

### 3.1 – What is a Rational Number?

Focus: Compare and order rational numbers

#### Main Ideas

- Remember...if a fraction has a negative sign, you can put it where you want (on the top, middle, or bottom) and it's still the same fraction!

#### Warmup:

Draw a number line from -1 to 1.

a) Label  $-\frac{1}{2}$  and  $\frac{1}{2}$

b) Label  $-\frac{1}{4}$  and  $\frac{1}{4}$

c) Label  $-\frac{3}{4}$  and  $\frac{3}{4}$

d) Label  $-\frac{1}{8}$  and  $\frac{1}{8}$

e) Label  $-\frac{5}{8}$  and  $\frac{5}{8}$

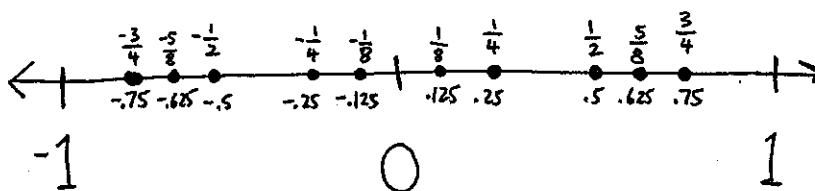
f) underneath each fraction, change to decimal

What is a rational number?

Are fractions rational numbers? What about mixed fractions?

Are whole numbers rational?

Which decimals are rational?



Any number that can be written as a fraction where the numerator and denominator are both integers (and denom. is not zero).

Fractions and mixed fractions are rational numbers.

Whole numbers are rational because any whole number can be made into a fraction by putting it over on  $\textcircled{\text{ex}}$  3 can be  $\frac{3}{1}$

Decimals that terminate (end) or repeat can be made into fractions and so are rational.

What is an irrational number and give two examples:

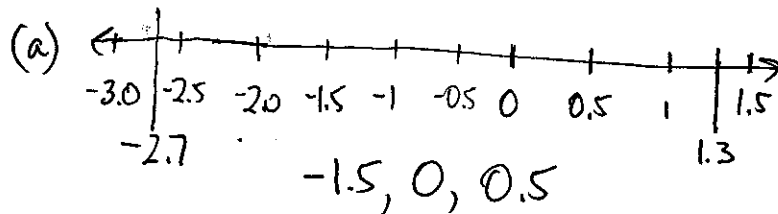
An irrational number is a decimal that neither terminates or repeats. These numbers cannot be made into fractions and so are not rational.

examples:  $\pi$ ,  $\sqrt{2}$

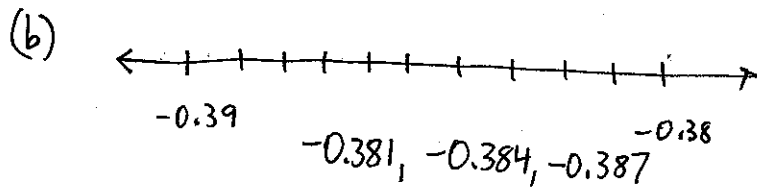
Ex1

Write 3 rational number between each pair of numbers. Start by sketching a number line.

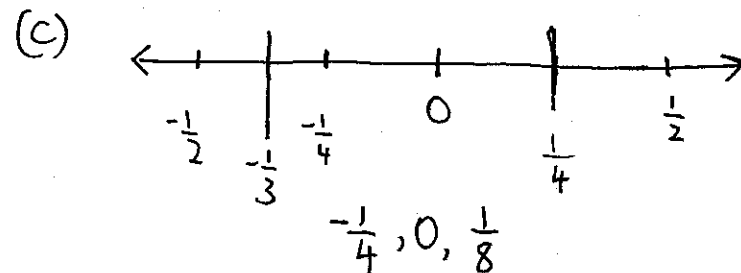
a) -2.7 and 1.3



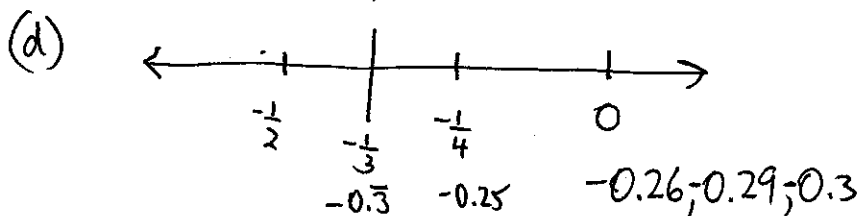
b) -0.38 and -0.39



c)  $-\frac{1}{3}$  and  $\frac{1}{4}$



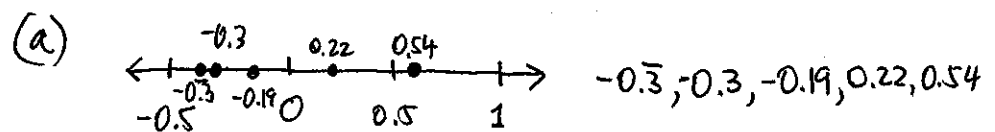
d)  $-\frac{1}{3}$  and  $-\frac{1}{4}$



Ex2

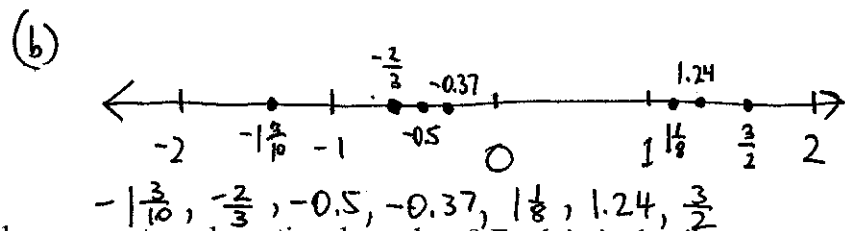
Order these numbers from least to greatest using a number line for assistance:

a) 0.54, -0.3,  $-0.\bar{3}$ , -0.19, 0, .22



b)  $-\frac{2}{3}$ , -0.5, 1.24,  $1\frac{1}{8}$ ,

$\frac{3}{2}$ , -0.37,  $-1\frac{3}{10}$



**Reflection:** What strategy do you use to order rational numbers? Explain in detail.

### 3.2 – Adding Rational Numbers

Focus: Solve problems that require adding rational numbers.

#### Main Ideas:

#### Warmup:

Complete and try to remember your strategies from last year.

- a)  $-3 + 5$
- b)  $4 + -3$
- c)  $-1 + -5$
- d)  $3 - 7$
- e)  $-2 - 6$
- f)  $3 - (-6)$

In a few sentences, jot down your strategies for adding and subtracting integers.

Ex1

How do you find

$$\frac{1}{2} + \frac{1}{3}$$

Find a way to verify your solution using the fraction pies

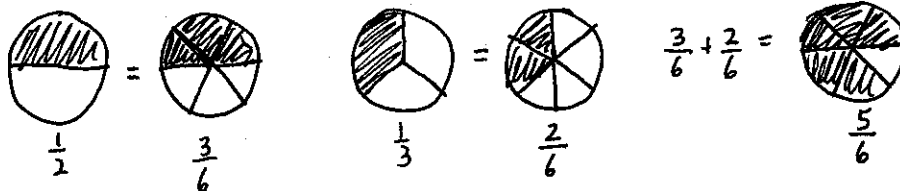
What are the steps involved with adding fractions?

How do you deal with negative fractions?

(a)  $-3 + 5 = 2$  (b)  $4 + -3 = 1$  (c)  $-1 + -5 = -6$   
 (d)  $3 - 7 = -4$  (e)  $-2 - 6 = -8$  (f)  $3 - (-6) = 3 + 6 = 9$

Strategies:

$$\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$



- ① Get common denominators
- ② Add numerators (leave denominators the same)
- ③ Reduce if possible.

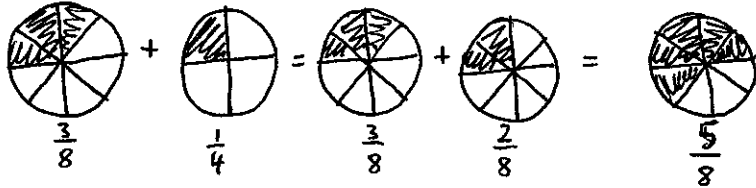
for a negative fraction, the negative sign can be written on the numerator, in the middle of the fraction, or in the denominator. Put the negative onto the numerator every time.

ex.  $-\frac{1}{3} = -\frac{1}{3} = \frac{1}{-3}$  but use  $-\frac{1}{3}$

$$\frac{3}{8} + \frac{1^{x^2}}{4^{x^2}} = \frac{3}{8} + \frac{2}{8} = \left(\frac{5}{8}\right)$$

Ex2 - Simplify and  
Verify with fraction pies

$$\frac{3}{8} + \frac{1}{4}$$



Ex3 - Simplify:

a)  $-\frac{2}{3} + \frac{1}{6}$

(a)  $\downarrow$  neg on top  
 $-\frac{2^{x^2}}{3^{x^2}} + \frac{1}{6} = \frac{-4}{6} + \frac{1}{6} = \frac{-3^{x^3}}{6^{x^3}} = \left(\frac{-1}{2}\right)$

b)  $-\frac{5}{8} + \left(-\frac{2}{7}\right)$

(b)  $-\frac{5^{x^7}}{8^{x^7}} + \frac{-2^{x^8}}{7^{x^8}} = \frac{-35}{56} + \frac{-16}{56} = \left(\frac{-51}{56}\right)$

c)  $-\frac{1}{3} + \left(-\frac{7}{8}\right)$

(c)  $-\frac{1^{x^8}}{3^{x^8}} + \frac{-7^{x^3}}{8^{x^3}} = \frac{-8}{24} + \frac{-21}{24} = \left(\frac{-29}{24} \text{ or } -\frac{5}{24}\right)$

d)  $-3\frac{1}{3} + 2\frac{5}{6}$

(d)  $-3\frac{1}{3} + 2\frac{5}{6}$  change to improper...

$$-\frac{10^{x^2}}{3^{x^2}} + \frac{17}{6} = \frac{-20}{6} + \frac{17}{6} = \frac{-3^{x^3}}{6^{x^3}} = \left(\frac{-1}{2}\right)$$

**Reflection:** What are the steps to adding fractions?

### 3.3 – Subtracting Rational Numbers

Focus: Solve problems that require subtracting rational numbers.

Main Ideas:

#### Warmup:

Solve:

- a)  $2 - 5$
- b)  $-3 - 7$
- c)  $8 - (+2)$
- d)  $6 - (-2)$
- e)  $-1.2 - (-4.9)$

What strategies did you use to solve the problems above?

#### Ex1

Use the fraction pies to help you solve the following:

$$\frac{1}{2} - \frac{1}{3}$$

Then, show the work.

What are the steps to subtracting fractions?

#### Ex2

Solve and then verify using fraction pies:

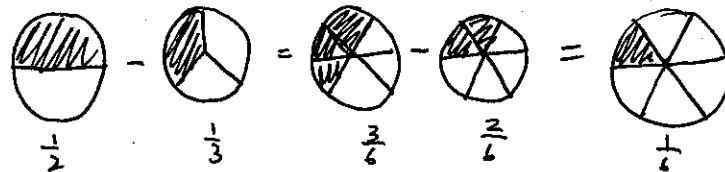
$$\frac{5}{6} - \frac{2}{3}$$

$$(a) 2 - 5 = -3 \quad (b) -3 - 7 = -10 \quad (c) 8 - (+2) = 6$$

$$(d) 6 - (-2) = 8 \quad (e) -1.2 - (-4.9) = 3.7$$

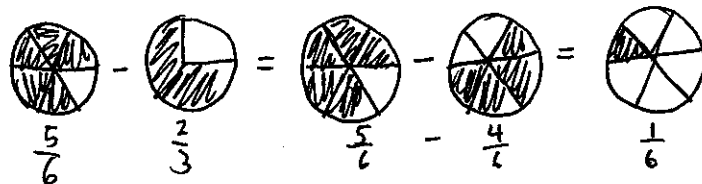
Strategies:

$$\frac{1^{x3}}{2^{x3}} - \frac{1^{x2}}{3^{x2}} = \frac{3}{6} - \frac{2}{6} = \frac{1}{6}$$



- ① Get common denominators
- ② Subtract numerators (leave denominators the same)
- ③ Reduce if possible.

$$\frac{5}{6} - \frac{2^{x2}}{3^{x2}} = \frac{5}{6} - \frac{4}{6} = \frac{1}{6}$$



Ex3 - Simplify

a)  $-\frac{3}{4} - \frac{5}{6}$

b)  $\frac{4}{3} - \left(-\frac{2}{5}\right)$

c)  $\frac{5}{-4} - \left(-3\frac{1}{5}\right)$

(a)  $-\frac{3^{x3}}{4^{x3}} - \frac{5^{x2}}{6^{x2}} = -\frac{9}{12} - \frac{10}{12} = \frac{-19}{12}$  or  $-1\frac{7}{12}$

$-9 - 10 = -19$

(b)  $\frac{4^{x5}}{3^{x5}} - \left(-\frac{2^{x3}}{5^{x3}}\right) = \frac{20}{15} + \frac{6}{15} = \frac{26}{15}$  or  $1\frac{11}{15}$

(c)  $\frac{-5}{4} - \left(-3\frac{1}{5}\right) = \frac{-5^{x5}}{4^{x5}} + \frac{16^{x4}}{5^{x4}} = \frac{-25}{20} + \frac{64}{20}$

$-25 + 64 = 39$

$= \frac{39}{20}$  or  $1\frac{19}{20}$

**Reflection:** What are the steps to subtracting fractions?

### 3.4 – Multiplying Rational Numbers

Focus: Solve problems that require multiplying rational numbers

Main Ideas:

#### Warmup:

Multiply:

- a)  $-9 \times 4$
- b)  $3 \times -3$
- c)  $-1 \times -4$
- d)  $-3 \times 2 \times -1$
- e)  $-1 \times -1 \times -1$

Explain how you found your answers to the questions above.

When multiplying

Fractions such as

$\frac{1}{2} \times \frac{1}{4}$ , you may want to

think: 'What's half of a quarter?' Use fraction pies for assistance.

How do you multiply

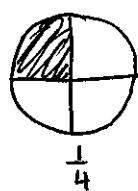
$\frac{1}{2} \times \frac{1}{4}$ ?

What are the steps to multiplying fractions?

(a)  $-9 \times 4 = -36$  (b)  $3 \times -3 = -9$  (c)  $-1 \times -4 = 4$   
(d)  $-3 \times 2 \times -1 = 6$  (e)  $-1 \times -1 \times -1 = -1$

If there are an odd number of negative signs (1, 3, 5, 7 etc), the answer is negative.

If there are an even number of negative signs (0, 2, 4, 6 etc), the answer is positive.



What's half of  $\frac{1}{4}$ ?



$\frac{1}{8}$  is half of a quarter!

$$\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$$

- multiply numerators  
- multiply denominators

- ① change to improper if fractions are mixed.
- ② multiply numerators
- ③ multiply denominators
- ④ Reduce if possible

Ex1 - Multiply

a)  $\frac{3}{2} \times \frac{1}{5}$

b)  $-\frac{3}{2} \times \frac{1}{-5}$

c)  $\frac{4}{7}(-3)$

Ex2 - Multiply

a)  $\left(\frac{-11}{7}\right)\left(-\frac{21}{44}\right)$

b)  $\left(2\frac{2}{3}\right)\left(-1\frac{3}{4}\right)$

Ex3

On Feb. 5, 2009, the price of a share in CIBC changed by  $-\$1.64$ . A person owns 54 shares. By how much did those shares change in value that day?

(a)  $\frac{3}{2} \times \frac{1}{5} = \left(\frac{3}{10}\right)$  (b)  $\overset{\text{negative on top}}{\downarrow} \frac{-3}{2} \times \frac{-1}{5} = \left(\frac{3}{10}\right)$

(c)  $\frac{4}{7}(-3) = \frac{4}{7} \times -3 = \frac{4}{7} \times \frac{-3}{1} = \left(\frac{-12}{7} \text{ or } -1\frac{5}{7}\right)$

(a)  $\frac{-11}{7} \times \frac{-21}{44} = \frac{-\cancel{11}^1}{\cancel{7}_1} \times \frac{-\cancel{21}^3}{\cancel{44}^4} = \frac{-1}{1} \times \frac{3}{4} = \left(\frac{-3}{4}\right)$

\*Sometimes, it's possible to reduce before multiplying.

(b)  $2\frac{2}{3} \times -1\frac{3}{4} = \frac{8}{3} \times -\frac{7}{4} = \frac{\cancel{8}^2}{3} \times -\frac{7}{\cancel{4}_1} = \left(\frac{-14}{3} \text{ or } -4\frac{2}{3}\right)$

changed by  $-1.64$  means the share value decreased by  $\$1.64$ .

$$54 \times -1.64 = -88.56$$

The shares decreased by  $\$88.56$  that day.

**Reflection:** What are the steps for multiplying fractions?



### 3.5 – Dividing Rational Numbers

Focus: Solve problems that require dividing rational numbers.

#### Main Ideas:

#### Warmup:

Use fraction pies:

a)  $\frac{1}{4} \times \frac{1}{2}$

think: what's half of a quarter?

b)  $\frac{1}{4} \div 2$

think: what's a quarter divided by 2?

What did you learn from a and b together?

c)  $\frac{1}{4} \times 2$

think: what does one quarter become when doubled?

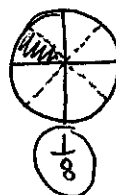
d)  $\frac{1}{4} \div \frac{1}{2}$

Predict the answer to d using what you've learned from a, b, & c.

What are the set of steps for dividing fractions? Use d as an example



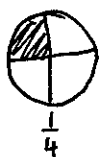
half of  $\frac{1}{4}$  ...



$\frac{1}{8}$  is half of a quarter.

$\frac{1}{4}$  divided into 2 equal pieces is  $\frac{1}{8}$ , as can be seen above.

Multiplying by  $\frac{1}{2}$  is the same as dividing by 2



double it...



$$\frac{1}{4} \times 2 = \frac{2}{4} = \frac{1}{2}$$

if multiplying by  $\frac{1}{2}$  is the same as dividing by 2, then multiplying by 2 must be the same as dividing by a  $\frac{1}{2}$

so  $\frac{1}{4} \div \frac{1}{2}$  must equal  $\left(\frac{1}{2}\right)$

- ① Change any mixed fractions to improper
- ② Flip the second fraction and change  $\div$  to  $\times$

$$\frac{1}{4} \div \frac{1}{2} = \frac{1}{4} \times \frac{2}{1}$$

- ③ Multiply numerators, multiply denominators

$$\frac{1}{4} \times \frac{2}{1} = \frac{2}{4}$$

- ④ Reduce if possible.  $\frac{2 \div 2}{4 \div 2} = \left(\frac{1}{2}\right)$

What is a fraction that has been flipped over called?

Ex1 - Divide

a)  $\frac{3}{2} \div \left(-\frac{1}{5}\right)$

b)  $\left(-\frac{4}{3}\right) \div \left(\frac{-8}{3}\right)$

c)  $-4\frac{2}{3} \div 1\frac{4}{5}$

Ex2 - Divide using a calculator  
 $(-0.25) \div (-0.3)$

Ex3 - Simplify

$\left(-\frac{1}{3}\right) \times \frac{4}{3} \div \left(\frac{5}{-6}\right)$

A fraction that's been flipped over is called a RECIPROCAL.

put negative on top

(a)  $\frac{3}{2} \div -\frac{1}{5} = \frac{3}{2} \times \frac{-5}{1} = \frac{-15}{2} \text{ or } -7\frac{1}{2}$

(b)  $-\frac{4}{3} \div \frac{-8}{3} = -\frac{4}{3} \times \frac{-3}{8} = \frac{12}{24} = \frac{1}{2}$

(c)  $-4\frac{2}{3} \div 1\frac{4}{5} = -\frac{14}{3} \div \frac{9}{5} = -\frac{14}{3} \times \frac{5}{9} = \frac{-70}{27} \text{ or } -2\frac{16}{27}$

$-0.25 \div -0.3 = 0.8\bar{3}$   
 use  $\boxed{+/-}$  button or  $\boxed{(-)}$

$\left(-\frac{1}{3}\right) \times \frac{4}{3} \div -\frac{5}{6}$

$= -\frac{4}{9} \div -\frac{5}{6}$

$= -\frac{4}{9} \times -\frac{6}{5}$

$= \frac{24}{45}$

$= \frac{8}{15}$

**Reflection:** What are the steps for dividing fractions?

### 3.6 – Order of Operations with Rational Numbers

Focus: Explain and apply the order of operations with rational numbers.

#### Main Ideas:

#### Warmup:

Do the problem at the very top of p.137 and answer the two questions below the student answers.

What is the key word for order of operations?

Ex1 – Evaluate

$$(-3.2) - 0.9 \div [0.7 - (-1.2)]^2$$

Ex2 – To convert Fahrenheit To Celsius, use the formula

$$C = \frac{F - 32}{1.8}$$

If the temperature is  $-5.6^\circ\text{F}$ , what is it in Celsius?

Student #1: did subtraction first step which is incorrect

$$(-8) - 2 \left( \frac{24}{-8} \right)^2 = (-8) - 2(-3)^2$$

Student #2: multiplied  $-2(-3)$  before doing the exponent

$$= (-8) - 2(9) = (-8) - 18 = \underline{\underline{-26}}$$

**BEDMAS**    B: brackets    DM: division and multiplication (whichever comes first)    AS: add and subtract (whichever comes first)  
                   E: exponents

$$\begin{aligned} & (-3.2) - 0.9 \div [0.7 - (-1.2)]^2 \\ &= (-3.2) - 0.9 \div [1.9]^2 \\ &= (-3.2) - 0.9 \div 3.61 \\ &= (-3.2) - 0.25 \\ &= \underline{\underline{-3.45}} \end{aligned}$$

$$C = \frac{F - 32}{1.8} \quad F = -5.6$$

$$C = \frac{(-5.6) - 32}{1.8}$$

$$C = \frac{-37.6}{1.8}$$

$$C = \underline{\underline{-20.9^\circ\text{C}}}$$

Ex3 - Evaluate

$$\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) - \left(-\frac{2}{3}\right) \div \left[\frac{1}{3} + \left(-\frac{3}{12}\right)\right]$$

$$-\frac{1}{2} \times -\frac{1}{2} - \left(-\frac{2}{3}\right) \div \left[\frac{1}{3} + \frac{-3}{12}\right]$$

$$-\frac{1}{2} \times -\frac{1}{2} - \left(-\frac{2}{3}\right) \div \left[\frac{4}{12} + \frac{-3}{12}\right]$$

$$-\frac{1}{2} \times -\frac{1}{2} - \left(-\frac{2}{3}\right) \div \frac{1}{12}$$

$$-\frac{1}{2} \times -\frac{1}{2} - \left(-\frac{2}{3}\right) \times \frac{12}{1}$$

$$-\frac{1}{2} \times -\frac{1}{2} - \frac{-24}{3}$$

$$\frac{-1}{2} \times \frac{-1}{2} - \frac{-8}{1}$$

$$\frac{1}{4} - \frac{-8}{1}$$

$$\frac{1}{4} + \frac{32}{4}$$

$$= \frac{33}{4} \text{ or } 8\frac{1}{4}$$

**Reflection:** As the number of operations increases and the expressions become more complex, it is easy to make mistakes. What can you do to prevent yourself from making mistakes?