) + (x^2 + 2x + 3)$

$2x^2 \boxed{+3x} +2 +1x^2 \boxed{+2x} +3$

What steps are involved in adding polynomials?

1. Identify like terms
2. Add like terms (or subtract)

$3x^2 + 5x + 5$

ex2 – Simplify
a) $(3y^2 - 8y + 3) + (2y^2 + 8y - 9)$

b) $(5x^3 + 7x - 9) + (-8x + 11 + 4x^3)$

(a) $3y^2 \boxed{-8y} +3 +2y^2 \boxed{+8y} -9$

$5y^2 - 6$

(b) $5x^3 + 7x \boxed{-9} - 8x^2 \boxed{+11} +4x^3$

$9x^3 - 8x^2 + 7x - 9$

Subtracting Polynomials

ex3 - Simplify
$(4x^2 - 5x + 7) - (3x^2 + 2x - 5)$

$(4x^2 - 5x + 7) - 1(3x^2 + 2x - 5)$

$4x^2 \boxed{-5x} +7 -3x^2 \boxed{-2x} +5$

$x^2 - 7x + 12$

What is the extra step needed when subtracting polynomials?	the subtraction sign must be distributed to every term in the second set of brackets.
ex4 - Simplify a) $(3y^2 - y + 5) - (4y^2 - y + 1)$ b) $(-x^2y + 3xy) - (4xy^2 + xy - 2x^2y)$	(a) $(3y^2 - y + 5) - 1(4y^2 - y + 1)$ $3y^2 \boxed{-1y} + 5 - 4y^2 \boxed{+1y} - 1$ $\underline{-y^2 + 4}$ (b) $(-1x^2y + 3xy) - 1(4xy^2 + 1xy - 2x^2y)$ $-1x^2y \boxed{+3xy} - 4xy^2 \boxed{-1xy} + 2x^2y$ $\underline{x^2y + 2xy - 4xy^2}$

Reflection: What is different about adding polynomials compared to subtracting them?

4.5 – Multiplying Monomials by Monomials

Focus: How to multiply two monomials so only one monomial results.

Main Ideas:

Warmup:
Simplify in power form:
a) $3^2 \times 3^5$
b) $(-7)^4(-7)$
c) $y^6 \times y^3$
What is the shortcut for multiplying powers with the same base?

(a) $3^2 \times 3^5 = 3^{2+5} = 3^7$ (b) $(-7)^4(-7)^1 = (-7)^5$
(c) $y^6 \times y^3 = y^9$

Add exponents

ex1 - Multiply
$(6x^2)(-3x)$

$(6x^2)(-3x^1)$
$= -18x^3$

What are the steps to multiplying monomials?

① Multiply coefficients
② Add exponents of like variables.

ex2 - Multiply
a) $(3a)(4b^3)$
b) $-x^2(2xy)$
c) $-2y^3(4y^2)$

(a) $(3a)(4b^3)$ $= 12ab^3$
(b) $-1x^2(2x^1y)$ $= -2x^3y$
(c) $-2y^3(4y^2)$ $= -8y^5$

ex3 – Simplify
a) $(2x^2y)(-3xy)$
b) $(x^3)(-3xy^3)(4y)$
c) $(-2a^3bc^2)(-5ac^4y^2)$

(a) $(2x^2y^1)(-3x^1y^1)$ $= -6x^3y^2$
(b) $(1x^3)(-3x^1y^3)(4y^1)$ $-12x^4y^4$
(c) $(-2a^3b^1c^2)(-5a^1c^4y^2)$
$= 10a^4bc^6y^2$

Reflection: What are the steps to multiplying monomials?

5.3 – Multiplying a Polynomial by a Monomial

Focus: How to multiply a monomial to a polynomial.

Main Ideas:

Warmup:
Use distributive property
To expand the following:
x(x – 3)

$x^1(x^1-3)$

$= x^2-3x$

ex2 – Expand
a) 2x(x – 6)
b) –y(y^2 – 7)
c) m(m + 2) - 3m(m – 5)

(a) $2x(x-6)$

$2x^2-12x$

(b) $-1y(y^2-7)$

$-1y^3+7y$

$=-y^3+7y$

(c) $m(m+2)-3m(m-5)$

$m^2+2m-3m^2+15$

$-2m^2+2m+15$

ex3 - Simplify
a) 4x(x^2 – x + 7)
b) y(y + 3) – y(y – 2)
c) 3(x^2 + 2x – 5) – x(x + 1)

(a) $4x(x^2-x+7)$

$4x^3-4x^2+28x$

(b) $y(y+3)-y(y-2)$

$y^2\boxed{+3y}-y^2\boxed{+2y}$

$=\underline{5y}$

(c) $3(x^2+2x-5)-1x(x+1)$

$3x^2\boxed{+6x}-15-x^2\boxed{-x}$

$\underline{2x^2+5x-15}$

Reflection: When distributing into brackets, what exponent rule do you often use? Then, when adding and subtracting in the last step, explain how to collect like terms?

1.9 – Powers of Monomials

Focus: How to work with monomials that have an exponent.

Main Ideas:

Warmup:
Simplify in power form:
a) $(3^4)^5$
b) $(9^2)^3$
c) $(x^6)^2$
What is the shortcut when you have an exponent raised to an exponent?

(a) $(3^4)^5 = 3^{4\times5} = 3^{20}$ (b) $(9^2)^3 = 9^6$

(c) $(x^6)^2 = x^{12}$

multiply exponents

ex1 - Simplify
a) $(x^2)^3$
b) $(y)^4$

(a) $(x^2)^3 = x^{2\times3} = x^6$

(b) $(y^1)^4 = y^{1\times4} = y^4$

ex2 – Simplify
$(2x^2y^3)^2$

$(2x^2y^3)^2 = (2x^2y^3)(2x^2y^3) = 2\times2\times x^2\times x^2\times y^3\times y^3$
$= 4x^4y^6$

shortcut: $(2x^2y^3)^2 = 4x^4y^6$

ex3 - Simplify
a) $(-x^2y)^2$
b) $(-2a^2bc^3)^3$

(a) $(-1x^2y^1)^2 = 1x^4y^2 = x^4y^2$

(b) $(-2a^2b^1c^3)^3 = (-2)^3a^6b^3c^9 = -8a^6b^3c^9$

ex4 - Simplify
$-(m^2n^5)^0$

$-1(m^2n^5)^0 = -1(m^0n^0) = -1(1\times1) = -1$

ex5 - Simplify
a) $(2x^2y^3)^4(-xy^2)^3$
b) $(ab^3c)^2(2a^2bc^2)^3$

(a) $(2x^2y^3)^4(-1x^1y^2)^3$
$= (2^4x^8y^{12})(-1x^3y^6)$
$= 16x^8y^{12}(-1x^3y^6)$
$= -16x^{11}y^{18}$

(b) $(a^1b^3c^1)^2(2a^2b^1c^2)^3$
$= a^2b^6c^2(2^3a^6b^3c^6)$
$= a^2b^6c^2(8a^6b^3c^6)$
$= 8a^8b^9c^8$

ex6 - Simplify
$$\left(\frac{2x}{3y^2}\right)^2$$

$$\left(\frac{2x}{3y^2}\right)^2 = \left(\frac{2x}{3y^2}\right)\left(\frac{2x}{3y^2}\right) = \frac{4x^2}{9y^4}$$

OR ... shortcut...

$$\left(\frac{2x^1}{3y^2}\right)^2 = \frac{2^2x^2}{3^2y^4} = \frac{4x^2}{9y^4}$$

Reflection: What is the most difficult part for you and what can you do to improve upon it?

4.7 – Dividing Monomials by Monomials

Focus: How to divide monomials so only one monomial results.

Main Ideas:

Warmup:
Simplify in power form:
a) $4^5 \div 4^3$
b) $\frac{2^7}{2^6}$
c) $y^3 \div y$

(a) $4^5 \div 4^3 = 4^{5-3} = 4^2$ (b) $\frac{2^7}{2^6} = 2^{7-6} = 2^1 = 2$

(c) $y^3 \div y^1 = y^{3-1} = y^2$

What is the shortcut for dividing powers with the same base?

subtract exponents.

ex1 - Divide
$$\frac{8x^2}{2x}$$

$$\frac{8x^2}{2x^1} = \overset{8 \div 2}{4}\overset{x^2 \div x^1}{x}$$

What are the steps for dividing monomials?

(1) divide coefficients
(2) divide like variables by subtracting exponents

ex2 - Simplify
a) $16y \div 4y$
b) $-12abc \div -2bc$
c) $18x^5y^2 \div -6x^2y$

(a) $16\cancel{y} \div 4\cancel{y}$ $= 4$

(b) $-12a\cancel{b}\cancel{c} \div -2\cancel{b}\cancel{c}$ $= 6a$

(c) $18x^5y^2 \div -6x^2y^1 = -3x^3y$

ex3 – Divide
a) $\frac{21x^5yz^2}{-7xyz}$
b) $\frac{10m^8n^5}{5m^3n^2}$

(a) $\frac{21x^5y^1z^2}{-7x^1y^1z^1}$ $= -3x^4z$

(b) $\frac{10m^8n^5}{5m^3n^2}$ $= 2m^5n^3$

ex4 – Simplify

a) $\dfrac{2x^2y}{6x^{-2}y^4}$

b) $\dfrac{8a^2b^4c^3}{a^4bc^4}$

(a) $\dfrac{2x^2y^1}{6x^{-2}y^4}$ $\qquad \dfrac{2^{\div 2}}{6_{\div 2}} = \dfrac{1}{3} \qquad \dfrac{x^2}{x^{-2}} = x^4 \qquad \dfrac{y^1}{y^4} = y^{-3}$

$2-(-2) = 2+2 = 4 \qquad 1-4 = -3$

$= \dfrac{1}{3}x^4y^{-3}$ OR $\dfrac{x^4y^{-3}}{3}$

remember $y^{-3} = \dfrac{1}{y^3}$

so... $\dfrac{x^4}{3y^3}$

(b) $\dfrac{8a^2b^4c^3}{1a^4b^1c^4} = 8a^{-2}b^3c^{-1} = \dfrac{8b^3}{a^2c}$

Reflection: Describe the steps for multiplying monomials by monomials.

5.4 – Dividing Polynomials by Monomials

Focus: How to divide a polynomial by a monomial

Main Ideas:

Warmup:

Divide $\dfrac{12x^3yz^2}{-3xz^2}$

$$\frac{12x^3y^1z^2}{-3x^1z^2} = -4x^2y$$

What steps did you use to solve the above division?

① Multiply coefficients
② Divide like variables by subtracting exponents.

ex1 - Divide $\dfrac{5xyz+10xy}{5xy}$

$$\frac{5x^1y^1z^1 + 10x^1y^1}{5x^1y^1} = z + 2$$

What are the steps involved?

① Divide every term on top by the term on the bottom
② Use the same 2 steps as above.

ex2 - Simplify

a) $\dfrac{18x^2y^2z^2-12x^3yz^2}{6xyz}$

b) $\dfrac{-10a^5b^4+5a^6b^8}{-5ab^4}$

c) $\dfrac{27y^3-9y+18y^2}{9y}$

(a) $$\frac{18x^2y^2z^2 - 12x^3y^1z^2}{6x^1y^1z^1}$$

$$= 3xyz - 2x^2z$$

(b) $$\frac{-10a^5b^4 + 5a^6b^8}{-5a^1b^4}$$

$$= 2a^4 - a^5b^4$$

(c) $$\frac{27y^3 - 9y^1 + 18y^2}{9y^1}$$

$$= 3y^2 - 1 + 2y$$

Reflection: What are the steps to dividing a polynomial by a monomial?
What results when a top term and bottom term perfectly cancel?