UNIT 2

Powers and Exponent Laws

What You'll Learn

- Use powers to show repeated multiplication.
- Evaluate powers with exponent 0.
- Write numbers using powers of 10.
- Use the order of operations with exponents.
- Use the exponent laws to simplify and evaluate expressions.

Why It's Important

Powers are used by

- lab technicians, when they interpret a patient's test results
- reporters, when they write large numbers in a news story

Key Words

integer
opposite
positive
negative
factor
power
base
exponent
squared
cubed
standard form
product
quotient

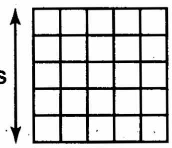
1.1 Skill Builder

Side Lengths and Areas of Squares

The side length and area of a square are related.

- The area is the **square** of the side length.

5 units

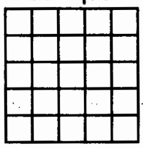


Area = $(\text{Length})^2$
$= 5^2$
$= 5 \times 5$
$= 25$

The area is 25 square units.

- The side length is the **square root** of the area.

Area = 25 square units

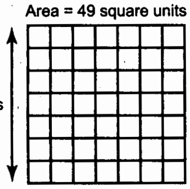
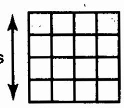
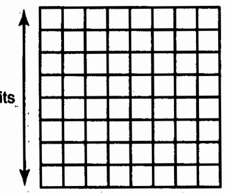
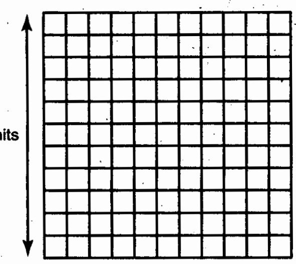


Length = $\sqrt{\text{Area}}$
$= \sqrt{25}$
$= \sqrt{5 \times 5}$
$= 5$

The side length is 5 units.

Check

1. Which square and square root are modelled by each diagram?

Diagram	Square Modelled	Square Root Modelled
a) Area = 49 square units 7 units 	$(\text{Length})^2 = \text{Area}$ $7^2 =$ _____ The area is 49 square units.	$\sqrt{\text{Area}} = \text{Length}$ $\sqrt{49} =$ _____ The side length is 7 units.
b) 4 units 	_____ = _____ The area is _____ square units.	$\sqrt{\_\_\_\_\_}$ = _____ The side length is _____ units.
c) 8 units 	_____ = _____ The area is _____ square units.	$\sqrt{\_\_\_\_\_}$ = _____ The side length is _____ units.
d) 11 units 	_____ = _____ The area is _____ square units.	$\sqrt{\_\_\_\_\_}$ = _____ The side length is _____ units.

Whole Number Squares and Square Roots

- The square of a number is the number multiplied by itself.
- A square root of a number is one of 2 equal factors of the number.
- Squaring and taking a square root are inverse operations.

$5^2 = 5 \times 5$
$= 25$
$\sqrt{25} = \sqrt{5 \times 5}$
$= 5$
$5^2 = 25$ and $\sqrt{25} = 5$

Check

1. Complete each sentence.

a) $4^2 = 16$, so $\sqrt{16} =$ ____ **b)** $12^2 =$ ____, so $\sqrt{\_\_\_\_} =$ ____

c) $\sqrt{25} =$ ____, since ____ $= 25$ **d)** $\sqrt{100} =$ ____, since ____ $=$ ____

Perfect Squares

A number is a **perfect square** if it is the product of 2 equal factors.
25 is a perfect square because $25 = 5 \times 5$.
24 is a **non-perfect square.** It is not the product of 2 equal factors.

Check

1. Complete each sentence.

First 12 Whole-Number Perfect Squares			
Perfect Square	**Square Root**	**Perfect Square**	**Square Root**
$1^2 = 1 \times 1 = 1$	$\sqrt{1} = 1$	$7^2 =$ ___ $\times$ ___ $=$ ___	$\sqrt{\_\_\_} =$ ___
$2^2 = 2 \times 2 = 4$	$\sqrt{4} = 2$	$8^2 =$ ___ $\times$ ___ $=$ ___	$\sqrt{\_\_\_} =$ ___
$3^2 =$ ___ $\times$ ___ $=$ ___	$\sqrt{\_\_\_} =$ ___	$9^2 =$ ___ $\times$ ___ $=$ ___	$\sqrt{\_\_\_} =$ ___
$4^2 =$ ___ $\times$ ___ $=$ ___	$\sqrt{\_\_\_} =$ ___	$10^2 =$ ___ $\times$ ___ $=$ ___	$\sqrt{\_\_\_} =$ ___
$5^2 =$ ___ $\times$ ___ $=$ ___	$\sqrt{\_\_\_} =$ ___	$11^2 =$ ___ $\times$ ___ $=$ ___	$\sqrt{\_\_\_} =$ ___
$6^2 =$ ___ $\times$ ___ $=$ ___	$\sqrt{\_\_\_} =$ ___	$12^2 =$ ___ $\times$ ___ $=$ ___	$\sqrt{\_\_\_} =$ ___

1.1 Square Roots of Perfect Squares

FOCUS **Find the square roots of decimals and fractions that are perfect squares.**

The square of a fraction or decimal is the number multiplied by itself.

$$\left(\frac{2}{3}\right)^2 = \frac{2}{3} \times \frac{2}{3} = \frac{2 \times 2}{3 \times 3} = \frac{4}{9}$$

$$(1.5)^2 = 1.5 \times 1.5 = 2.25$$

$\frac{4}{9}$ and 2.25 are perfect squares because they are the product of 2 equal factors.

$\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$, so

$\frac{2}{3}$ is a square root of $\frac{4}{9}$.

We write: $\sqrt{\frac{4}{9}} = \frac{2}{3}$

$2.25 = 1.5 \times 1.5$, so

1.5 is a square root of 2.25.

We write: $\sqrt{2.25} = 1.5$

Each equal factor is a square root of the perfect square.

Example 1 Finding a Perfect Square Given Its Square Root

Calculate the number whose square root is:

a) $\frac{5}{8}$

b) 1.2

Solution

A square root of a number is one of two equal factors of the number.

a) $\frac{5}{8}$

$$\frac{5}{8} \times \frac{5}{8} = \frac{5 \times 5}{8 \times 8} = \frac{25}{64}$$

So, $\frac{5}{8}$ is a square root of $\frac{25}{64}$.

b) 1.2

Use a calculator.

$1.2 \times 1.2 = 1.44$

So, 1.2 is a square root of 1.44.

Check

1. Calculate the perfect square with the given square root.

a) $\frac{3}{8}$

$\frac{3}{8} \times \frac{3}{8} = \frac{\_\_\_ \times \_\_\_}{\_\_\_ \times \_\_\_}$

$= \frac{\_\_\_}{\_\_\_}$

$\frac{3}{8}$ is a square root of _____.

b) $\frac{3}{2}$

$\frac{\_\_}{\_\_} \times \frac{\_\_}{\_\_} = \frac{\_\_\_\_\_}{\_\_\_\_\_}$

$= \frac{\_\_\_\_\_}{\_\_\_\_\_}$

$\frac{3}{2}$ is a square root of _____.

c) 0.5

$0.5 \times 0.5 =$ ______

0.5 is a square root of ______.

d) 2.5

$2.5 \times 2.5 =$ ______

2.5 is a square root of ______.

Example 2 Identifying Fractions that Are Perfect Squares

Is each fraction a perfect square? If so, find its square root.

a) $\frac{16}{25}$

b) $\frac{9}{20}$

Solution

Check if the numerator and denominator are perfect squares.

a) $\frac{16}{25}$

$16 = 4 \times 4$, so 16 is a perfect square.

$25 = 5 \times 5$, so 25 is a perfect square.

So, $\frac{16}{25}$ is a perfect square.

b) $\frac{9}{20}$

$9 = 3 \times 3$, so 9 is a perfect square.

20 is not a perfect square.

So, $\frac{9}{20}$ is not a perfect square.

Check

1. Determine whether the fraction is or is not a perfect square. How do you know?

a) $\frac{9}{49}$

9 _______ a perfect square because _______________________________________.

49 _______ a perfect square because _______________________________________.

So, $\frac{9}{49}$ _______ a perfect square.

b) $\frac{25}{13}$

25 _______ a perfect square because _______________________________________.

13 _______ a perfect square because _______________________________________.

So, $\frac{25}{13}$ _______ a perfect square.

c) $\frac{64}{81}$ 64 ________ a perfect square because ________________________________.

81 ________ a perfect square because ________________________________.

So, $\frac{64}{81}$ ________ a perfect square.

2. Find the value of each square root.

a) $\sqrt{\frac{9}{4}} = \sqrt{\frac{\_\_\_ \times \_\_\_}{\_\_\_ \times \_\_\_}} = \frac{\_\_\_}{\_\_\_}$

b) $\sqrt{\frac{16}{81}} = \sqrt{\frac{\_\_\_ \times \_\_\_}{\_\_\_ \times \_\_\_}} = \frac{\_\_\_}{\_\_\_}$

A **terminating decimal** ends after a certain number of decimal places.
A **repeating decimal** has a repeating pattern of digits in the decimal expansion.
The bar shows the digits that repeat.

Terminating	Repeating	Non-terminating and non-repeating
0.5 0.28	0.333 333 … = $0.\overline{3}$ 0.191 919 … = $0.\overline{19}$	1.414 213 56 … 7.071 067 812 …

You can use a calculator to find out if a decimal is a perfect square.
The square root of a perfect square decimal is either a terminating decimal or a repeating decimal.

Example 3 Identifying Decimals that Are Perfect Squares

Is each decimal a perfect square? How do you know?

a) 1.69 **b)** 3.5

Solution

Use a calculator to find the square root of each number.

a) $\sqrt{1.69} = 1.3$
The square root is the terminating decimal 1.3.
So, 1.69 is a perfect square.

b) $\sqrt{3.5} \doteq 1.870\ 828\ 693$
The square root appears to be a decimal that neither repeats nor terminates.
So, 3.5 is not a perfect square.

The symbol $\doteq$ means "approximately equal to".

Check

1. Complete the table to find whether each decimal is a perfect square.
The first one is done for you.

	Decimal	Value of square root	Type of decimal	Is decimal a perfect square?
a)	70.5	8.396 427 811 …	Non-repeating Non-terminating	No
b)	5.76	______	______ ______	______
c)	0.25	______	______ ______	______
d)	2.5	______	______ ______	______

Practice

1. Calculate the number whose square root is:

a) $\frac{1}{4}$

$\frac{1}{4} \times \frac{1}{4} = \frac{\_\_ \times \_\_}{\_\_ \times \_\_}$

$= \frac{\_\_}{\_\_}$

$\frac{1}{4}$ is a square root of ____.

b) $\frac{2}{7}$

$\frac{\_\_}{\_\_} \times \frac{\_\_}{\_\_} = \frac{\_\_ \times \_\_}{\_\_ \times \_\_}$

$= \frac{\_\_}{\_\_}$

$\frac{2}{7}$ is a square root of ____.

c) 0.6

_____ × _____ = _____

0.6 is a square root of _____.

d) 1.1

_____ × _____ = _____

1.1 is a square root of _____.

2. Identify the fractions that are perfect squares. The first one has been done for you.

	Fraction	Is numerator a perfect square?	Is denominator a perfect square?	Is fraction a perfect square?
a)	$\frac{81}{125}$	Yes; 9 × 9 = 81	No	No
b)	$\frac{25}{49}$	______	______	______
c)	$\frac{36}{121}$	______	______	______
d)	$\frac{17}{25}$	______	______	______
e)	$\frac{9}{100}$	______	______	______

3. Find each square root.

a) $\sqrt{\frac{49}{100}} = \sqrt{\frac{\_\_\_ \times \_\_\_}{\_\_\_ \times \_\_\_}}$

$= \_\_\_$

b) $\sqrt{\frac{25}{144}} = \sqrt{\frac{\_\_\_ \times \_\_\_}{\_\_\_ \times \_\_\_}}$

$= \_\_\_$

c) $\sqrt{\frac{1}{16}} = \sqrt{\frac{\_\_\_ \times \_\_\_}{\_\_\_ \times \_\_\_}}$

$= \_\_\_$

d) $\sqrt{\frac{9}{400}} = \sqrt{\frac{\_\_\_ \times \_\_\_}{\_\_\_ \times \_\_\_}}$

$= \_\_\_$

4. Use a calculator. Find each square root.

a) $\sqrt{8.41} = \_\_\_$ **b)** $\sqrt{0.0676} = \_\_\_$ **c)** $\sqrt{51.125} = \_\_\_$ **d)** $\sqrt{6.25} = \_\_\_$

5. Which decimals are perfect squares?

a) 1.44

$\sqrt{1.44} = \_\_\_\_\_\_$

The square root is a decimal that ________________.

So, 1.44 ____ a perfect square.

b) 30.25

$\sqrt{30.25} = \_\_\_\_\_\_$

The square root is a decimal that ________________.

So, 30.25 ____ a perfect square.

c) 8.5

$\sqrt{8.5} \doteq \_\_\_\_\_\_$

The square root is a decimal that ________________.

So, 8.5 ____ a perfect square.

d) 0.0256

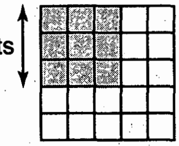
$\sqrt{0.0256} = \_\_\_\_\_\_$

The square root is a decimal that ________________.

So, 0.0256 ____ a perfect square.

6. Find the area of each square.

a) $\frac{\_\_}{\_\_}$ units

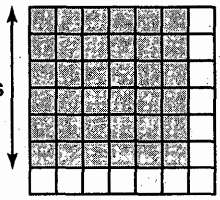


Area = ____

= ____

= ____

b) $\frac{\_\_}{\_\_}$ units



Area = ____

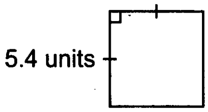
= ____

= ____

Area = (Length)^2

The area is __________ ________________

c)

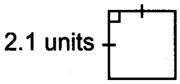


Area = _____

= _____ × _____

= _______

d)



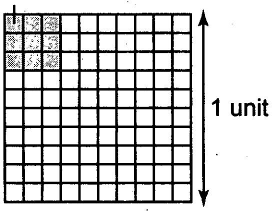
Area = _____

= _____ × _____

= _______

7. Find the side length of each square.

a) Area = $\frac{9}{100}$ square units



Side Length = $\sqrt{}$

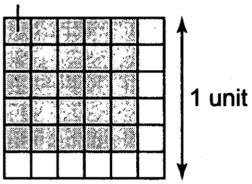
= $\sqrt{}$

= _______

Length = $\sqrt{\text{Area}}$

The side length is ___ units.

b) Area = $\frac{25}{36}$ square units

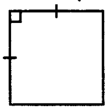


Length = $\sqrt{}$

= $\sqrt{}$

= _______

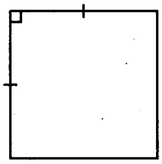
c) Area = 0.01 square units



Length = $\sqrt{}$

= _______

d) Area = 46.24 square units



Length = $\sqrt{}$

= _______

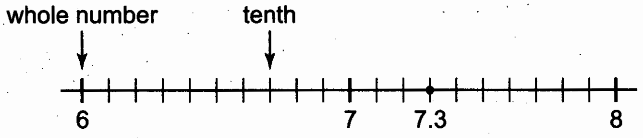
1.2 Skill Builder

Degree of Accuracy

We are often asked to write an answer to a given decimal place.
To do this, we can use a number line.

To write 7.3 to the nearest whole number:
Place 7.3 on a number line in tenths.

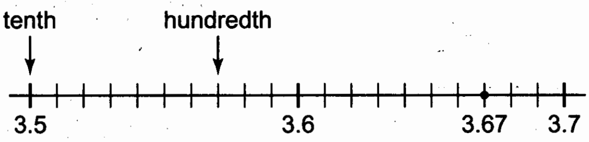
3 is the last digit. It is in the tenths position. So, use a number line in tenths.



7.3 is closer to 7 than to 8.
So, 7.3 to the nearest whole number is: 7

To write 3.67 to the nearest tenth:
Place 3.67 on a number line in hundredths.

7 is the last digit. It is in the hundredths position. So, use a number line in hundredths.

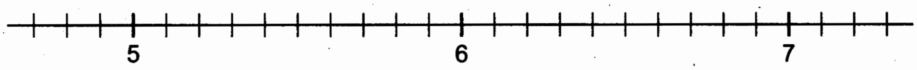


3.67 is closer to 3.7 than to 3.6.
So, 3.67 to the nearest tenth is: 3.7

Check

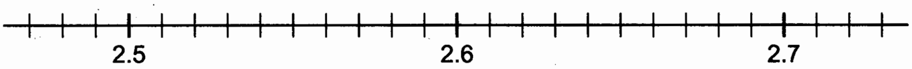
1. Write each number to the nearest whole number.
Mark it on the number line.

a) 5.3 _____ **b)** 6.8 _____ **c)** 7.1 _____ **d)** 6.4 _____



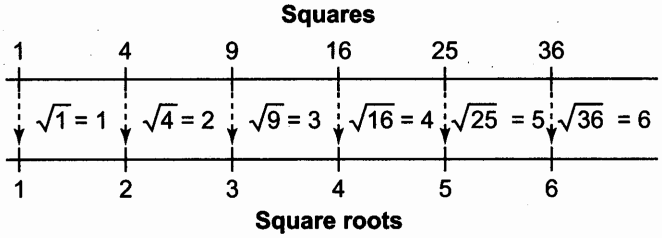
2. Write each number to the nearest tenth.
Mark it on the number line.

a) 2.53 _____ **b)** 2.64 _____ **c)** 2.58 _____ **d)** 2.66 _____

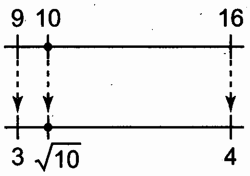


Squares and Square Roots on Number Lines

Most numbers are not perfect squares.
You can use number lines to estimate the square roots of these numbers.



10 is between the perfect squares 9 and 16.
So, $\sqrt{10}$ is between $\sqrt{9}$ and $\sqrt{16}$.
$\sqrt{9} = 3$ and $\sqrt{16} = 4$
So, $\sqrt{10}$ is between 3 and 4.



10 is closer to 9 than 16, so $\sqrt{10}$ is closer to 3 than 4.

Check with a calculator.
$\sqrt{10} \doteq 3.2$, which is between 3 and 4.

Check

1. Between which 2 consecutive whole numbers is each square root? Explain.

Refer to the squares and square roots number lines.

a) $\sqrt{22}$

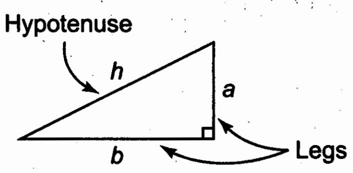
22 is between the perfect squares 16 and 25.
So, $\sqrt{22}$ is between $\sqrt{\_\_\_\_}$ and $\sqrt{\_\_\_\_}$.
$\sqrt{\_\_\_\_} = \_\_\_\_$ and $\sqrt{\_\_\_\_} = \_\_\_\_$
So, $\sqrt{22}$ is between _____ and _____.

b) $\sqrt{6}$

6 is between the perfect squares _____ and _____.
So, $\sqrt{6}$ is between $\sqrt{\_\_\_\_}$ and $\sqrt{\_\_\_\_}$.
$\sqrt{\_\_\_\_} = \_\_\_\_$ and $\sqrt{\_\_\_\_} = \_\_\_\_$
So, $\sqrt{6}$ is between _____ and _____.

The Pythagorean Theorem

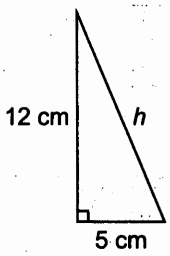
You can use the Pythagorean Theorem to find unknown lengths in right triangles.



Pythagorean Theorem

$h^2 = a^2 + b^2$

To find the length of the hypotenuse, h, in this triangle:



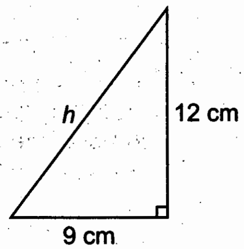
$$h^2 = 5^2 + 12^2$$
$$h^2 = 25 + 144$$
$$h^2 = 169$$
$$h = \sqrt{169}$$
$$h = 13$$

The length of the hypotenuse is 13 cm.

Check

1. Use the Pythagorean Theorem to find the length of each hypotenuse, h.

a)



h^2 = _____ + _____

h^2 = _____ + _____

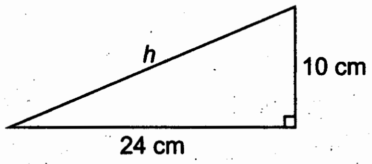
h^2 = _____

h = $\sqrt{}$_____

h = _____

The length of the hypotenuse is _____ cm.

b)



h^2 = _____ + _____

h^2 = _____ + _____

h^2 = _____

h = $\sqrt{}$_____

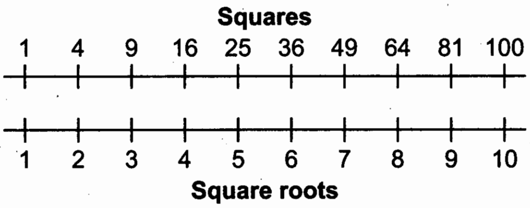
h = _____

The length of the hypotenuse is _____ cm.

1.2 Square Roots of Non-Perfect Squares

FOCUS **Approximate the square roots of decimals and fractions that are not perfect squares.**

The top number line shows all the perfect squares from 1 to 100.



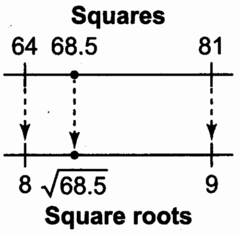
The bottom number line shows the square root of each number in the top line. You can use these lines to estimate the square roots of fractions and decimals that are not perfect squares.

Example 1 Estimating a Square Root of a Decimal

Estimate: $\sqrt{68.5}$

Solution

68.5 is between the perfect squares 64 and 81.
So, $\sqrt{68.5}$ is between $\sqrt{64}$ and $\sqrt{81}$.
That is, $\sqrt{68.5}$ is between 8 and 9.
Since 68.5 is closer to 64 than 81, $\sqrt{68.5}$ is closer to 8 than 9.
So, $\sqrt{68.5}$ is between 8 and 9, and closer to 8.



Check

1. Estimate each square root.
Explain your estimate.

a) $\sqrt{13.5}$

13.5 is between the perfect squares ____ and ____.
So, $\sqrt{13.5}$ is between $\sqrt{\quad}$ and $\sqrt{\quad}$.
That is, $\sqrt{13.5}$ is between ____ and ____.
Since 13.5 is closer to ____ than ____, $\sqrt{13.5}$ is closer to ____ than ____.
So, $\sqrt{13.5}$ is between ____ and ____, and closer to ____.

b) $\sqrt{51.5}$

51.5 is between the perfect squares ______ and ______.
So, $\sqrt{51.5}$ is between $\sqrt{\_\_\_\_}$ and $\sqrt{\_\_\_\_}$.
That is, $\sqrt{51.5}$ is between ____ and ____.
Since 51.5 is closer to ____ than ____, $\sqrt{51.5}$ is closer to ____ than ____.
So, $\sqrt{51.5}$ is between ____ and ____, and closer to ____.

Example 2 Estimating a Square Root of a Fraction

Estimate: $\sqrt{\frac{3}{10}}$

Solution

Find the closest perfect square to the numerator and denominator.

In the fraction $\frac{3}{10}$:
3 is close to the perfect square 4.
10 is close to the perfect square 9.

So, $\sqrt{\frac{3}{10}} \doteq \sqrt{\frac{4}{9}}$ and $\sqrt{\frac{4}{9}} = \frac{2}{3}$

So, $\sqrt{\frac{3}{10}} \doteq \frac{2}{3}$

Check

1. Estimate each square root.

a) $\sqrt{\frac{23}{80}}$

23 is close to the perfect square ____.
80 is close to the perfect square ____.

So, $\sqrt{\frac{23}{80}} \doteq \sqrt{\frac{\_\_\_\_}{\_\_\_\_}}$

$\sqrt{\frac{\_\_\_\_}{\_\_\_\_}} = \frac{\_\_\_}{\_\_\_}$

So, $\sqrt{\frac{23}{80}} \doteq$ ___

b) $\sqrt{\frac{8}{17}}$

8 is close to the perfect square ____.
17 is close to the perfect square ____.

So, $\sqrt{\frac{8}{17}} \doteq \sqrt{\frac{\_\_\_\_}{\_\_\_\_}}$

$\sqrt{\frac{\_\_\_\_}{\_\_\_\_}} = \frac{\_\_\_}{\_\_\_}$

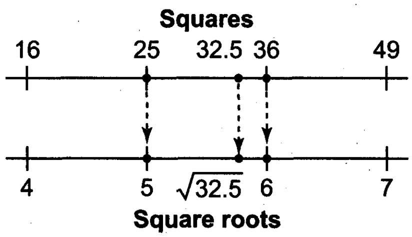
So, $\sqrt{\frac{8}{17}} \doteq$ _____

Example 3 Finding a Number with a Square Root between Two Given Numbers

Identify a decimal that has a square root between 5 and 6.

Solution

$5^2 = 25$, so 5 is a square root of 25.
$6^2 = 36$, so 6 is a square root of 36.
So, any decimal between 25 and 36 has a square root between 5 and 6.
Choose 32.5.



Check the answer by using a calculator.
$\sqrt{32.5} \doteq 5.7$, which is between 5 and 6.
So, the decimal 32.5 is one correct answer.
There are many more correct answers.

Check

1. a) Identify a decimal that has a square root between 7 and 8.
Check the answer.
$7^2 =$ _____ and $8^2 =$ _____
So, any decimal between _____ and _____ has a square root between 7 and 8.
Choose _____.
Check the answer on a calculator.
$\sqrt{\_\_\_\_} \doteq$ _____
The decimal _____ is one correct answer.

b) Identify a decimal that has a square root between 11 and 12.
_____ = _____ and _____ = _____
So, any decimal between _____ and _____ has a square root between 11 and 12.
Choose ______.
$\sqrt{\_\_\_\_} \doteq$ _____
So, ______ is one correct answer.

Practice

1. For each number, name the 2 closest perfect squares and their square roots.

	Number	Two closest perfect squares	Their square roots
a)	44.4	____ and ____	____ and ____
b)	10.8	____ and ____	____ and ____
c)	125.9	____ and ____	____ and ____
d)	87.5	____ and ____	____ and ____

2. For each fraction, name the closest perfect square and its square root for the numerator and for the denominator.

	Fraction	Closest perfect squares	Their square roots
a)	$\frac{5}{11}$	Numerator: ____; denominator: ____	____ and ____
b)	$\frac{17}{45}$	Numerator: ____; denominator: ____	____ and ____
c)	$\frac{3}{24}$	Numerator: ____; denominator: ____	____ and ____
d)	$\frac{11}{62}$	Numerator: ____; denominator: ____	____ and ____

3. Estimate each square root.
Explain.

a) $\sqrt{1.6}$

1.6 is between ____ and ____.
So, $\sqrt{1.6}$ is between $\sqrt{\_\_\_\_}$ and $\sqrt{\_\_\_\_}$.
That is, $\sqrt{1.6}$ is between ____ and ____.
Since 1.6 is closer to ____ than ____, $\sqrt{1.6}$ is closer to ____ than ____.
So, $\sqrt{1.6}$ is between ____ and ____, and closer to ____.

b) $\sqrt{44.5}$

44.5 is between ____ and ____.
So, $\sqrt{44.5}$ is between $\sqrt{\_\_\_\_}$ and $\sqrt{\_\_\_\_}$.
That is, $\sqrt{44.5}$ is between ____ and ____.
Since 44.5 is closer to ____ than ____, $\sqrt{44.5}$ is closer to ____ than ____.
So, $\sqrt{44.5}$ is between ____ and ____, and closer to ____.

c) $\sqrt{75.8}$

75.8 is between ____ and ____.

So, $\sqrt{75.8}$ is between $\sqrt{\_\_\_\_}$ and $\sqrt{\_\_\_\_}$.

That is, $\sqrt{75.8}$ is between ____ and ____.

Since 75.8 is closer to ____ than ____, $\sqrt{75.8}$ is closer to ____ than ____.

So, $\sqrt{75.8}$ is between ____ and ____, and closer to ____.

4. Estimate each square root. Explain.

a) $\sqrt{\frac{7}{15}}$

7 is close to ____; 15 is close to ____.

So, $\sqrt{\frac{7}{15}} \doteq \sqrt{\frac{\_\_\_}{\_\_\_}}$

$\doteq$ ____

b) $\sqrt{\frac{2}{7}}$

2 is close to ____; 7 is close to ____.

So, $\sqrt{\frac{2}{7}} \doteq \sqrt{\frac{\_\_\_}{\_\_\_}}$

$\doteq$ ____

c) $\sqrt{\frac{35}{37}}$

35 is close to ____; 37 is close to ____.

So, $\sqrt{\frac{35}{37}} \doteq \sqrt{\frac{\_\_\_}{\_\_\_}}$

$\doteq$ ____

d) $\sqrt{\frac{99}{122}}$

99 is close to ____; 122 is close to ____.

So, $\sqrt{\frac{99}{122}} \doteq \sqrt{\frac{\_\_\_}{\_\_\_}}$

$\doteq$ ____

5. Identify a decimal that has a square root between the two given numbers. Check the answer.

a) 1 and 2

$1^2 =$ _____ and $2^2 =$ _____

So, any number between _____ and _____ has a square root between 1 and 2.

Choose _____.

Check: $\sqrt{\_\_\_\_\_} \doteq$ _____

The decimal _____ is one possible answer.

b) 8 and 9

$8^2 =$ _____ and $9^2 =$ _____

So, any number between _____ and _____ has a square root between 8 and 9.

Choose _____.

Check: $\sqrt{\_\_\_\_\_} \doteq$ _____

The decimal _____ is one possible answer.

c) 2.5 and 3.5

_______ = _______ and _______ = _______

So, any number between _______ and _______ has a square root between 2.5 and 3.5.

Choose _______.

Check: $\sqrt{\_\_\_\_\_\_\_}$ ≐ _______

The decimal _______ is one correct answer.

d) 20 and 21

_______ = _______ and _______ = _______

So, any number between _______ and _______ has a square root between 20 and 21.

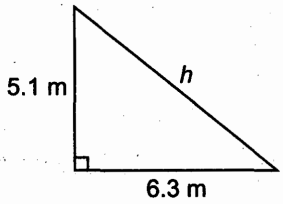
Choose _______.

Check: $\sqrt{\_\_\_\_\_\_\_}$ ≐ _______

The decimal _______ is one correct answer.

6. Determine the length of the hypotenuse in each right triangle.
Write each answer to the nearest tenth.

a)



$h^2 = 5.1^2 + 6.3^2$

h^2 = _____ + _____

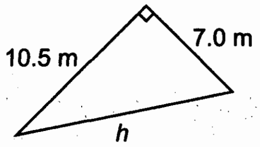
h^2 = _____

$h = \sqrt{\_\_\_\_\_}$

h ≐ _____

So, h is about _____ m.

b)



h^2 = _______ + _______

h^2 = _______ + _______

h^2 = _______

$h = \sqrt{\_\_\_\_\_\_\_}$

h ≐ _______

So, h is about _______ m.

CHECKPOINT

Can you ...

- Identify decimals and fractions that are perfect squares?
- Find the square roots of decimals and fractions that are perfect squares?
- Approximate the square roots of decimals and fractions that are not perfect squares?

1.1 **1.** Calculate the number whose square root is:

a) $\frac{2}{7}$

$\frac{2}{7} \times \frac{2}{7} = \frac{\_\_}{\_\_}$

$\frac{2}{7}$ is a square root of ____.

b) $\frac{8}{11}$

$\frac{\_\_}{\_\_} \times \frac{\_\_}{\_\_} = \frac{\_\_}{\_\_}$

$\frac{8}{11}$ is a square root of ____.

c) 0.1

____ × ____ = ____

0.1 is a square root of ____.

d) 1.4

1.4 × 1.4 = ____

1.4 is a square root of ____.

2. Identify the fractions that are perfect squares.
The first one has been done for you.

	Fraction	Is numerator a perfect square?	Is denominator a perfect square?	Is fraction a perfect square?
a)	$\frac{64}{75}$	Yes; 8 × 8 = 64	No	No
b)	$\frac{9}{25}$	____	____	____
c)	$\frac{25}{55}$	____	____	____

3. Find each square root.

a) $\sqrt{\frac{9}{49}} = \sqrt{\frac{\_\_ \times \_\_}{\_\_ \times \_\_}}$
$= \frac{\_\_}{\_\_}$

b) $\sqrt{\frac{16}{25}} = \sqrt{\frac{\_\_ \times \_\_}{\_\_ \times \_\_}}$
$= \frac{\_\_}{\_\_}$

c) $\sqrt{\frac{36}{121}} = \sqrt{\frac{\_\_ \times \_\_}{\_\_ \times \_\_}}$
$= \frac{\_\_}{\_\_}$

4. a) Put a check mark beside each decimal that is a perfect square.

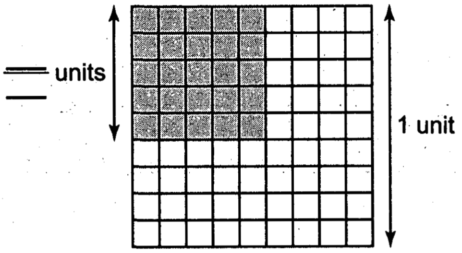
i) 4.84 ____ **ii)** 3.63 ____ **iii)** 98.01 ____ **iv)** 67.24 ____

b) Explain how you identified the perfect squares in part a.

____

____

5. a) Find the area of the shaded square.



Area = (Length)^2

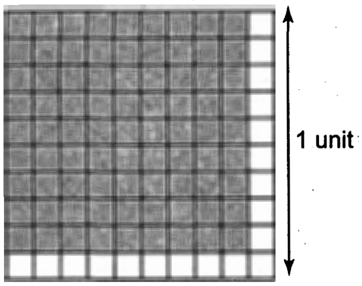
$= (\_\_)^2$

$= \_\_ \times \_\_$

$= \_\_$

The area is ____ square units.

b) Find the side length of the shaded square.



Length = $\sqrt{\text{Area}}$

$= \sqrt{\_\_}$

$= \sqrt{\_\_ \times \_\_}$

$= \_\_$

The side length is ____ units.

1.2 **6.** Estimate each square root.
Explain.

a) $\sqrt{7.5}$

7.5 is between _____ and _____.
So, $\sqrt{7.5}$ is between $\sqrt{\_\_\_\_\_}$ and $\sqrt{\_\_\_\_\_}$.
That is, $\sqrt{7.5}$ is between _____ and _____.
Since 7.5 is closer to _____ than _____, $\sqrt{7.5}$ is closer to _____ than _____.
So, $\sqrt{7.5}$ is between _____ and _____, and closer to _____.

b) $\sqrt{66.6}$

66.6 is between _____ and _____.
So, $\sqrt{66.6}$ is between $\sqrt{\_\_\_\_\_}$ and $\sqrt{\_\_\_\_\_}$.
That is, $\sqrt{66.6}$ is between _____ and _____.
Since 66.6 is closer to _____ than _____, $\sqrt{66.6}$ is closer to _____ than _____.
So, $\sqrt{66.6}$ is between _____ and _____, and closer to _____.

7. Estimate each square root.

a) $\sqrt{\frac{15}{79}}$

15 is close to ____; 79 is close to ____.

So, $\sqrt{\frac{15}{79}} \doteq \sqrt{\frac{\_\_\_\_}{\_\_\_\_}}$

$\doteq \frac{\_\_\_\_}{\_\_\_\_}$

b) $\sqrt{\frac{23}{50}}$

23 is close to ____; 50 is close to ____.

So, $\sqrt{\frac{23}{50}} \doteq \sqrt{\frac{\_\_\_\_}{\_\_\_\_}}$

$\doteq \frac{\_\_\_\_}{\_\_\_\_}$

8. Identify a decimal whose square root is between the given numbers.
Check your answer.

a) 2 and 3

$2^2 =$ _____ and $3^2 =$ _____

So, any number between _____ and _____ has a square root between 2 and 3.

Choose _____.

Check: $\sqrt{\_\_\_\_} \doteq$ _____

The decimal _____ is one correct answer.

b) 6 and 7

$6^2 =$ _____ and $7^2 =$ _____

So, any number between _____ and _____ has a square root between 6 and 7.

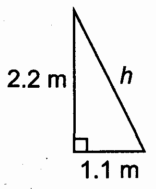
Choose _____.

$\sqrt{\_\_\_\_} \doteq$ _____

The decimal _____ is one correct answer.

9. Find the length of each hypotenuse.

a)



$h^2 =$ _______ + _______

$h^2 =$ _______ + _______

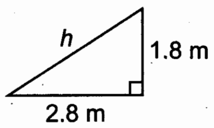
$h^2 =$ _______

$h = \sqrt{\_\_\_\_\_\_\_}$

$h \doteq$ _______

The length of the hypotenuse is about _______ m.

b)



$h^2 =$ _______ + _______

$h^2 =$ _______ + _______

$h^2 =$ _______

$h = \sqrt{\_\_\_\_\_\_\_}$

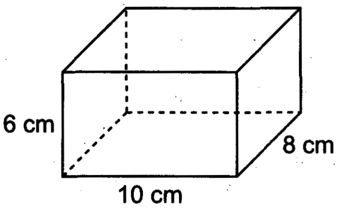
$h \doteq$ _______

The length of the hypotenuse is about _______ m.

1.3 Skill Builder

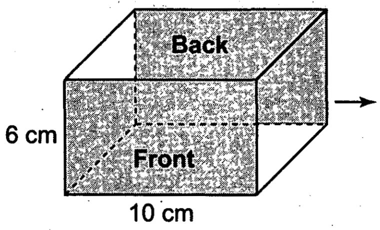
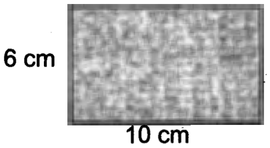
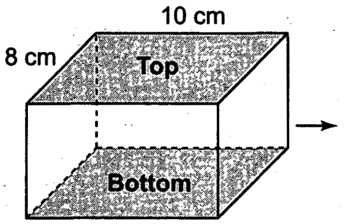
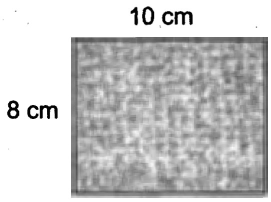
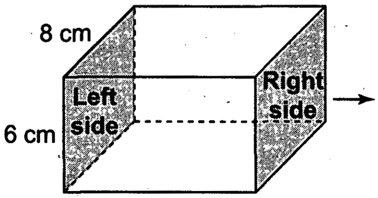
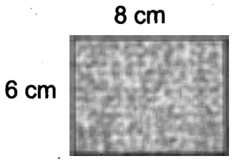
Surface Areas of Rectangular Prisms

The **surface area** of a rectangular prism is the sum of the areas of its 6 rectangular faces. Look for matching faces with the same areas.



The matching faces in each pair have the same area. We find the area of one face and multiply by 2.

For each rectangular face, area equals its length times its width.

Matching Faces	Diagram	Corresponding Area (cm^2)
		$2(10 \times 6) = 120$
		$2(10 \times 8) = 160$
		$2(8 \times 6) = 96$
Total		376

The surface area is 376 cm^2.

2.1 Skill Builder

Multiplying Integers

When multiplying 2 integers, look at the sign of each integer:

- When the integers have the same sign, their product is positive.
- When the integers have different signs, their product is negative.

×	(−)	(+)
(−)	(+)	(−)
(+)	(−)	(+)

$6 \times (-3)$ These 2 integers have different signs, so their product is negative.

$6 \times (-3) = -18$

$(-10) \times (-2)$ These 2 integers have the same sign, so their product is positive.

$(-10) \times (-2) = 20$

When an integer is positive, we do not have to write the + sign in front.

Check

1. Will the product be positive or negative?

a) 7×4 __________ **b)** $3 \times (-6)$ __________

c) $(-9) \times 10$ __________ **d)** $(-5) \times (-9)$ __________

2. Multiply.

a) $7 \times 4 =$ ______ **b)** $3 \times (-6) =$ ______

c) $(-9) \times 10 =$ ______ **d)** $(-5) \times (-9) =$ ______

e) $(-3) \times (-5) =$ ______ **f)** $2 \times (-5) =$ ______

g) $(-8) \times 2 =$ ______ **h)** $(-4) \times 3 =$ ______

Multiplying More than 2 Integers

We can multiply more than 2 integers.
Multiply pairs of integers, from left to right.

$\underbrace{(-1) \times (-2)} \times (-3)$
$= 2 \times (-3)$
$= -6$

The product of 3 negative factors is negative.

$\underbrace{(-1) \times (-2)} \times (-3) \times (-4)$
$= \underbrace{2 \times (-3)} \times (-4)$
$= (-6) \times (-4)$
$= 24$

The product of 4 negative factors is positive.

Multiplying Integers
When the number of negative factors is *even*, the product is positive.
When the number of negative factors is *odd*, the product is negative.

We can show products of integers in different ways:
$(-2) \times (-2) \times 3 \times (-2)$ is the same as $(-2)(-2)(3)(-2)$.

So, $(-2) \times (-2) \times 3 \times (-2) = (-2)(-2)(3)(-2)$
$= -24$

Check

1. Multiply.

a) $(-3) \times (-2) \times (-1) \times 1$ ______

b) $(-2)(-1)(-2)(-2)(2)$ ______

c) $(-2)(-2)(-1)(-2)(-2)$ ______

d) $3 \times 3 \times 2$ ______

Is the answer positive or negative? How can you tell?

2.1 What Is a Power?

FOCUS **Show repeated multiplication as a power.**

We can use powers to show repeated multiplication.

$$\underbrace{2 \times 2 \times 2 \times 2 \times 2}_{\text{Repeated multiplication, 5 factors of 2}} = \underbrace{2^5}_{\text{Power}}$$

Repeated multiplication
5 **factors** of 2

Power

*2 is the **base**.*
*5 is the **exponent**.*
*2^5 is a **power**.*

We read 2^5 as "2 to the 5th."
Here are some other powers of 2.

Repeated Multiplication	Power	Read as...
$\underbrace{2}_{\text{1 factor of 2}}$	2^1	2 to the 1st
$\underbrace{2 \times 2}_{\text{2 factors of 2}}$	2^2	2 to the 2nd, or 2 squared
$\underbrace{2 \times 2 \times 2}_{\text{3 factors of 2}}$	2^3	2 to the 3rd, or 2 cubed
$\underbrace{2 \times 2 \times 2 \times 2}_{\text{4 factors of 2}}$	2^4	2 to the 4th

In each case, the exponent in the power is equal to the number of factors in the repeated multiplication.

Example 1 Writing Powers

Write as a power.

a) $4 \times 4 \times 4 \times 4 \times 4 \times 4$

b) 3

Solution

a) The base is 4.

$$\underbrace{4 \times 4 \times 4 \times 4 \times 4 \times 4}_{\text{6 factors of 4}} = 4^6$$

So, $4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^6$

b) The base is 3.

$$\underbrace{3}_{\text{1 factor of 3}}$$

So, $3 = 3^1$

Check

1. Write as a power.

a) $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^{\_\_\_}$

b) $5 \times 5 \times 5 \times 5 = 5^{\_\_\_}$

c) $(-10)(-10)(-10) =$ _______

d) $4 \times 4 =$ _____

e) $(-7)(-7)(-7)(-7)(-7)(-7)(-7)(-7) =$ _______

2. Complete the table.

	Repeated Multiplication	Power	Read as…
a)	$8 \times 8 \times 8 \times 8$	______	8 to the 4th
b)	7×7	______	____________
c)	$3 \times 3 \times 3 \times 3 \times 3 \times 3$	______	3 to the 6th
d)	$2 \times 2 \times 2$	______	____________

Power	Repeated Multiplication	Standard Form
2^5	$2 \times 2 \times 2 \times 2 \times 2$	32

Example 2 Evaluating Powers

Write as repeated multiplication and in standard form.

a) 2^4

b) 5^3

Solution

a) $2^4 = 2 \times 2 \times 2 \times 2$ As repeated multiplication
$= 16$ Standard form

b) $5^3 = 5 \times 5 \times 5$ As repeated multiplication
$= 125$ Standard form

Check

1. Complete the table.

Power	Repeated Multiplication	Standard Form
2^3	$2 \times 2 \times 2$	_____
6^2	_______________	36
3^4	_______________	_____
10^4	_______________	_____
8 squared	_______________	_____
7 cubed	_______________	_____

To evaluate a power that contains negative integers, identify the base of the power. Then, apply the rules for multiplying integers.

Example 3 Evaluating Expressions Involving Negative Signs

Identify the base, then evaluate each power.

a) $(-5)^4$

b) -5^4

Solution

a) $(-5)^4$

The brackets tell us that the base of this power is (-5).

$$(-5)^4 = (-5) \times (-5) \times (-5) \times (-5)$$
$$= 625$$

There is an even number of negative integers, so the product is positive.

b) -5^4

There are no brackets. So, the base of this power is 5. The negative sign applies to the whole expression.

$$-5^4 = -(5 \times 5 \times 5 \times 5)$$
$$= -625$$

Check

1. Identify the base of each power, then evaluate.

a) $(-1)^3$

The base is _____.

$(-1)^3 =$ ____________

$=$ ____________

b) -10^3

The base is _____.

$-10^3 =$ ____________

$=$ ____________

c) $(-7)^2$

The base is _____.

$(-7)^2 =$ ____________

$=$ ____________

d) $-(-5)^4$

The base is _____.

$-(-5)^4 =$ ____________

$=$ ____________

The first negative sign applies to the whole expression.

Practice

1. Write as a power.

a) $\underbrace{8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8}_{\text{7 factors of 8}}$

The base is 8. There are _____ equal factors, so the exponent is _____.

$8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 = 8^{\_\_}$

b) $\underbrace{10 \times 10 \times 10 \times 10 \times 10}_{\text{5 factors of 10}}$

The base is _____. There are _____ equal factors, so the exponent is _____.

So, $10 \times 10 \times 10 \times 10 \times 10 =$ _____

c) $\underbrace{(-2)(-2)(-2)}_{\text{3 factors of \_\_\_\_\_}}$

The base is _____. There are _____ equal factors, so the exponent is _____.

So, $(-2)(-2)(-2) =$ _______

d) $(-13)(-13)(-13)(-13)(-13)(-13)$

_______ factors of _______

The base is _______. There are _______ equal factors, so the exponent is _______.

So, $(-13)(-13)(-13)(-13)(-13)(-13) =$ _______

2. Write each expression as a power.

a) $9 \times 9 \times 9 \times 9 = \_\_\_\_\_^{4}$

b) $(5)(5)(5)(5)(5)(5) = 5^{\_\_}$

c) $11 \times 11 =$ _____

d) $(-12)(-12)(-12)(-12)(-12) =$ _______

3. Write each power as repeated multiplication.

a) $3^2 =$ ____________ **b)** $3^4 =$ ____________

c) $2^7 =$ ______________________

d) $10^8 =$ __________________________________

Identify the base first.

4. State whether the answer will be positive or negative.

a) $(-3)^2$ ____________ **b)** 6^3 ____________

c) $(-10)^3$ ____________ **d)** -4^3 ____________

5. Write each power as repeated multiplication and in standard form.

Predict. Will the answer be positive or negative?

a) $(-3)^2 =$ ____________
$=$ ______

b) $6^3 =$ ______________
$=$ ______

c) $(-10)^3 =$ ______________
$=$ ______________

d) $-4^3 =$ ______________
$=$ ______

6. Write each product as a power and in standard form.

a) $(-3)(-3)(-3) =$ ______
$=$ ______

b) $(-8)(-8) =$ ______
$=$ ______

c) $-(8 \times 8 \times 8) =$ ______
$=$ ______

d) $-(-1)(-1)(-1)(-1)(-1)(-1)(-1) =$ ______
$=$ ______

7. Identify any errors and correct them.

a) $4^3 = 12$ ______________________________
____

b) $(-2)^9$ is negative. ______________________________
____

c) $(-9)^2$ is negative. ______________________________
____

d) $3^2 = 2^3$ ______________________________
____

e) $(-10)^2 = 100$ ______________________________
____

2.2 Skill Builder

Patterns and Relationships in Tables

Look at the patterns in this table.

Input		Output
1	×2 →	2
2	×2 →	4
3	×2 →	6
4	×2 →	8
5	×2 →	10

+1 +1 +1 +1

+2 +2 +2 +2

The input starts at 1 and increases by 1 each time.

The output starts at 2 and increases by 2 each time.

The input and output are also related.
Double the input to get the output.

Check

1. a) Describe the patterns in the table.
b) What is the input in the last row?
What is the output in the last row?

Input	Output
1	5
2	10
3	15
4	20
_____	_____

+1 +5

a) The input starts at _____, and increases by _____ each time.
The output starts at _____, and increases by _____ each time.
You can also multiply the input by _____ to get the output.

b) The input in the last row is 4 + _____ = _____.
The output in the last row is 20 + _____ = _____.

2. a) Describe the patterns in the table.

b) Extend the table 3 more rows.

Input	Output
10	100
9	90
8	80
7	70
6	60

a) The input starts at 10, and decreases by _____ each time.
The output starts at 100, and decreases by _____ each time.
You can also multiply the input by _____ to get the output.

b) To extend the table 3 more rows, continue to decrease the input by _____ each time.
Decrease the output by _____ each time.

Input	Output
5	_____
_____	_____
_____	_____

Writing Numbers in Expanded Form

8000 is 8 thousands, or 8×1000
600 is 6 hundreds, or 6×100
50 is 5 tens, or 5×10

Read it aloud.

Check

1. Write each number in expanded form.

a) 7000 __________

b) 900 __________

c) 400 __________

d) 30 __________

2.2 Powers of Ten and the Zero Exponent

FOCUS **Explore patterns and powers of 10 to develop a meaning for the exponent 0.**

This table shows decreasing powers of 3.

Power	Repeated Multiplication	Standard Form
3^5	$3 \times 3 \times 3 \times 3 \times 3$	243
3^4	$3 \times 3 \times 3 \times 3$	81
3^3	$3 \times 3 \times 3$	27
3^2	3×3	9
3^1	3	3

$\div 3$

$\div 3$

$\div 3$

$\div 3$

Look for patterns in the columns.

The exponent decreases by 1 each time. Divide by 3 each time.

The patterns suggest $3^0 = 1$ because $3 \div 3 = 1$.

We can make a similar table for the powers of any integer base except 0.

The Zero Exponent
A power with exponent 0 is equal to 1.

The base of the power can be any integer except 0.

Example 1 Powers with Exponent Zero

Evaluate each expression.

a) 6^0 **b)** $(-5)^0$

Solution

A power with exponent 0 is equal to 1.

a) $6^0 = 1$ **b)** $(-5)^0 = 1$

The zero exponent applies to the number in the brackets.

Check

1. Evaluate each expression.

a) $8^0 =$ _____ **b)** $-4^0 =$ _____

c) $4^0 =$ _____ **d)** $(-10)^0 =$ _____

If there are no brackets, the zero exponent applies only to the base.

Example 2 Powers of Ten

Write as a power of 10.

a) 10 000 **b)** 1000 **c)** 100 **d)** 10 **e)** 1

Solution

a) $10\ 000 = 10 \times 10 \times 10 \times 10$
$= 10^4$

b) $1000 = 10 \times 10 \times 10$
$= 10^3$

c) $100 = 10 \times 10$
$= 10^2$

d) $10 = 10^1$

e) $1 = 10^0$

Notice that the exponent is equal to the number of zeros.

Check

1. a) $5^1 =$ _____ **b)** $(-7)^1 =$ _____

c) $10^1 =$ _____ **d)** $10^0 =$ _____

Practice

1. a) Complete the table below.

Power	Repeated Multiplication	Standard Form
5^4	$5 \times 5 \times 5 \times 5$	625
5^3	$5 \times 5 \times 5$	_____
5^2	_______________	_____
5^1	_______________	_____

b) What is the value of 5^1? _____

c) Use the table. What is the value of 5^0? _____

2. Evaluate each power.

a) $2^0 =$ _____ **b)** $9^0 =$ _____

c) $(-2)^0 =$ _____ **d)** $-2^0 =$ _____

e) $10^1 =$ _____ **f)** $(-8)^1 =$ _____

If there are no brackets, the exponent applies only to the base.

3. Write each number as a power of 10.

a) 10 000 = 10___ **b)** 1 000 000 = 10___

c) Ten million = _____ **d)** One = _____

e) 1 000 000 000 = _____ **f)** 10 = _____

4. Evaluate each power of 10.

a) $-10^6 =$ ______________ **b)** $-10^0 =$ _____

c) $-10^8 =$ ______________ **d)** $-10^1 =$ _____

5. One trillion is written as 1 000 000 000 000.
Write each number as a power of 10.

a) One trillion = 1 000 000 000 000 = ______

b) Ten trillion = 10 × ________________ = ______

c) One hundred trillion = ____________________ = ______

6. Write each number in standard form.

a) $5 \times 10^4 = 5 \times 10\,000$
= ______________

b) $(4 \times 10^2) + (3 \times 10^1) + (7 \times 10^0) = (4 \times 100) +$ ______________
= ______________
= ______________

c) $(2 \times 10^3) + (6 \times 10^2) + (4 \times 10^1) + (9 \times 10^0)$
= ______________________________
= ______________________________
= ______________________________

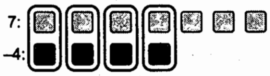
d) $(7 \times 10^3) + (8 \times 10^0) =$ ______________
= ______________
= ______________

2.3 Skill Builder

Adding Integers

To add a positive integer and a negative integer: $7 + (-4)$

- Model each integer with tiles.
- Circle zero pairs.



There are 4 zero pairs.
There are 3 ▩ tiles left.
They model 3.
So, $7 + (-4) = 3$

Each pair of 1 ■ tile and 1 ▩ tile makes a zero pair. The pair models 0.

To add 2 negative integers: $(-4) + (-2)$

- Model each integer with tiles.
- Combine the tiles.

There are 6 ■ tiles.
They model -6.
So, $(-4) + (-2) = -6$

Check

1. Add.

a) $(-3) + (-4) =$ _____ **b)** $6 + (-2) =$ _____

c) $(-5) + 2 =$ _____ **d)** $(-4) + (-4) =$ _____

2. a) Kerry borrows \$5. Then she borrows another \$5.
Add to show what Kerry owes.
$(-5) + (-5) =$ _____ Kerry owes \$_____.

When an amount of money is negative, it is owed.

b) The temperature was 8°C. It fell 10°C.
Add to show the new temperature.
$8 + ($_____$) =$ _____ The new temperature is _____°C.

Subtracting Integers

To subtract 2 integers: $3 - 6$

- Model the first integer.
- Take away the number of tiles equal to the second integer.

Model 3.

Adding zero pairs does not change the value. Zero pairs represent 0.

There are not enough tiles to take away 6.
To take away 6, we need 3 more ▩ tiles.
We add zero pairs. Add 3 ▩ tiles and 3 ■ tiles.

Now take away the 6 ▩ tiles.

Since 3 ■ tiles remain, we write: $3 - 6 = -3$

When tiles are not available, think of subtraction as the opposite of addition.
To subtract an integer, add its opposite integer.
For example,

$(-3) - (+2) = -5$ — Subtract +2.

$(-3) + (-2) = -5$ — Add –2.

Check

1. Subtract.

a) $(-6) - 2 =$ _____

b) $2 - (-6) =$ _____

c) $(-8) - 9 =$ _____

d) $8 - (-9) =$ _____

Dividing Integers

When dividing 2 integers, look at the sign of each integer:

- When the integers have the same sign, their quotient is positive.
- When the integers have different signs, their quotient is negative.

The same rule applies to the multiplication of integers.

$6 \div (-3)$ These 2 integers have different signs, so their quotient is negative.

$6 \div (-3) = -2$

$(-10) \div (-2)$ These 2 integers have the same sign, so their quotient is positive.

$(-10) \div (-2) = 5$

Check

1. Calculate.

a) $(-4) \div 2$

$=$ _____

b) $(-6) \div (-3)$

$=$ _____

c) $15 \div (-3)$

$=$ _____

2.3 Order of Operations with Powers

FOCUS **Explain and apply the order of operations with exponents.**

We use this order of operations when evaluating an expression with powers:

- Do the operations in brackets first.
- Evaluate the powers.
- Multiply and divide, in order, from left to right.
- Add and subtract, in order, from left to right.

We can use the word BEDMAS to help us remember the order of operations:

B **B**rackets
E **E**xponents
D **D**ivision
M **M**ultiplication
A **A**ddition
S **S**ubtraction

Example 1 Adding and Subtracting with Powers

Evaluate.

a) $2^3 + 1$ **b)** $8 - 3^2$ **c)** $(3 - 1)^3$

Solution

a) $2^3 + 1$ — Evaluate the power first: 2^3
$= (2)(2)(2) + 1$ — Multiply: $(2)(2)(2)$
$= 8 + 1$ — Then add: $8 + 1$
$= 9$

b) $8 - 3^2$ — Evaluate the power first: 3^2
$= 8 - (3)(3)$ — Multiply: $(3)(3)$
$= 8 - 9$ — Then subtract: $8 - 9$
$= -1$

To subtract, add the opposite: $8 + (-9)$

c) $(3 - 1)^3$ — Subtract inside the brackets first: $3 - 1$
$= 2^3$ — Evaluate the power: 2^3
$= (2)(2)(2)$ — Multiply: $(2)(2)(2)$
$= 8$

Check

1. Evaluate.

a) $4^2 + 3 =$ _____ $+ 3$
$=$ _____
$=$ _____

b) $5^2 - 2^2 =$ _____ $- (2)(2)$
$=$ _____
$=$ _____

c) $(2 + 1)^2 =$ _____^{2}
$=$ _____
$=$ _____

d) $(5 - 6)^2 =$ _____
$=$ _____
$=$ _____

Example 2 Multiplying and Dividing with Powers

Evaluate.

a) $[2 \times (-2)^3]^2$

Curved brackets   Square brackets

b) $(7^2 + 5^0) \div (-5)^1$

When we need curved brackets for integers, we use square brackets to show the order of operations.

Solution

a) $[2 \times (-2)^3]^2$ — Evaluate what is inside the square brackets first: $2 \times (-2)^3$
$= [2 \times (-8)]^2$ — Start with $(-2)^3 = -8$.
$= (-16)^2$
$= 256$

b) $(7^2 + 5^0) \div (-5)^1$ — Evaluate what is inside the brackets first: $7^2 + 5^0$
$= (49 + 1) \div (-5)^1$ — Add inside the brackets: $49 + 1$
$= 50 \div (-5)^1$ — Evaluate the power: $(-5)^1$
$= 50 \div (-5)$
$= -10$

Check

1. Evaluate.

a) $5 \times 3^2 = 5 \times \underline{\qquad}\ \underline{\qquad}$
$= 5 \times \underline{\qquad}$
$= \underline{\qquad}$

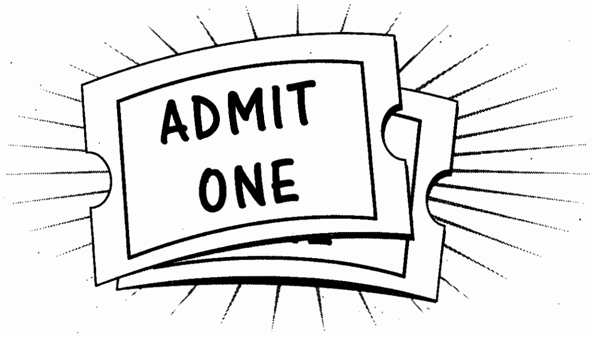
b) $8^2 \div 4 = \underline{\qquad}\ \underline{\qquad} \div 4$
$= \underline{\qquad} \div 4$
$= \underline{\qquad}$

c) $(3^2 + 6^0)^2 \div 2^1$
$= (\underline{\qquad} + \underline{\qquad})^2 \div 2^1$
$= \underline{\qquad} \div 2^1$
$= \underline{\qquad} \div \underline{\qquad}$
$= \underline{\qquad}$

d) $10^2 + (2 \times 2^2)^2 = 10^2 + (2 \times \underline{\qquad})^2$
$= 10^2 + \underline{\qquad}$
$= \underline{\qquad} + \underline{\qquad}$
$= \underline{\qquad}$

Example 3 Solving Problems Using Powers

Corin answered the following skill-testing question to win free movie tickets:
$120 + 20^3 \div 10^3 + 12 \times 120$
His answer was 1568.
Did Corin win the movie tickets? Show your work.



Solution

$120 + 20^3 \div 10^3 + 12 \times 120$ — Evaluate the powers first: 20^3 and 10^3
$= 120 + 8000 \div 1000 + 12 \times 120$ — Divide and multiply.
$= 120 + 8 + 1440$ — Add: $120 + 8 + 1440$
$= 1568$

Corin won the movie tickets.

Check

1. Answer the following skill-testing question to enter a draw for a Caribbean cruise.

$(6 + 4) + 3^2 \times 10 - 10^2 \div 4$
$= \underline{\qquad\qquad\qquad\qquad}$
$= \underline{\qquad\qquad\qquad\qquad}$
$= \underline{\qquad\qquad\qquad\qquad}$

Practice

1. Evaluate.

a) $2^2 + 1 = \_\_\_\_\_ + 1$
$= \_\_\_\_\_ + 1$
$= \_\_\_\_\_$

b) $2^2 - 1 = \_\_\_\_\_ - 1$
$= \_\_\_\_\_ - 1$
$= \_\_\_\_\_$

c) $(2 + 1)^2 = \_\_\_\_\_$
$= \_\_\_\_\_$
$= \_\_\_\_\_$

d) $(2 - 1)^2 = \_\_\_\_\_$
$= \_\_\_\_\_$
$= \_\_\_\_\_$

2. Evaluate.

a) $4 \times 2^2 = 4 \times \_\_\_\_\_$
$= 4 \times \_\_\_\_\_$
$= \_\_\_\_\_$

b) $4^2 \times 2 = \_\_\_\_\_ \times 2$
$= \_\_\_\_\_ \times 2$
$= \_\_\_\_\_$

c) $(4 \times 2)^2 = \_\_\_\_\_$
$= \_\_\_\_\_$
$= \_\_\_\_\_$

d) $(-4)^2 \div 2 = \_\_\_\_\_ \div 2$
$= \_\_\_\_\_ \div 2$
$= \_\_\_\_\_$

3. Evaluate.

a) $2^3 + (-1)^3 = \_\_\_\_\_ + (-1)^3$
$= \_\_\_\_\_ + (-1)^3$
$= \_\_\_\_\_ + \_\_\_\_\_\ \_\_\_\_\_\ \_\_\_\_\_$
$= \_\_\_\_\_ + \_\_\_\_\_$
$= \_\_\_\_\_$

b) $(2 - 1)^3 = \_\_\_\_\_$
$= \_\_\_\_\_$
$= \_\_\_\_\_$

c) $2^3 - (-1)^3 = \_\_\_\_\_ - (-1)^3$
$= \_\_\_\_\_ - (-1)^3$
$= \_\_\_\_\_ - \_\_\_\_\_\ \_\_\_\_\_\ \_\_\_\_\_$
$= \_\_\_\_\_ - \_\_\_\_\_$
$= \_\_\_\_\_$

d) $(2 + 1)^3 = \_\_\_\_\_$
$= \_\_\_\_\_$
$= \_\_\_\_\_$

4. Evaluate.

a) $3^2 \div (-1)^2 = \_\_\_\_\_ \div (-1)^2$
$= \_\_\_\_\_ \div (-1)^2$
$= \_\_\_\_\_ \div \_\_\_\_\_\ \_\_\_\_\_$
$= \_\_\_\_\_ \div \_\_\_\_\_$
$= \_\_\_\_\_$

b) $(3 \div 1)^2 = \_\_\_\_\_^2$
$= \_\_\_\_\_$
$= \_\_\_\_\_$

c) $3^2 \times (-2)^2 = \_\_\_\_\_ \times (-2)^2$
$= \_\_\_\_\_ \times (-2)^2$
$= \_\_\_\_\_ \times \_\_\_\_\_$
$= \_\_\_\_\_ \times \_\_\_\_\_$
$= \_\_\_\_\_$

d) $5^2 \div (-5)^1 = \_\_\_\_\_ \div (-5)^1$
$= \_\_\_\_\_ \div (-5)^1$
$= \_\_\_\_\_ \div \_\_\_\_\_$
$= \_\_\_\_\_$

5. Evaluate.

a) $(-2)^0 \times (-2) =$ _____ $\times (-2)$
$=$ _____

b) $2^3 \div (-2)^2 =$ ________ $\div (-2)^2$
$=$ _____ $\div (-2)^2$
$=$ _____ $\div$ _________
$=$ _____ $\div$ _____
$=$ _____

c) $(3 + 2)^0 + (3 \times 2)^0 =$ _____ $+$ _____
$=$ _____

d) $(3 \times 5^2)^0 =$ _____

A power with exponent 0 is equal to 1.

e) $(2)(3) - (4)^2 = (2)(3) -$ ______
$= (2)(3) -$ _____
$=$ _____ $-$ _____
$=$ _____

f) $3(2 - 1)^2 = 3$_____
$= 3$_____
$=$ _____

g) $(-2)^2 + (3)(4) =$ _________ $+ (3)(4)$
$=$ _____ $+ (3)(4)$
$=$ _____ $+$ _____
$=$ _____

h) $(-2) + 3^0 \times (-2) = (-2) +$ _____ $\times (-2)$
$= (-2) +$ _____
$=$ _____

6. Amaya wants to replace the hardwood floor in her house.
Here is how she calculates the cost, in dollars:
$70 \times 6^2 + 60 \times 6^2$
How much will it cost Amaya to replace the hardwood floor?

$70 \times$ ______ $+ 60 \times$ ______

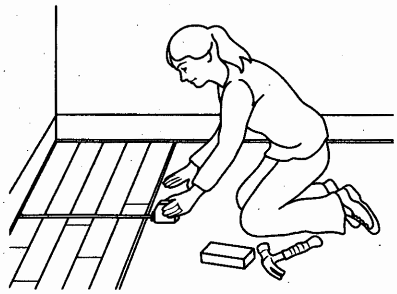
$= 70 \times$ _____ $+ 60 \times$ _____

$=$ ______ $+$ ______

$=$ ______

It will cost Amaya $______ to replace the hardwood floor.

Remember the order of operations: BEDMAS



CHECKPOINT

Can you ...

- Use powers to show repeated multiplication?
- Use patterns to evaluate a power with exponent zero, such as 50?
- Use the correct order of operations with powers?

2.1 **1.** Give the base and exponent of each power.

a) 6^2 Base: _____ Exponent: _____
There are _____ factors of _____.

b) 4^5 Base: _____ Exponent: _____
There are _____ factors of _____.

c) $(-3)^8$ Base: _____ Exponent: _____
There are _____ factors of _____.

d) -3^8 Base: _____ Exponent: _____
There are _____ factors of _____.

2. Write as a power.

a) $7 \times 7 \times 7 \times 7 \times 7 \times 7 = 7^{\_\_}$

b) $2 \times 2 \times 2 \times 2 = 2^{\_\_}$

c) $5 =$ _____

d) $(-5)(-5)(-5)(-5)(-5) =$ _____

3. Write each power as repeated multiplication and in standard form.

a) $5^2 = 5 \times$ _____ = _____

b) $2^3 =$ _______________ = _____

c) $3^4 =$ __________________ = _____

2.2 **4. a)** Complete the table.

Power	Repeated Multiplication	Standard Form
7^3	$7 \times 7 \times 7$	343
7^2	7×7	
7^1		

b) What is the value of 7^0? _____

5. Write each number in standard form and as a power of 10.

a) One hundred = 100
= 10___

b) Ten thousand = ____________
= 10___

c) One million = ____________
= 10___

d) One = __
= 10___

6. Evaluate.

a) $6^0 =$ _____

b) $(-8)^0 =$ _____

c) $12^1 =$ _____

d) $-8^0 =$ _____

7. Write each number in standard form.

a) 4×10^3
$= 4 \times$ ________
= ________

b) $(1 \times 10^3) + (3 \times 10^2) + (2 \times 10^1) + (1 \times 10^0)$
$= (1 \times 1000) + (3 \times$ _____) + (________) + (________)
= ________ + _____ + _____ + _____
= ________

c) $(4 \times 10^3) + (2 \times 10^2) + (3 \times 10^1) + (6 \times 10^0)$
$= (4 \times$ ________) + (________) + (________) + (________)
= ________ + _____ + _____ + _____
= ________

d) $(8 \times 10^2) + (1 \times 10^1) + (9 \times 10^0)$
= ________ + _______ + ______
= ______________
= ______________

2.3 **8.** Evaluate.

a) $3^2 + 5 = \_\_\_\_\_ + 5$
$= \_\_\_\_\_ + 5$
$= \_\_\_\_\_$

b) $5^2 - 2^3 = \_\_\_\_\_ - 2^3$
$= \_\_\_\_\_ - 2^3$
$= \_\_\_\_\_ - \_\_\_\_\_$
$= \_\_\_\_\_$
$= \_\_\_\_\_$

c) $(2 + 3)^3 = (\_)^3$
$= \_\_\_\_\_$
$= \_\_\_\_\_$

d) $2^3 + (-3)^3 = \_\_\_\_\_ + (-3)^3$
$= \_\_\_\_\_ + (-3)^3$
$= \_\_\_\_\_ + \_\_\_\_\_$
$= \_\_\_\_\_$
$= \_\_\_\_\_$

9. Evaluate.

a) $5 \times 3^2 = 5 \times \_\_\_\_\_$
$= \_\_\_\_\_$

b) $8^2 \div 4 = \_\_\_\_\_ \div 4$
$= \_\_\_\_\_$

c) $(10 + 2) \div 2^2 = \_\_\_\_\_ \div 2^2$
$= \_\_\_\_\_ \div \_\_\_\_\_$
$= \_\_\_\_\_$

d) $(7^2 + 1) \div (2^3 + 2)$
$= (\_\_\_\_\_ + 1) \div (\_\_\_\_\_ + 2)$
$= \_\_\_\_\_ \div \_\_\_\_\_$
$= \_\_\_\_\_$

10. Evaluate. State which operation you do first.

a) $3^2 + 4^2$ ____________________
$= \_\_\_\_\_ + \_\_\_\_\_$
$= \_\_\_\_\_ + \_\_\_\_\_$
$= \_\_\_\_\_$

b) $[(-3) - 2]^3$ ____________________
$= (\_\_\_\_\_)^3$
$= \_\_\_\_\_$
$= \_\_\_\_\_$

c) $(-2)^3 + (-3)^0$ ____________________
$= \_\_\_\_\_ + \_\_\_\_\_$
$= \_\_\_\_\_$
$= \_\_\_\_\_$

d) $[(6 - 3)^3 \times (2 + 2)^2]^0$ ____________________
$= \_\_\_\_\_$

2.4 Skill Builder

Simplifying Fractions

To simplify a fraction, divide the numerator and denominator by their common factors.

To simplify $\frac{5 \times 5 \times 5 \times 5}{5 \times 5}$:

This fraction shows repeated multiplication.

Divide the numerator and denominator by their common factors: 5×5.

$$\frac{\cancel{5}^{1} \times \cancel{5}^{1} \times 5 \times 5}{\cancel{5}^{1} \times \cancel{5}^{1}}$$

$$= \frac{5 \times 5}{1}$$

$$= 25$$

Check

1. Simplify each fraction.

What are the common factors?

a) $\frac{\cancel{3} \times 3 \times 3}{\cancel{3}}$

= ______

= ______

b) $\frac{\cancel{8} \times \cancel{8} \times \cancel{8} \times \cancel{8} \times \cancel{8}}{\cancel{8} \times \cancel{8} \times \cancel{8} \times \cancel{8} \times \cancel{8}}$

= ______

c) $\frac{\cancel{5} \times \cancel{5} \times \cancel{5} \times 5 \times 5}{\cancel{5} \times \cancel{5} \times \cancel{5}}$

= ______

= ______

d) $\frac{\cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times 2 \times 2 \times 2}{\cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2}}$

= ______

= ______

2.4 Exponent Laws I

FOCUS **Understand and apply the exponent laws for products and quotients of powers.**

Multiply $3^2 \times 3^4$.

$3^2 \times 3^4$ Write as repeated multiplication.

$= \underbrace{(3 \times 3)}_{\text{2 factors of 3}} \times \underbrace{(3 \times 3 \times 3 \times 3)}_{\text{4 factors of 3}}$

$= \underbrace{3 \times 3 \times 3 \times 3 \times 3 \times 3}_{\text{6 factors of 3}}$

$= 3^6$

Base Exponent

So, $3^2 \times 3^4 = 3^6$ Look at the pattern in the exponents.

2 + 4 = 6

We write: $3^2 \times 3^4 = 3^{(2 + 4)}$

$= 3^6$

This relationship is true when you multiply any 2 powers with the same base.

Exponent Law for a Product of Powers

To multiply powers with the same base, add the exponents.

Example 1 Simplifying Products with the Same Base

Write as a power.

a) $5^3 \times 5^4$ **b)** $(-6)^2 \times (-6)^3$ **c)** $(7^2)(7)$

Solution

a) The powers have the same base: 5

Use the exponent law for products: add the exponents.

$5^3 \times 5^4 = 5^{(3 + 4)}$

$= 5^7$

To check your work, you can write the powers as repeated multiplication.

b) The powers have the same base: -6

$(-6)^2 \times (-6)^3 = (-6)^{(2+3)}$ Add the exponents.

$= (-6)^5$

c) $(7^2)(7) = 7^2 \times 7^1$ Use the exponent law for products.

$= 7^{(2+1)}$ Add the exponents.

$= 7^3$

7 can be written as 7^1.

Check

1. Write as a power.

a) $2^5 \times 2^4 = 2^{(\_\_ + \_\_)}$

$= 2^{\_\_}$

b) $5^2 \times 5^5 = 5^{\_\_\_}$

$= 5^{\_\_}$

c) $(-3)^2 \times (-3)^3 =$ __________

$=$ __________

d) $10^5 \times 10 =$ __________

$=$ __________

Divide $3^4 \div 3^2$.

We can show division in fraction form.

$3^4 \div 3^2 = \frac{3^4}{3^2}$

$= \frac{3 \times 3 \times 3 \times 3}{3 \times 3}$ Simplify.

$= \frac{\cancel{3}^1 \times \cancel{3}^1 \times 3 \times 3}{\cancel{3}_1 \times \cancel{3}_1}$

$= \frac{3 \times 3}{1}$

$= 3 \times 3$

$= 3^2$

So, $3^{\mathbf{4}} \div 3^{\mathbf{2}} = 3^{\mathbf{2}}$ Look at the pattern in the exponents.

$\mathbf{4 - 2 = 2}$

We write: $3^{\mathbf{4}} \div 3^{\mathbf{2}} = 3^{(\mathbf{4-2})}$

$= 3^{\mathbf{2}}$

This relationship is true when you divide any 2 powers with the same base.

Exponent Law for a Quotient of Powers
To divide powers with the same base, subtract the exponents.

Example 2 Simplifying Quotients with the Same Base

Write as a power.

a) $4^5 \div 4^3$

b) $(-2)^7 \div (-2)^2$

Solution

Use the exponent law for quotients: subtract the exponents.

a) $4^5 \div 4^3 = 4^{(5-3)}$
$= 4^2$

The powers have the same base: 4

To check your work, you can write the powers as repeated multiplication.

b) $(-2)^7 \div (-2)^2 = (-2)^{(7-2)}$
$= (-2)^5$

The powers have the same base: -2

Check

1. Write as a power.

a) $(-5)^6 \div (-5)^3 = (-5)^{\_\_\_\_}$
$= \_\_\_\_$

b) $\frac{(-3)^9}{(-3)^5} = (-3)^{\_\_\_\_}$
$= \_\_\_\_$

$\frac{(-3)^9}{(-3)^5}$ is the same as $(-3)^9 \div (-3)^5$

c) $8^4 \div 8^3 = \_\_\_\_$
$= \_\_\_\_$

d) $9^8 \div 9^2 = \_\_\_\_$
$= \_\_\_\_$

Example 3 Evaluating Expressions Using Exponent Laws

Evaluate.

a) $2^2 \times 2^3 \div 2^4$

b) $(-2)^5 \div (-2)^3 \times (-2)$

Solution

a) $2^2 \times 2^3 \div 2^4$
$= 2^{(2+3)} \div 2^4$
$= 2^5 \div 2^4$
$= 2^{(5-4)}$
$= 2^1$
$= 2$

Add the exponents of the 2 powers that are multiplied. Then, subtract the exponent of the power that is divided.

b) $(-2)^5 \div (-2)^3 \times (-2)$
$= (-2)^{(5-3)} \times (-2)$
$= (-2)^2 \times (-2)$
$= (-2)^{(2+1)}$
$= (-2)^{(3)}$
$= (-2)(-2)(-2)$
$= -8$

Subtract the exponents of the 2 powers that are divided.

Multiply: add the exponents.

Check

1. Evaluate.

a) $4 \times 4^3 \div 4^2 = 4^{(\_\_+\_\_)} \div 4^2$
$= 4^{\_\_} \div 4^2$
$= 4^{(\_\_-\_\_)}$
$= 4^{\_\_}$
$= \_\_\_$

b) $(-3) \div (-3) \times (-3)$
$= (-3)^{\_\_\_} \times (-3)$
$= (-3)^{\_\_} \times (-3)$
$= (-3)^{\_\_\_}$
$= (-3)^{\_\_}$
$= \_\_\_$

$(-3) = (-3)^1$

Practice

1. Write each product as a single power.

a) $7^6 \times 7^2 = 7^{(\underline{\quad} + \underline{\quad})}$
$= 7^{\underline{\quad}}$

b) $(-4)^5 \times (-4)^3 = (-4)^{\underline{\quad\quad}}$
$= (-4)^{\underline{\quad}}$

To multiply powers with the same base, add the exponents.

c) $(-2) \times (-2)^3 = \underline{\quad\quad\quad\quad}$
$= \underline{\quad\quad\quad\quad}$

d) $10^5 \times 10^5 = \underline{\quad\quad\quad\quad}$
$= \underline{\quad\quad\quad\quad}$

e) $7^0 \times 7^1 = \underline{\quad\quad\quad\quad}$
$= \underline{\quad\quad\quad\quad}$

f) $(-3)^4 \times (-3)^5 = \underline{\quad\quad\quad\quad}$
$= \underline{\quad\quad\quad\quad}$

2. Write each quotient as a power.

a) $(-3)^5 \div (-3)^2 = (-3)^{(\underline{\quad} - \underline{\quad})}$
$= (-3)^{\underline{\quad}}$

b) $5^6 \div 5^4 = 5^{\underline{\quad\quad}}$
$= 5^{\underline{\quad}}$

To divide powers with the same base, subtract the exponents.

c) $\frac{4^7}{4^4} = 4^{\underline{\quad\quad}}$
$= 4^{\underline{\quad}}$

d) $\frac{5^8}{5^6} = \underline{\quad\quad\quad}$
$= \underline{\quad\quad}$

e) $6^4 \div 6^4 = \underline{\quad\quad}$
$= \underline{\quad\quad}$

f) $\frac{(-6)^8}{(-6)^7} = \underline{\quad\quad\quad\quad}$
$= \underline{\quad\quad\quad\quad}$

3. Write as a single power.

a) $2^3 \times 2^4 \times 2^5 = 2^{(\underline{\quad} + \underline{\quad})} \times 2^5$
$= 2^{\underline{\quad}} \times 2^5$
$= 2^{\underline{\quad\quad}}$
$= 2^{\underline{\quad\quad}}$

b) $\frac{3^2 \times 3^2}{3^2 \times 3^2} = \frac{3^{\underline{\quad\quad}}}{3^{\underline{\quad\quad}}}$
$= \frac{3^{\underline{\quad}}}{3^{\underline{\quad}}}$
$= \underline{\quad\quad\quad\quad}$
$= \underline{\quad\quad\quad\quad}$

Which exponent law should you use?

c) $10^3 \times 10^5 \div 10^2 = \underline{\quad\quad\quad} \div 10^2$
$= \underline{\quad\quad} \div 10^2$
$= \underline{\quad\quad\quad}$
$= \underline{\quad\quad\quad}$

d) $(-1)^9 \div (-1)^5 \times (-1)^0$
$= \underline{\quad\quad\quad\quad} \times (-1)^0$
$= \underline{\quad\quad\quad} \times (-1)^0$
$= \underline{\quad\quad\quad\quad}$
$= \underline{\quad\quad\quad\quad}$

4. Simplify, then evaluate.

a) $(-3)^1 \times (-3)^2 \times 2$

$= \underline{\qquad\qquad} \times 2$

$= \underline{\qquad} \times 2$

$= \underline{\qquad} \times 2$

$= \underline{\qquad}$

b) $9^9 \div 9^7 \times 9^0 = \underline{\qquad\qquad} \times 9^0$

$= \underline{\qquad} \times 9^0$

$= \underline{\qquad\quad}$

$= \underline{\qquad}$

$= \underline{\qquad}$

See if you can use the exponent laws to simplify.

c) $\frac{5^2}{5^0} = \underline{\qquad\qquad}$

$= \underline{\qquad\qquad}$

$= \underline{\qquad\qquad}$

d) $\frac{5^5}{5^4} \times 5 = 5^{\underline{\qquad}} \times 5$

$= 5^{\underline{\quad}} \times 5$

$= 5^{\underline{\qquad}}$

$= 5^{\underline{\quad}}$

$= \underline{\qquad}$

5. Identify any errors and correct them.

a) $4^3 \times 4^5 = 4^8$

b) $2^5 \times 2^5 = 2^{25}$

c) $(-3)^6 \div (-3)^2 = (-3)^3$

d) $7^0 \times 7^2 = 7^0$

e) $6^2 + 6^2 = 6^4$

f) $10^6 \div 10 = 10^6$

g) $2^3 \times 5^2 = 10^5$

2.5 Skill Builder

Grouping Equal Factors

In multiplication, you can group equal factors.

Order does not matter in multiplication.

For example:

$3 \times 7 \times 7 \times 3 \times 7 \times 7 \times 3$ Group equal factors.

$= \underbrace{3 \times 3 \times 3}_{} \times \underbrace{7 \times 7 \times 7 \times 7}_{}$ Write repeated multiplication as powers.

$= \quad 3^3 \quad \times \quad 7^4$

Check

1. Group equal factors and write as powers.

a) $2 \times 10 \times 2 \times 10 \times 2 = 2 \times 2 \times 2 \times$ ____________

$=$ ____________

b) $2 \times 5 \times 2 \times 5 \times 2 \times 5 \times 2 \times 5 =$ ____________

$=$ ____________

Multiplying Fractions

To multiply fractions, first multiply the numerators, and then multiply the denominators.

$\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{2 \times 2 \times 2 \times 2}{3 \times 3 \times 3 \times 3}$ Write repeated multiplication as powers.

$= \frac{2^4}{3^4}$

There are 4 factors of 2, and 4 factors of 3.

Check

1. Multiply the fractions. Write as powers.

a) $\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} =$ ____________

$=$ ____________

b) $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$=$ ____________

$=$ ______

2.5 Exponent Laws II

FOCUS **Understand and apply exponent laws for powers of: products; quotients; and powers.**

Multiply $3^2 \times 3^2 \times 3^2$.
$3^2 \times 3^2 \times 3^2 = 3^{2+2+2}$
$= 3^6$

Use the exponent law for the product of powers.
Add the exponents.

We can write repeated multiplication as powers.

So, $\underbrace{3^2 \times 3^2 \times 3^2}_{\text{3 factors of } (3^2)}$

The base is 3^2. ← *This is also a power.*
The exponent is 3.
This is a **power of a power.**
Look at the pattern in the exponents.

$= (3^2)^3$
$= 3^6$
$\mathbf{2 \times 3 = 6}$
We write: $(3^2)^3 = 3^{2 \times 3}$
$= 3^6$

Exponent Law for a Power of a Power
To raise a power to a power, multiply the exponents.
For example: $(2^3)^5 = 2^{3 \times 5}$

Example 1 Simplifying a Power of a Power

Write as a power.

a) $(3^2)^4$ **b)** $[(-5)^3]^2$ **c)** $-(2^3)^4$

Solution

Use the exponent law for a power of a power: multiply the exponents.

a) $(3^2)^4 = 3^{2 \times 4}$
$= 3^8$

b) $[(-5)^3]^2 = (-5)^{3 \times 2}$
$= (-5)^6$
The base is -5.

c) $-(2^3)^4 = -(2^{3 \times 4})$
$= -2^{12}$
The base is 2.

Check

1. Write as a power.

a) $(9^3)^4 = 9^{\_\_ \times \_\_}$
$= 9^{\_\_}$

b) $[(-2)^5]^3 = (-2)^{\_\_\_}$
$= (-2)^{\_\_}$

c) $-(5^4)^2 = -(5^{\_\_\_})$
$= -5^{\_\_}$

Multiply $(3 \times 4)^2$.

The base of the power is a product: $\underbrace{3 \times 4}_{\textbf{base}}$

Write as repeated multiplication.

$(3 \times 4)^2 = (3 \times 4) \times (3 \times 4)$ Remove the brackets.

$= 3 \times 4 \times 3 \times 4$ Group equal factors.

$= \underbrace{(3 \times 3)}_{\text{2 factors of 3}} \times \underbrace{(4 \times 4)}_{\text{2 factors of 4}}$ Write as powers.

$= 3^2 \times 4^2$

So, $(3 \times 4)^2 = 3^2 \times 4^2$

power product power

> **Exponent Law for a Power of a Product**
> The power of a product is the product of powers.
> For example: $(2 \times 3)^4 = 2^4 \times 3^4$

Example 2 Evaluating Powers of Products

Evaluate.

a) $(2 \times 5)^2$

b) $[(-3) \times 4]^2$

Solution

Use the exponent law for a power of a product.

a) $(2 \times 5)^2 = 2^2 \times 5^2$
$= (2)(2) \times (5)(5)$
$= 4 \times 25$
$= 100$

b) $[(-3) \times 4]^2 = (-3)^2 \times 4^2$
$= (-3)(-3) \times (4)(4)$
$= 9 \times 16$
$= 144$

Or, use the order of operations and evaluate what is inside the brackets first.

a) $(2 \times 5)^2 = 10^2$
$= 100$

b) $[(-3) \times 4]^2 = (-12)^2$
$= 144$

Check

1. Write as a product of powers.

a) $(5 \times 7)^4 = \_\_\_\_ \times \_\_\_\_$

b) $(8 \times 2)^2 = \_\_\_\_ \times \_\_\_\_$

2. Evaluate.

a) $[(-1) \times 6]^2 = \_\_\_\_^2$
$= \_\_\_\_$

b) $[(-1) \times (-4)]^3 = \_\_\_\_^3$
$= \_\_\_\_$

Evaluate $\underbrace{\left(\frac{3}{4}\right)}_{\textbf{base}}^2$.

The base of the power is a quotient: $\frac{3}{4}$

Write as repeated multiplication.

$$\left(\frac{3}{4}\right)^2 = \left(\frac{3}{4}\right) \times \left(\frac{3}{4}\right)$$

$$= \frac{3}{4} \times \frac{3}{4}$$

Multiply the fractions.

$$= \frac{3 \times 3}{4 \times 4}$$

Write repeated multiplication as powers.

$$= \frac{3^2}{4^2}$$

So, $\left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2}$

power

quotient

power

Exponent Law for a Power of a Quotient

The power of a quotient is the quotient of powers.

For example: $\left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4}$

Example 3 Evaluating Powers of Quotients

Evaluate.

a) $[30 \div (-5)]^2$

b) $\left(\frac{20}{4}\right)^2$

Solution

Use the exponent law for a power of a quotient.

a) $[30 \div (-5)]^2 = \left(\frac{30}{-5}\right)^2$

$= \frac{30^2}{(-5)^2}$

$= \frac{900}{25}$

$= 36$

b) $\left(\frac{20}{4}\right)^2 = \frac{20^2}{4^2}$

$= \frac{400}{16}$

$= 25$

Or, use the order of operations and evaluate what is inside the brackets first.

a) $[30 \div (-5)]^2 = (-6)^2$

$= 36$

b) $\left(\frac{20}{4}\right)^2 = 5^2$

$= 25$

Check

1. Write as a quotient of powers.

a) $\left(\frac{3}{4}\right)^5 =$ ______

b) $[1 \div (-10)]^3 =$ ______

2. Evaluate.

a) $[(-16) \div (-4)]^2$

$=$ ____^{2} $=$ ____

b) $\left(\frac{36}{6}\right)^3 =$ ____

$=$ ____

You can evaluate what is inside the brackets first.

Practice

1. Write as a product of powers.

a) $(5 \times 2)^4 = 5$—— $\times\ 2$——

b) $(12 \times 13)^2 =$ __________

c) $[3 \times (-2)]^3 =$ __________

d) $[(-4) \times (-5)]^5 =$ __________

2. Write as a quotient of powers.

a) $(5 \div 8)^0 =$ ______

b) $[(-6) \div 5]^7 =$ ______

c) $\left(\frac{3}{5}\right)^2 =$ ______

d) $\left(\frac{-1}{-2}\right)^3 =$ ______

3. Write as a power.

a) $(5^2)^3 = 5^{\_\_ \times \_\_}$
$= 5^{\_\_}$

b) $[(-2)^3]^5 = (-2)^{\_\_}$
$= \_\_\_\_$

c) $(4^4)^1 = \_\_\_\_$
$= \_\_\_\_$

d) $(8^0)^3 = \_\_\_\_$
$= \_\_\_\_$

4. Evaluate.

a) $[(6 \times (-2)]^2 = \_\_\_\_$
$= \_\_\_\_$

b) $-(3 \times 4)^2 = -(\_\_\_)^{\_\_}$
$= \_\_\_\_$

c) $\left(\frac{-8}{-2}\right)^2 = \_\_\_\_$
$= \_\_\_\_$

d) $(10 \times 3)^1 = \_\_\_\_$
$= \_\_\_\_$

e) $[(-2)^1]^2 = \_\_\_\_$
$= \_\_\_\_$
$= \_\_\_\_$

f) $[(-2)^1]^3 = \_\_\_\_$
$= \_\_\_\_$
$= \_\_\_\_$

5. Find any errors and correct them.

a) $(3^2)^3 = 3^5$ ________

b) $(3 + 2)^2 = 3^2 + 2^2$ ________

c) $(5^3)^3 = 5^9$ ________

d) $\left(\frac{2}{3}\right)^8 = \frac{2^8}{3^8}$ ________

e) $(3 \times 2)^2 = 36$ ________

f) $\left(\frac{2}{3}\right)^2 = \frac{4}{6}$ ________

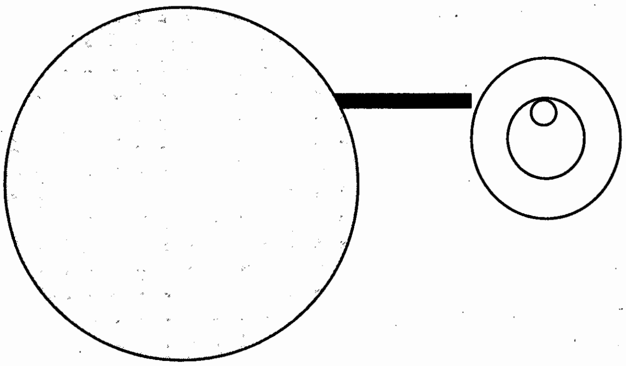
g) $[(-3)^3]^0 = (-3)^3$ ________

h) $[(-2) \times (-3)]^4 = -6^4$ ________

Unit 2 Puzzle

Bird's Eye View

This is a view through the eyes of a bird. What does the bird see?



To find out, simplify or evaluate each expression on the left, then find the answer on the right.
Write the corresponding letter beside the question number.
The numbers at the bottom of the page are question numbers.
Write the corresponding letter over each number.

1. $5 \times 5 \times 5 \times 5$ ____________

2. 2^3 ____________

3. $\frac{3^6}{3^2}$ ____________

4. $4 \times 4 \times 4 \times 4 \times 4$ ____________

5. $(-2)^3$ ____________

6. $(-2) + 4 \div 2$ ____________

7. $(5^2)^3$ ____________

8. $3^2 - 2^3$ ____________

9. $10^2 \times 10^3$ ____________

10. $5 + 3^0$ ____________

11. $4^7 \div 4$ ____________

A	100 000
P	5^6
S	0
E	1
F	3^4
G	6
I	8
O	4^6
N	4^5
R	5^4
Y	-8

9   7 8 1 6 11 4   3 1 5 2 4 10   9 4   8 10 10

Unit 2 Study Guide

Skill	Description	Example
Evaluate a power with an integer base.	Write the power as repeated multiplication, then evaluate.	$(-2)^3 = (-2) \times (-2) \times (-2)$ $= -8$
Evaluate a power with an exponent 0.	A power with an integer base and an exponent 0 is equal to 1.	$8^0 = 1$
Use the order of operations to evaluate expressions containing exponents.	Evaluate what is inside the brackets. Evaluate powers. Multiply and divide, in order, from left to right. Add and subtract, in order, from left to right.	$(3^2 + 2) \times (-5)$ $= (9 + 2) \times (-5)$ $= (11) \times (-5)$ $= -55$
Apply the exponent law for a product of powers.	To multiply powers with the same base, add the exponents.	$4^3 \times 4^6 = 4^{3+6}$ $= 4^9$
Apply the exponent law for a quotient of powers.	To divide powers with the same base, subtract the exponents.	$2^7 \div 2^4 = \frac{2^7}{2^4}$ $= 2^{7-4}$ $= 2^3$
Apply the exponent law for a power of a power.	To raise a power to a power, multiply the exponents.	$(5^3)^2 = 5^{3 \times 2}$ $= 5^6$
Apply the exponent law for a power of a product.	Write the power of a product as a product of powers.	$(6 \times 3)^5 = 6^5 \times 3^5$
Apply the exponent law for a power of a quotient.	Write the power of a quotient as a quotient of powers.	$\left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2}$

Unit 2 Review

2.1 **1.** Give the base and exponent of each power.

a) 6^2 Base _____ Exponent _____

b) $(-3)^8$ Base _____ Exponent _____

2. Write as a power.

a) $4 \times 4 \times 4 = 4$___

b) $(-3)(-3)(-3)(-3)(-3) =$ _______

3. Write each power as repeated multiplication and in standard form.

a) $(-2)^5 =$ ______________________
$=$ ______________________

b) $10^4 =$ ______________________
$=$ ______________________

c) Six squared $=$ ____________
$=$ ____________
$=$ ____________

d) Five cubed $=$ ____________
$=$ ____________
$=$ ____________

2.2 **4.** Evaluate.

a) $10^0 =$ _____

b) $(-4)^0 =$ _____

c) $8^1 =$ _____

d) $-4^0 =$ _____

5. Write each number in standard form.

a) 9×10^3
$= 9 \times$ _________ $\times$ _________ $\times$ _________
$= 9 \times$ _________
$=$ ____________

b) $(1 \times 10^2) + (3 \times 10^1) + (5 \times 10^0)$

$= (1 \times \_\_\_\_) + (3 \times \_\_\_\_) + (5 \times \_\_\_\_)$

$= \_\_\_\_\_\_\_\_\_\_\_\_$

$= \_\_\_\_\_\_\_\_\_\_\_\_$

c) $(2 \times 10^3) + (4 \times 10^2) + (1 \times 10^1) + (9 \times 10^0)$

$= (2 \times \_\_\_\_) + (4 \times \_\_\_\_) + (1 \times \_\_\_\_) + (9 \times \_\_\_\_)$

$= \_\_\_\_\_\_\_\_\_\_\_\_$

$= \_\_\_\_\_\_\_\_\_\_\_\_$

d) $(5 \times 10^4) + (3 \times 10^2) + (7 \times 10^1) + (2 \times 10^0)$

$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

2.3 **6.** Evaluate.

a) $3^2 + 3$

$= \_\_\_\_ + 3$

$= \_\_\_\_ + 3$

$= \_\_\_\_$

b) $[(-2) + 4)]^3$

$= \_\_\_\_^3$

$= \_\_\_\_\_\_\_\_$

$= \_\_\_\_$

c) $(20 + 5) \div 5^2 = \_\_\_\_ \div 5^2$

$= \_\_\_\_ \div \_\_\_\_$

$= \_\_\_\_$

d) $(8^2 - 4) \div (6^2 - 6)$

$= (\_\_\_\_ - 4) \div (\_\_\_\_ - 6)$

$= \_\_\_\_ \div \_\_\_\_$

$= \_\_\_\_$

7. Evaluate.

a) $5 \times 3^2 = 5 \times \_\_\_\_$

$= \_\_\_\_$

b) $10 \times (3^2 + 5^0) = 10 \times \_\_\_\_\_\_$

$= 10 \times \_\_\_\_$

$= \_\_\_\_$

c) $(-2)^3 + (-3)(4) = \_\_\_\_ + \_\_\_\_$

$= \_\_\_\_$

d) $(-3) + 4^0 \times (-3) = (-3) + \_\_\_\_ \times (-3)$

$= (-3) + \_\_\_\_$

$= \_\_\_\_$

2.4 **8.** Write as a power.

a) $6^3 \times 6^7 = 6^{(\_\_ + \_\_)}$
$= 6^{\_\_}$

b) $(-4)^2 \times (-4)^3 = (-4)^{\_\_\_}$
$= (-4)^{\_\_}$

c) $(-2)^5 \times (-2)^4 = (-2)^{\_\_\_}$
$= (-2)^{\_\_}$

d) $10^7 \times 10 =$ ________
$=$ ________

9. Write as a power.

a) $5^7 \div 5^3 = 5^{(\_\_ - \_\_)}$
$= 5^{\_\_}$

b) $\frac{10^5}{10^3} =$ ________
$=$ ________

c) $(-6)^8 \div (-6)^2 =$ ________
$=$ ________

d) $\frac{5^{10}}{5^6} =$ ________
$=$ ________

e) $8^3 \div 8 =$ ________
$=$ ________

f) $\frac{(-3)^4}{(-3)^0} =$ ________
$=$ ________

2.5 **10.** Write as a power.

a) $(5^3)^4 = 5^{\_\_ \times \_\_}$
$= 5^{\_\_}$

b) $[(-3)^2]^6 = (-3)^{\_\_ \times \_\_}$
$= (-3)^{\_\_}$

c) $(8^2)^4 =$ ________
$=$ ________

d) $[(-5)^5]^4 =$ ________
$=$ ________

11. Write as a product or quotient of powers.

a) $(3 \times 5)^2 = 3^{\_\_} \times 5^{\_\_}$

b) $(2 \times 10)^5 =$ ________

c) $[(-4) \times (-5)]^3 =$ ________

d) $\left(\frac{4}{3}\right)^5 = \frac{}{\_\_\_}$

e) $(12 \div 10)^4 = 12^{\_\_} \div 10^{\_\_}$

f) $[(-7) \div (-9)]^6 =$ ________